Neutron star core-quakes caused by a transition to the mixed-phase EOS

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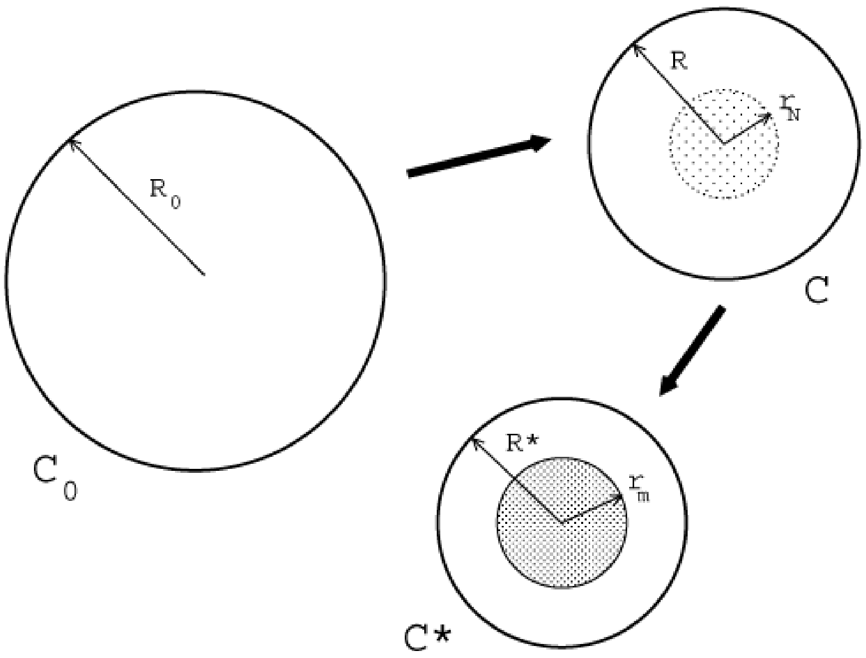
- mixed-phase
- linear response theory
- stellar models



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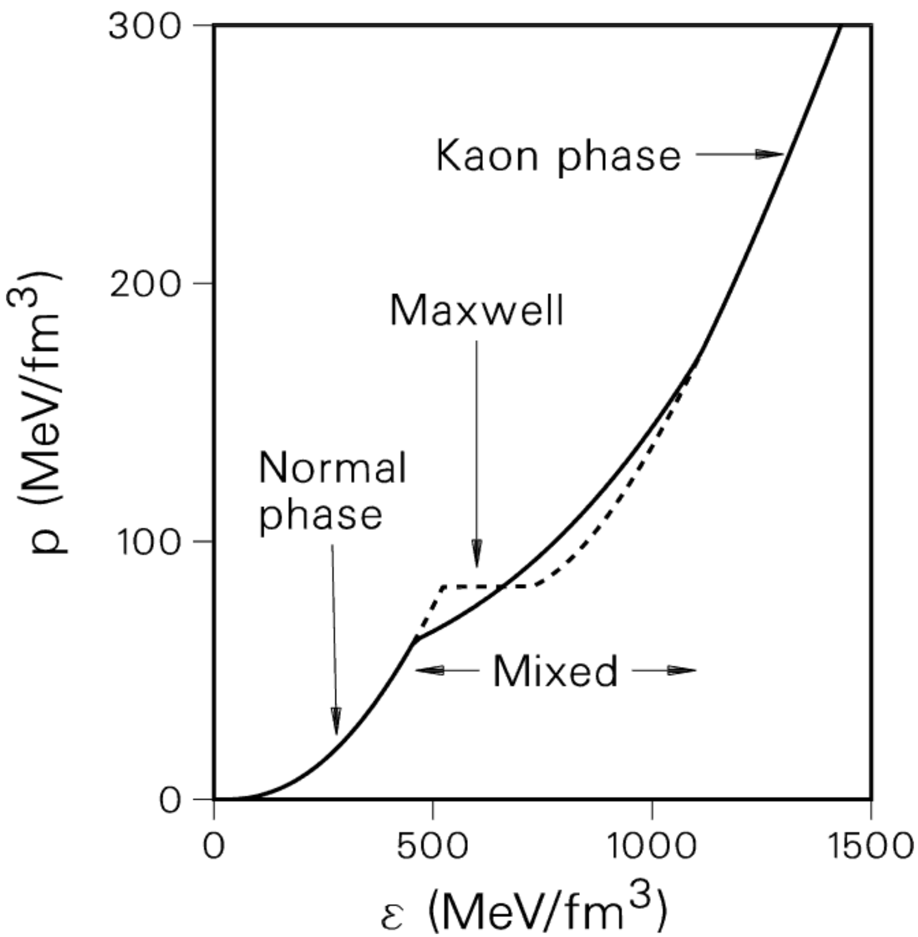
Mixed-phase

- First-order phase transition – mixed-phase appears via nucleation
- 2 phases in equilibrium for a set of pressures
- Preferred, if more than one conserved charge (i.e. baryon and electric), and if Coulomb and surface energy terms are small
- Global (not local) electric charge neutrality



(Glendenning 1992,
Glendenning & Schaffner-Bielich, 1998)

Mixed-phase

The thermodynamic equilibrium of a mixture of phases N and S, at a fixed average baryon density $n_{\rm b}$, will be calculated by minimizing the average energy density

$$\mathcal{E} = (1-\chi)\mathcal{E}^{\rm N} + \chi\mathcal{E}^{\rm S} \ .$$

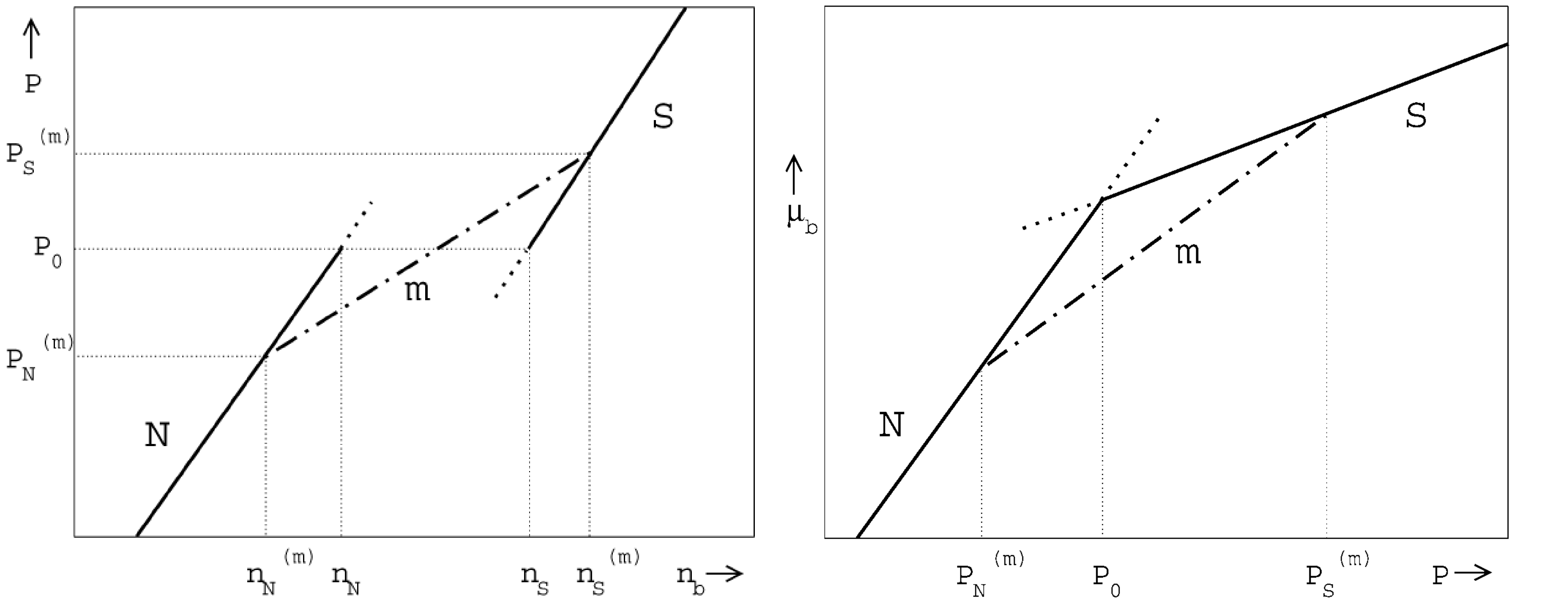
under the condition

$$n_{\rm b} = (1-\chi)n_{\rm b}^{\rm N} + \chi n_{\rm b}^{\rm S} \ ,$$

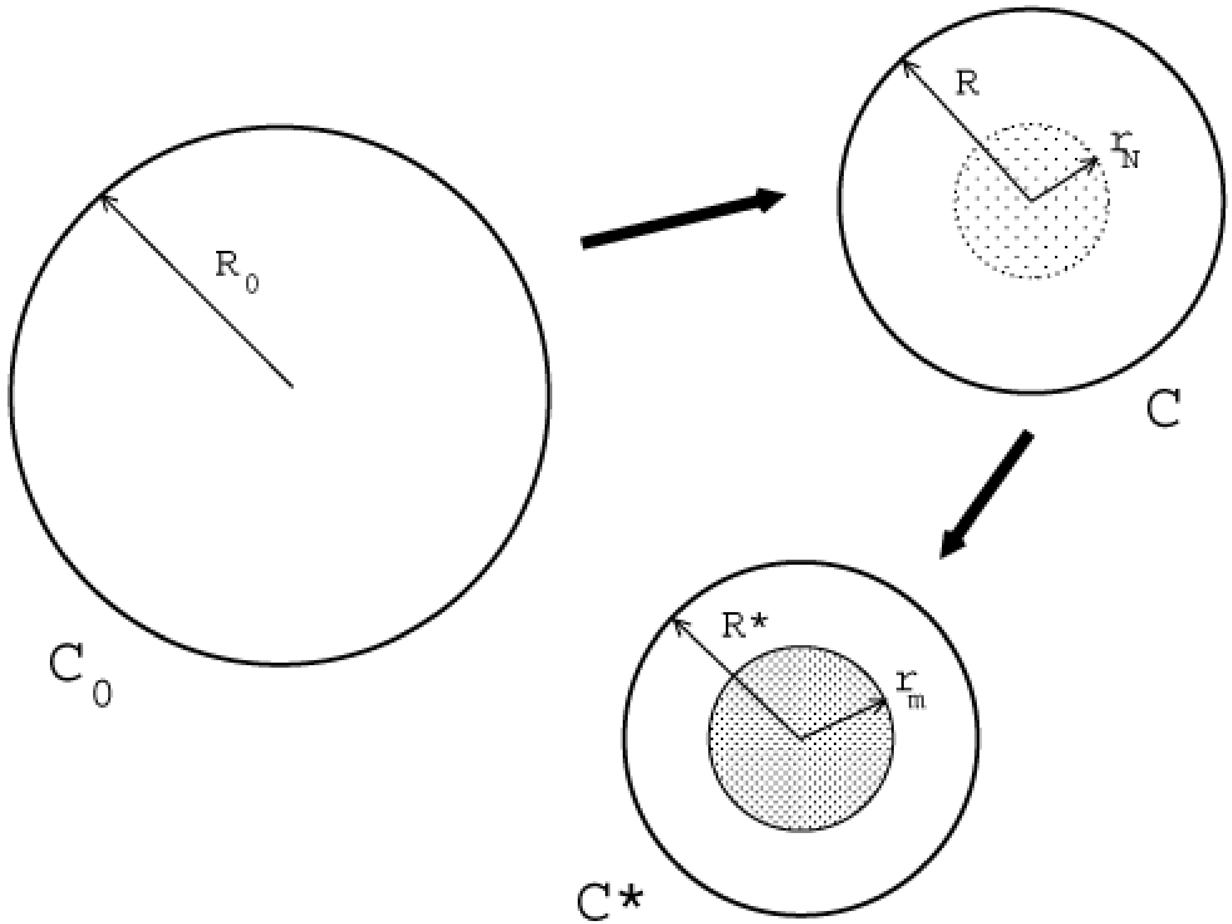
and under the constraint of average (macroscopic) electrical neutrality

$$\rho_{\rm el.} = (1-\chi)\rho_{\rm el.}^{\rm N} + \chi\rho_{\rm el.}^{\rm S} = 0.$$

Mixed-phase



Influence of the phase transition on the star structure



Linear response theory

TOV equation:

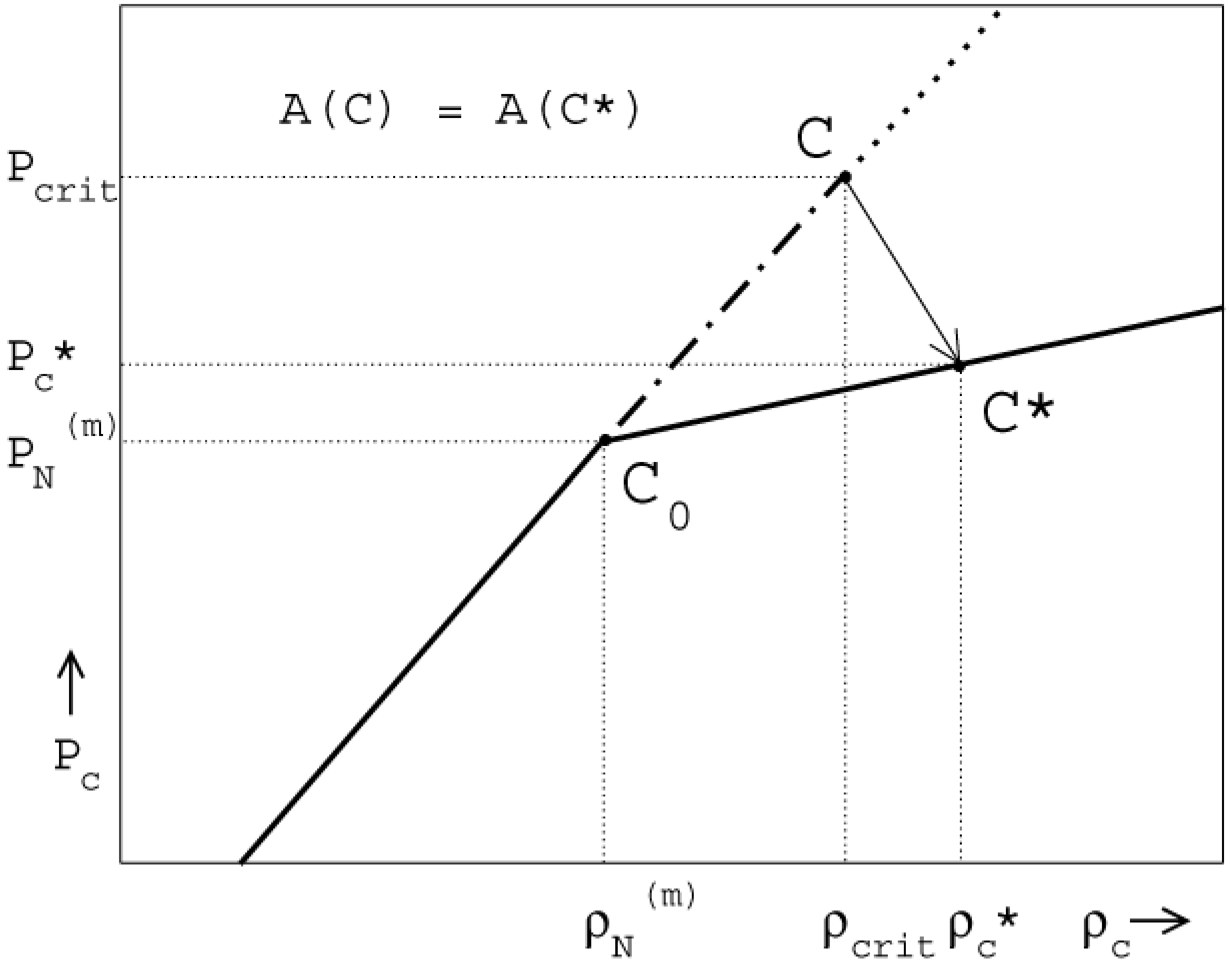
$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2}\left(1+\frac{p(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3 p(r)}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2\rho(r)$$

Linearization of the equation to compare the density profiles of stars with and without the dense core.

(Haensel et al. 1986, Zdunik et al. 1987)

From meta- to stable matter



Transition from **meta-stable** core configuration **C** to **stable** mixed-phase core configuration **C***

Global number of baryons is conserved, but other global parameters (M, R, I) are not.

Linear response theory – density jump

Core mass excess (density jump) $\delta m_{\rm core} = \frac{4}{3}\pi(\rho_{\rm S} - \rho_{\rm N}) r_{\rm S}^3 + \mathcal{O}(r_{\rm S}^5)$

Stellar response on the appearence of the new phase

$$\delta \bar{R} = \bar{R}^* - \bar{R} = -(\lambda - 1)\alpha_R \bar{r}_{\rm S}^3 + {\rm O}(\bar{r}_{\rm S}^5)$$

$$\delta \bar{I} = \bar{I}^* - \bar{I} = -(\lambda - 1)\alpha_I \bar{r}_{\rm S}^3 + {\rm O}(\bar{r}_{\rm S}^5)$$

$$\lambda = \rho_{\rm S}/\rho_{\rm N}$$

$$\delta \bar{M} = -(\lambda - 1)\alpha_M \bar{r}_{\rm S}^5 [3 - 2\lambda + 3x_{\rm N} - \tfrac{5}{4}(\lambda - 1) a_{\rm N} \bar{r}_{\rm S}]$$

(r_s – radius of the newly-born core)

Linear response theory – mixed-phase

Core mass excess (transition to a mixed-phase)

$$\delta m_{\rm core} = \frac{4}{45}\pi\rho_{\rm m}(1+x_{\rm m})(1+3x_{\rm m})(\kappa_{\rm m}^2-\kappa_{\rm N}^2)r_{\rm m}^5+\mathcal{O}(r_{\rm m}^7)$$

$$\kappa_{\rm N}^2 = 4\pi G\rho_{\rm N}/v_{\rm N}^2, \quad \kappa_{\rm m}^2 = 4\pi G\rho_{\rm m}/v_{\rm m}^2 \quad x_{\rm m} = P_{\rm m}/\rho_{\rm m}c^2$$

Stellar response on the appearence of the new phase

$$\delta\bar{Q} \equiv \frac{Q^*-Q}{Q_0} \simeq -(\gamma_{\rm N}/\gamma_{\rm m}-1)\cdot\beta_Q\cdot(\bar{r}_{\rm m})^l$$

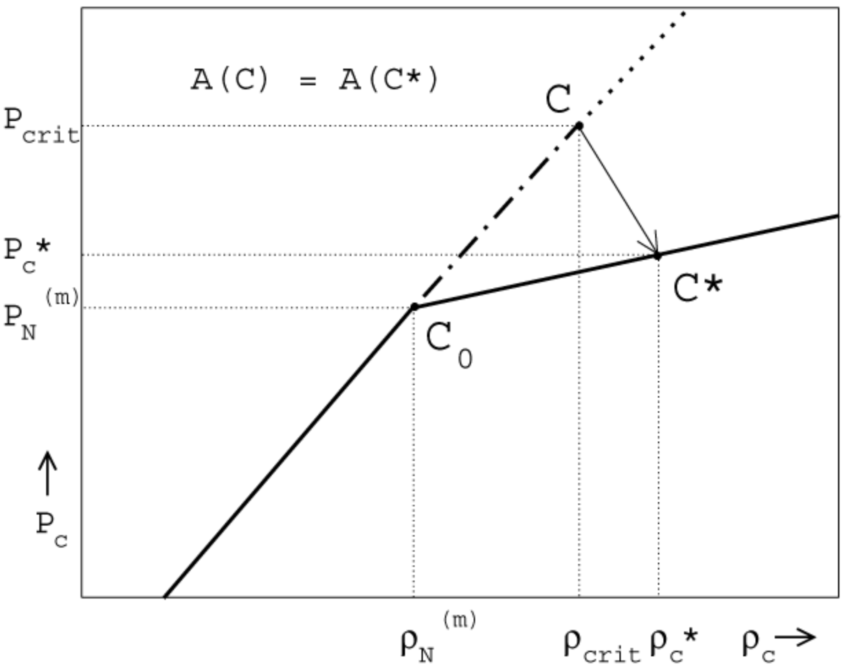
Q = radius R, moment of inertia I, mass-energy M

exponent l = 5 (R, I), 7 (M)

Linear response theory

Size of new core and critical density of the meta-stable core

$$\frac{\rho_{\rm crit} - \rho_{\rm N}^{(\rm m)}}{\rho_{\rm N}^{(\rm m)}} = \frac{1}{6}\kappa_{\rm N}^2 (1 + x_{\rm m})(1 + 3x_{\rm m}) r_{\rm m}^2$$



Transition to the **mixed-phase** and **density-jump** similarity:

$$\delta\bar{Q} \equiv \frac{Q^* - Q}{Q_0} \simeq -(\gamma_{\rm N}/\gamma_{\rm m} - 1) \cdot \beta_Q \cdot (\bar{r}_{\rm m})^l$$

$$\delta\bar{Q} \equiv \frac{Q^* - Q}{Q_0} \simeq -(\lambda - 1) \cdot \alpha_Q \cdot (\bar{r}_{\rm S})^l \qquad \boldsymbol{\lambda = \rho_S/\rho_N}$$

Response coefficients – polytropes

Relativistic polytrope EOS:

$$P(n_\mathrm{b}) = K_\mathrm{N} \cdot (n_\mathrm{b})^{\gamma_\mathrm{N}}$$

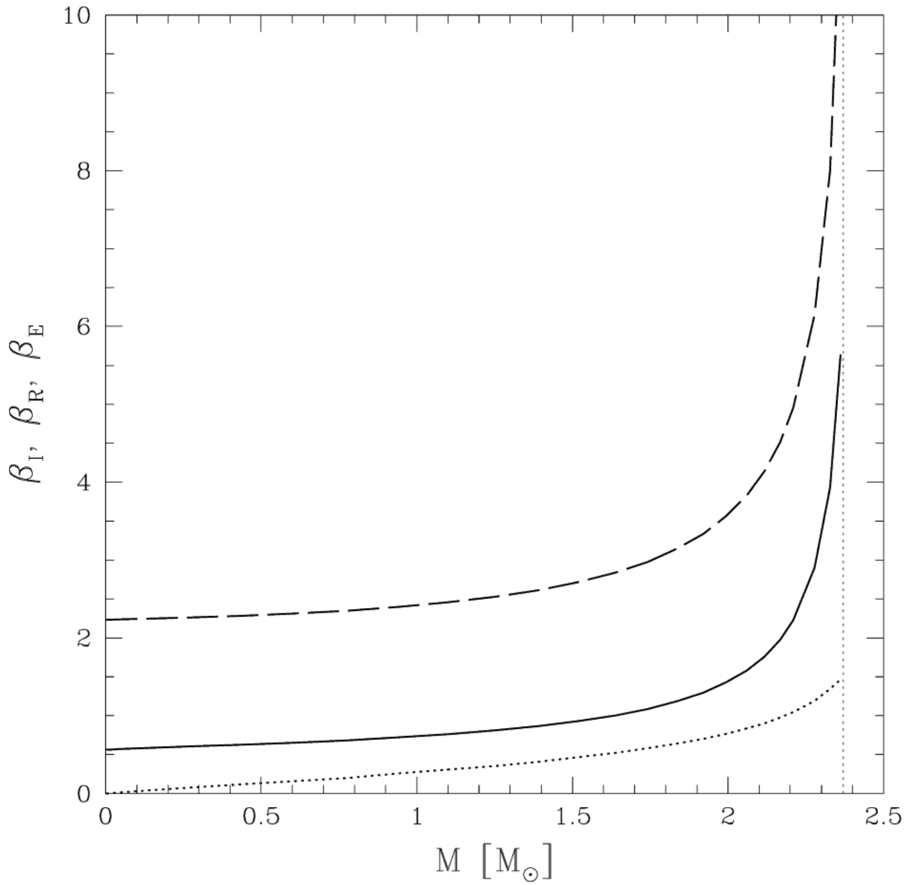
$$\mathcal{E}(n_\mathrm{b}) = \frac{K_\mathrm{N}}{\gamma_\mathrm{N} - 1}(n_\mathrm{b})^{\gamma_\mathrm{N}} + m_\mathrm{N} c^2 n_\mathrm{b}$$

Approximate formulae for the response:

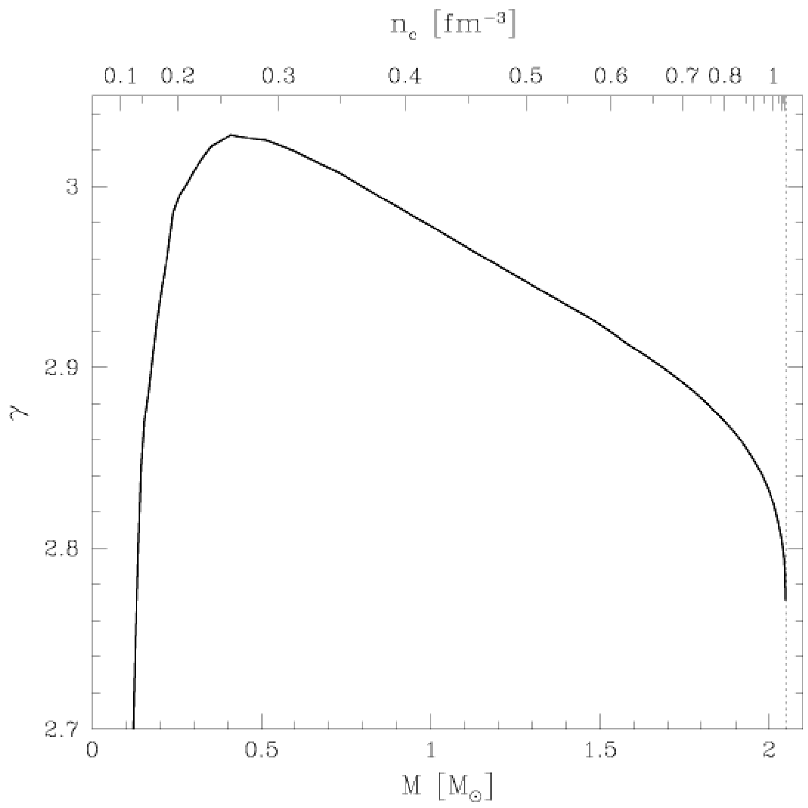
$$\beta_I[\mathcal{C}_0] \simeq 0.118 \cdot \gamma_\mathrm{N}^{9.1}/(\gamma_\mathrm{N}^{1.22} - 1)^{7.5}$$

$$\beta_R[\mathcal{C}_0] \simeq 0.014 \cdot \gamma_\mathrm{N}^{9.4}/(\gamma_\mathrm{N}^{1.13} - 1)^{8.2}$$

$$\beta_E[\mathcal{C}_0] \simeq \frac{0.085}{\gamma_\mathrm{N}^{2.56}\ K_\mathrm{N}^{1/2(\gamma_\mathrm{N}-1)}} \cdot \left(\frac{M}{M_\odot}\right)$$



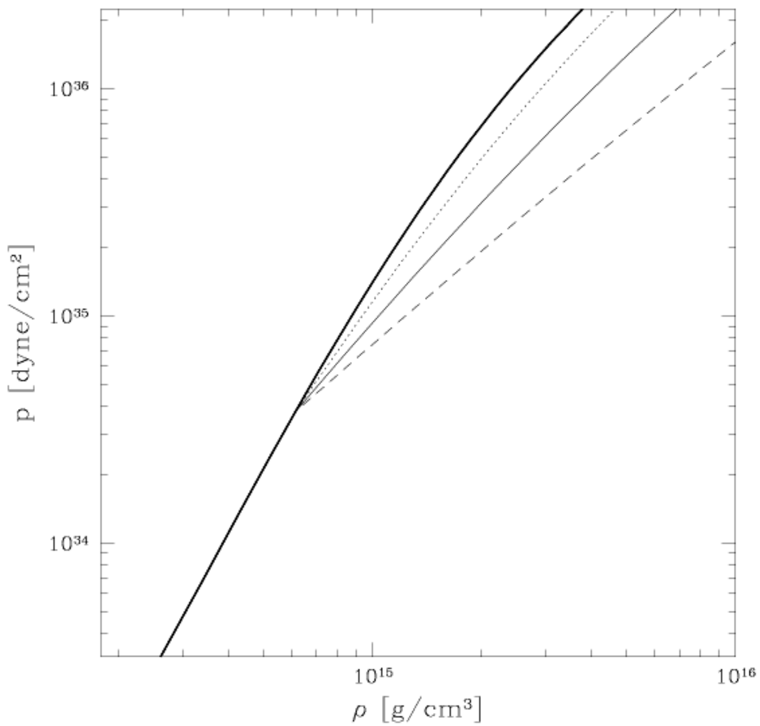
Response coefficients – realistic EOS



Adiabatic index:

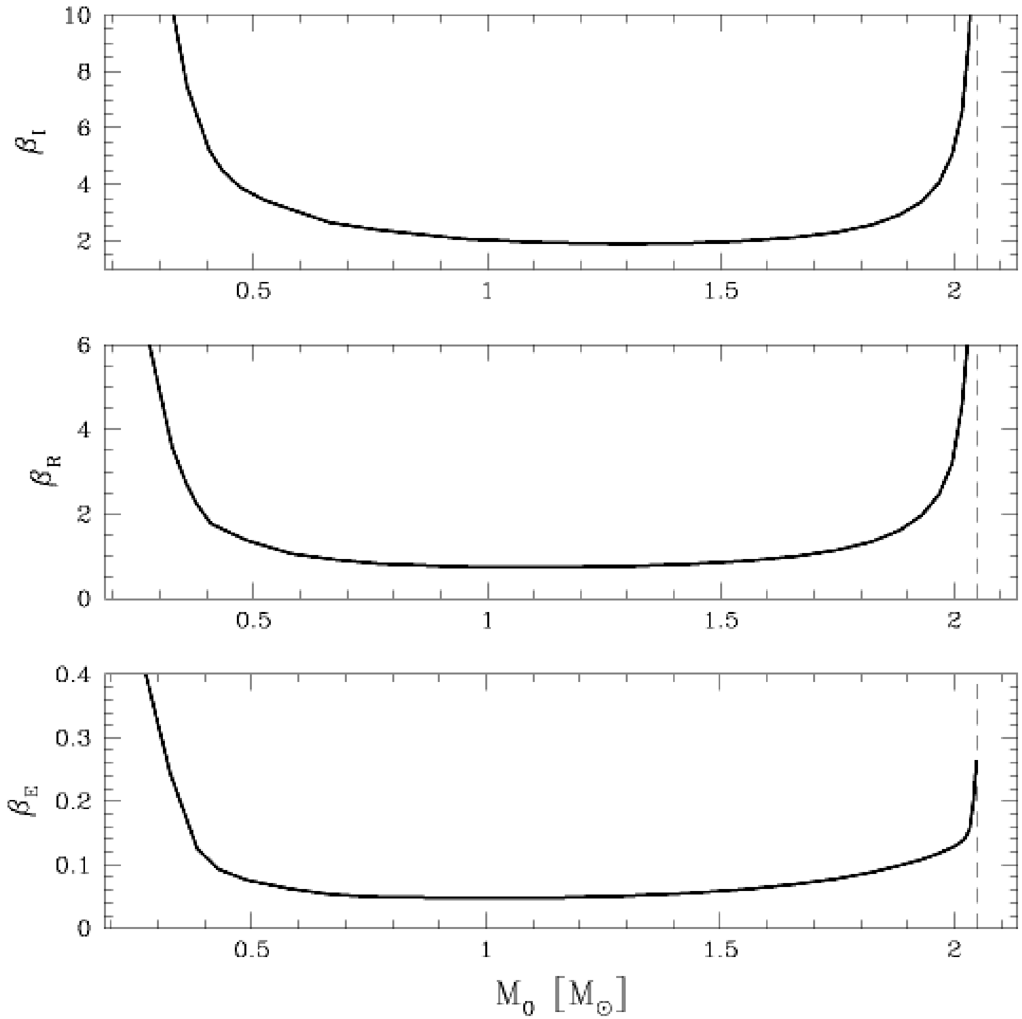
$$\gamma \equiv (n_b/P)dP/dn_b$$

SLy (Douchin & Haensel 2001)



Response coefficients – realistic EOS

$$\beta_I^{\rm plat} = 2.0,\ \beta_R^{\rm plat} = 0.8,\ \text{and}\ \beta_E^{\rm plat} = 0.05$$

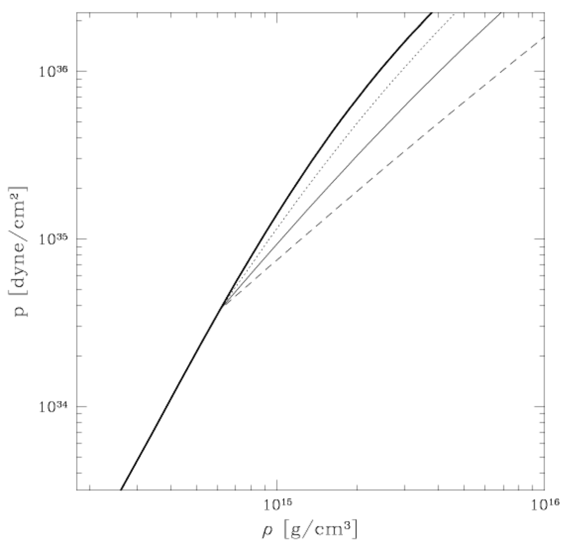


Realistic EOS - results

Realistic EOS SLy
(Douchin & Haensel 2001):

Reference configuration -
$M = 1.4\ M_{sun}$, $R = 11.73$ km,
$I_0 = 1.37 \cdot 10^{45}$ g cm^2

Transition to a mixed-phase
- γ_m=1.5 polytrope



Core-size: r_m = 1 km

$R \quad = 4$ cm, $\Omega \quad /\Omega = -I \quad /I_0 = 10^{-5}$,

$E \quad = 10^{45}$ erg

r_m = 4 km

$R \quad = 42$ m, $\Omega \quad /\Omega = -I \quad /I_0 = 10^{-2}$,

$E \quad = 10^{49}$ erg

Comparison:

Normal „glitch”: $\Omega \quad /\Omega = 10^{-8}$,
SGR: 10^{44}-10^{45} erg,
X-ray bursters: 10^{37}-10^{43} erg

Astrophysical scenarios

proto-neutron star

slow-down of a solitary pulsar $\dot{P}_{\rm c} \propto -\dot{\nu}\nu$

accretion in a binary system $\dot{P}_{\rm c} \propto \dot{M}$

Plans & prospects

- More realistic case of rotating stars: spin-up coefficient and the influence on the results
- Non-spherical stars
- Comparison of two independent conserved numbers: baryon number and angular momentum