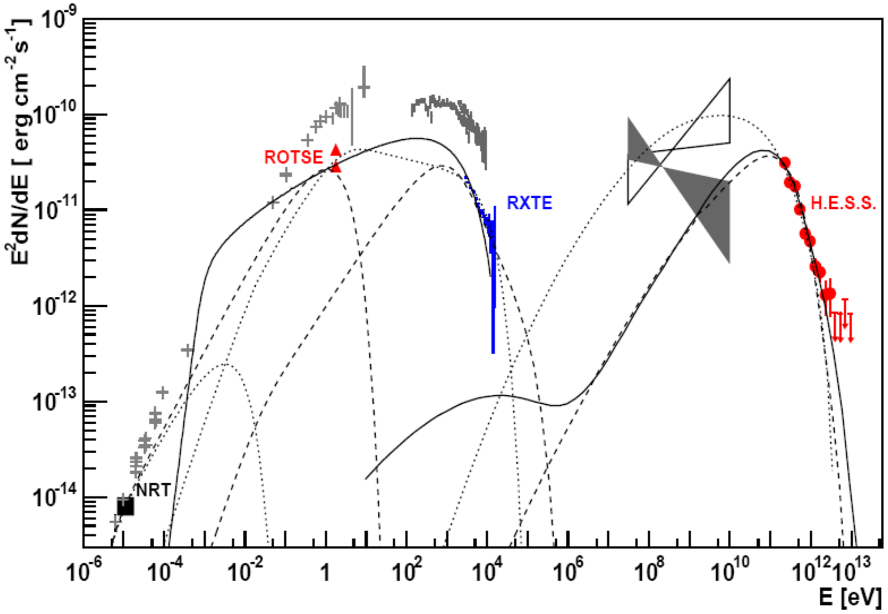
Spectral analysis in VHE γ-ray astronomy

Santiago Pita (CNRS/APC)

Warsaw, November 20th – 2007

Spectra at Very High Energies

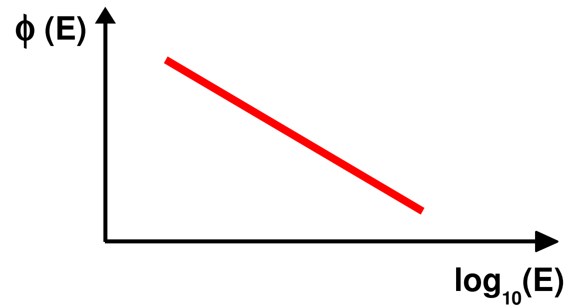
- Main features
  - Typical spectra
  - Ideal case : ideal instrument
  - Instrument response
- Spectrum determination
  - Classical approach
  - Maximum likelihood approach
- Spectral variations
  - Spatial variations
  - Time variations (shape, flux)



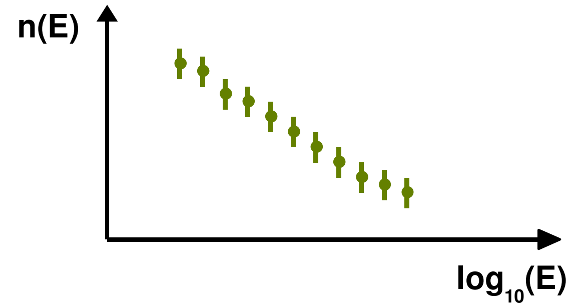
Spectrum determination : ideal case

- Flux at Earth $\phi(E)$

  [γ / cm^2.s.TeV]

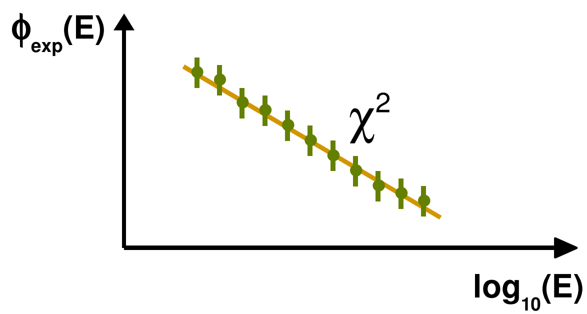


- Imagine an ideal case :
  - Constant efficiency $\Rightarrow A(E) = \text{const}$
  - Perfect energy determination $\Rightarrow R(E,\bar{E}) = \delta(E-\bar{E})$

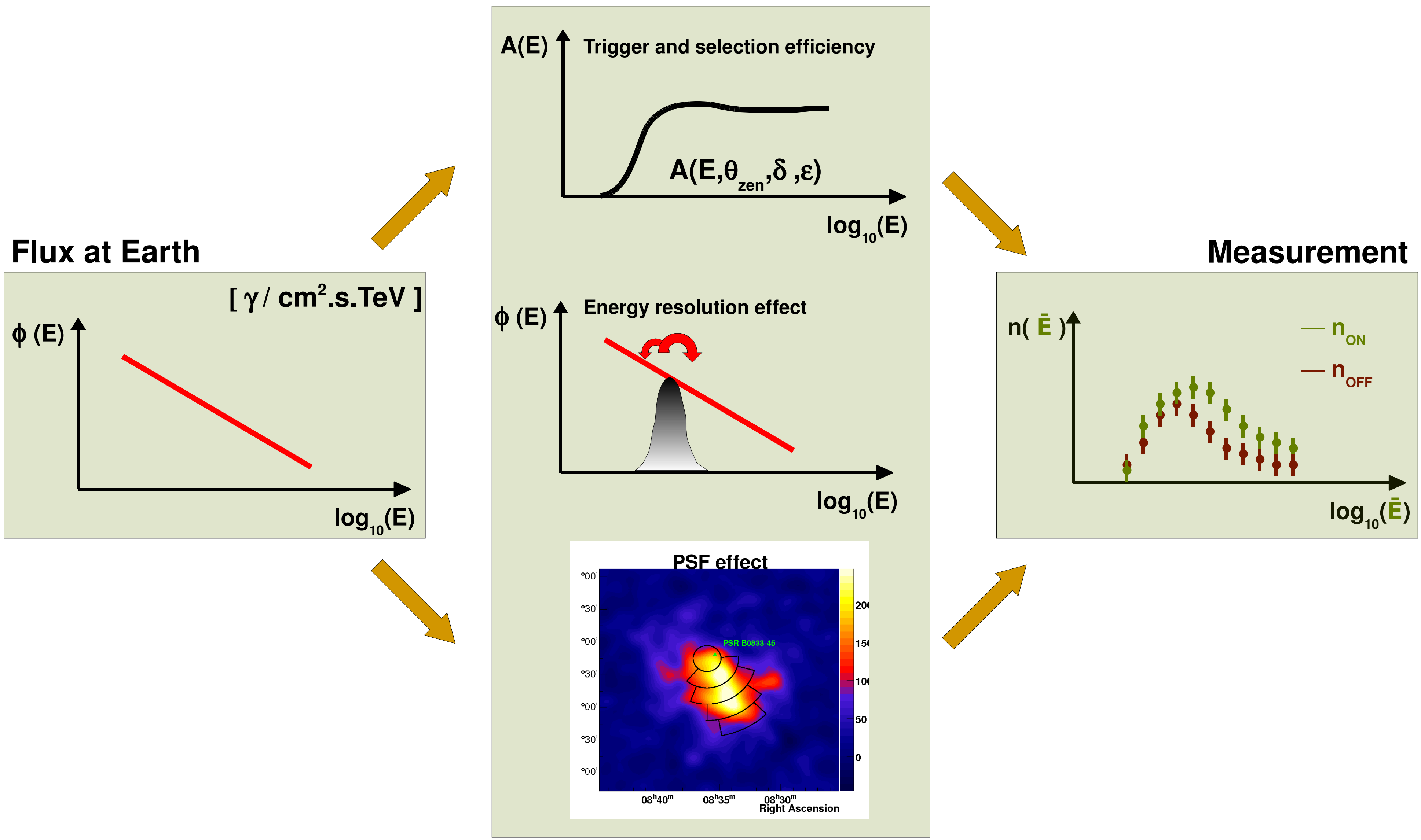


- Reconstructed flux :

$$\phi_{\exp}(E) = \frac{n(E)}{T.A(E)}$$

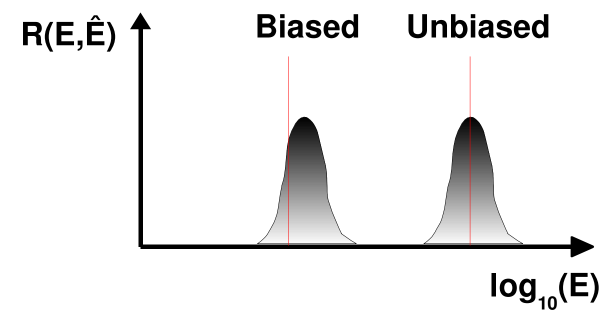


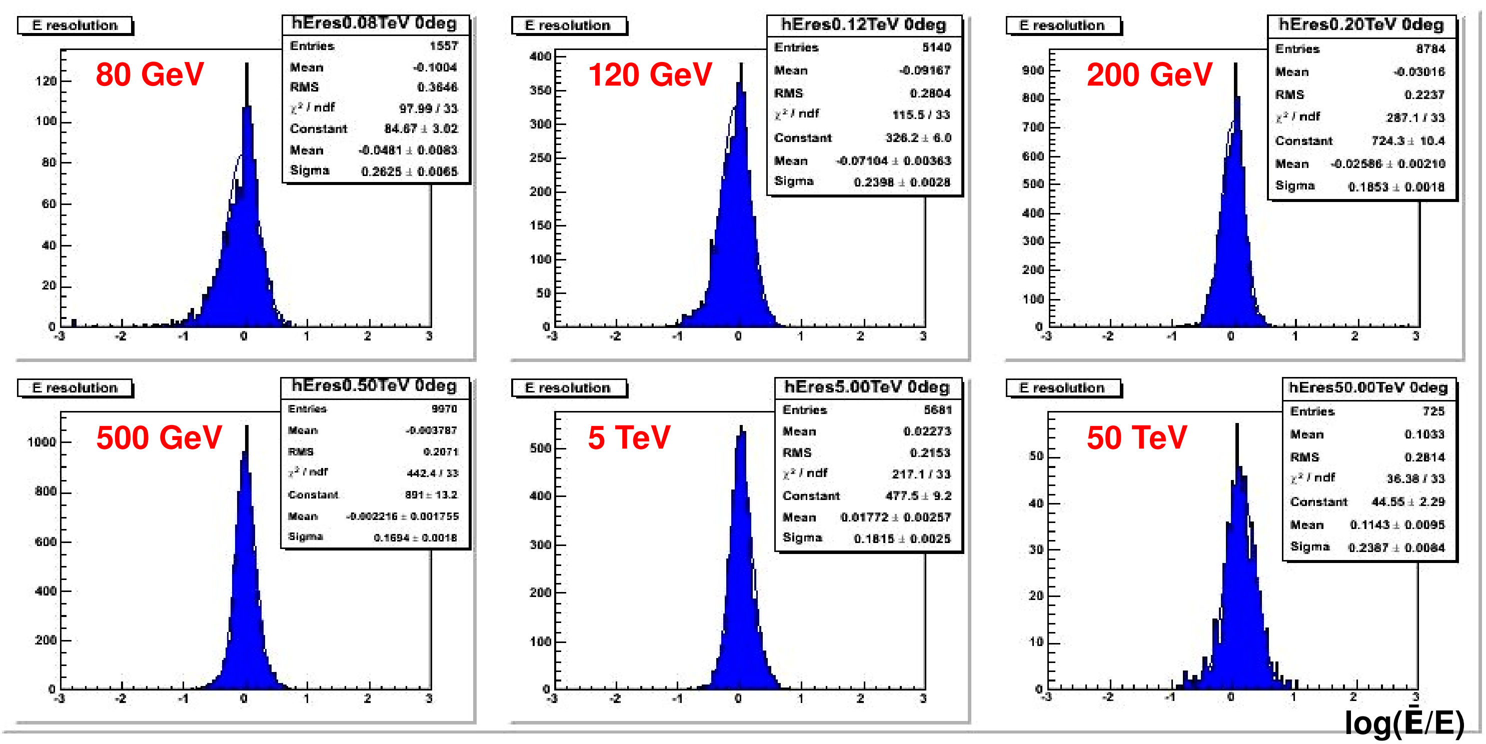
Instrument response



Instrument response : energy resolution

- See Mathieu's presentation for different methods
  - Hillas : $Q_{\text{meas}} = f(R, \theta_{\text{zen}}, \delta, \varepsilon)$
    - Hillas improvements at APC $Q_{\text{meas}} = f(R, \theta_{\text{zen}}, \delta, \varepsilon, \text{multiplicity}, H_{\max})$



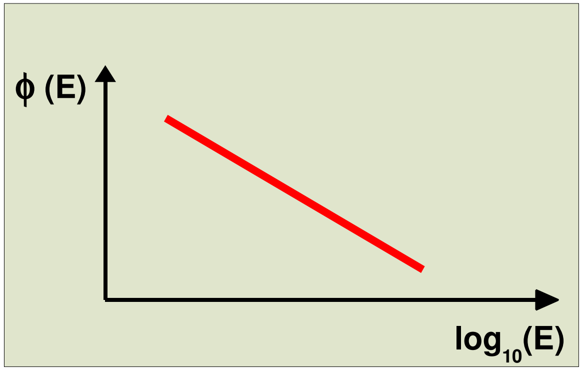


Instrument response : energy resolution

- See Mathieu's presentation for different methods
  - Hillas : $Q_{\mathrm{meas}} = f(R, \theta_{\mathrm{zen}}, \delta, \varepsilon)$
    - Hillas, improvements at APC $Q_{\mathrm{meas}} = f(R, \theta_{\mathrm{zen}}, \delta, \varepsilon, \mathrm{multiplicity}, H_{\max})$
      - No bias, even at threshold
  - Model 2D
    - Energy comes directly from fit
    - No bias, even at threshold
  - Model 3D
    - $N_{\mathrm{photosphere}}$ comes from fit
    - Energy determined with calibration tables
    - No bias, even at threshold

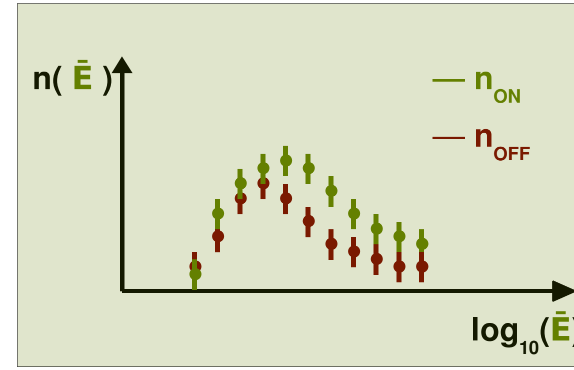
Two approaches

Flux at Earth



How to get ϕ(E) from n(Ē) ?

Measurement



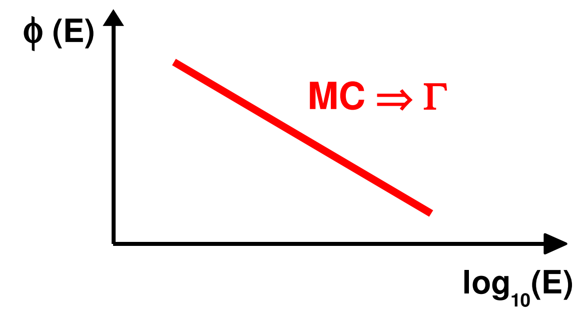
- “Classical”
  - Based on $n(\bar{E}) = n_{\mathrm{on}}(\bar{E}) - \alpha \cdot n_{\mathrm{off}}(\bar{E})$
  - Used by German groups
- Maximum likelihood
  - Based on $n_{\mathrm{on}}(\bar{E}), n_{\mathrm{off}}(\bar{E})$ and their statistical properties
  - Used by French groups (independently of reconstruction/discrimination methods)

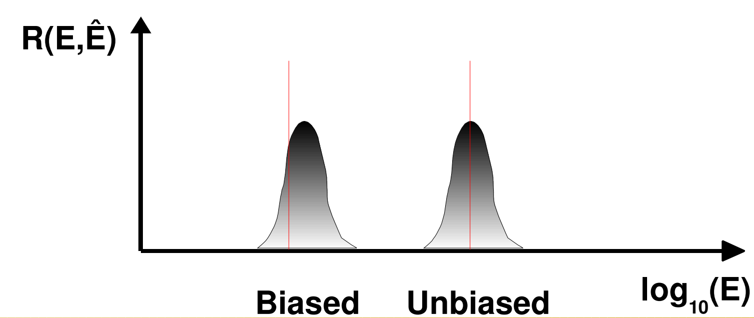
Classical approach

- Used by German groups
- Based on $n(\bar{E}) = n_{\text{on}}(\bar{E}) - \alpha \cdot n_{\text{off}}(\bar{E})$

$$\phi(E) = \frac{n_{\text{on}}(\bar{E}) - \alpha \cdot n_{\text{off}}(\bar{E})}{T \cdot A(\bar{E})}$$

- Area determined at measured energies
  - Based on simulated spectrum : power-law
  - a priori MC index Γ
  - Need iterative procedure
  - Possible bias when “true” spectrum ≠ pwl





- Method works very well if :
  - Iterative process convergent
  - No bias in resolution functions

Maximum likelihood approach

- Maximun likelihood, based on
  - $n_{\mathrm{on}}(\bar{E})$ and $n_{\mathrm{off}}(\bar{E})$ and their Poisson probabilities
  - ➢ A spectral hypothesis : $\{\Lambda\}$
    - power-law
    - power-law + exp. cutoff
    - log-parabolic
    - other

$$\phi(E)=\phi_o E^{-\Gamma}$$
$$\phi(E)=\phi_o E^{-\Gamma}\exp(-E/E_{cut})$$
$$\phi(E)=\phi_o E^{-(\Gamma+\beta\log_{10}(E))}$$
$$\phi(E)=\ldots$$

  - ➢ Instrument functions A(E), R(E,Ē)
    - Derived from fixed energies MC
- The best set of $\{\Lambda\}$ is derived from maximisation of :

$$L(\{\Lambda\})=\prod_{i_x,i_e} P(\bar{n}_{\mathrm{on}},n_{\mathrm{on}}).P(\bar{n}_{\mathrm{off}},n_{\mathrm{off}})$$

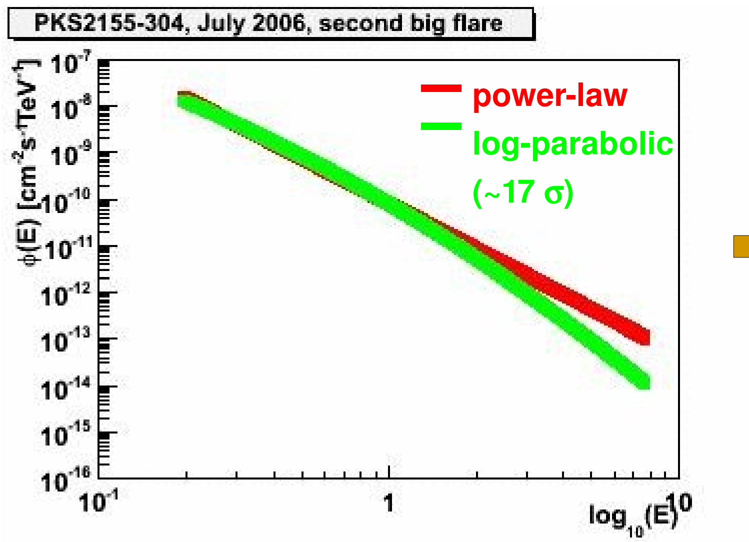
$\bar{n}_{\mathrm{on}}$ is a function of $\{\Lambda\}$ and $\bar{n}_{\mathrm{off}}$

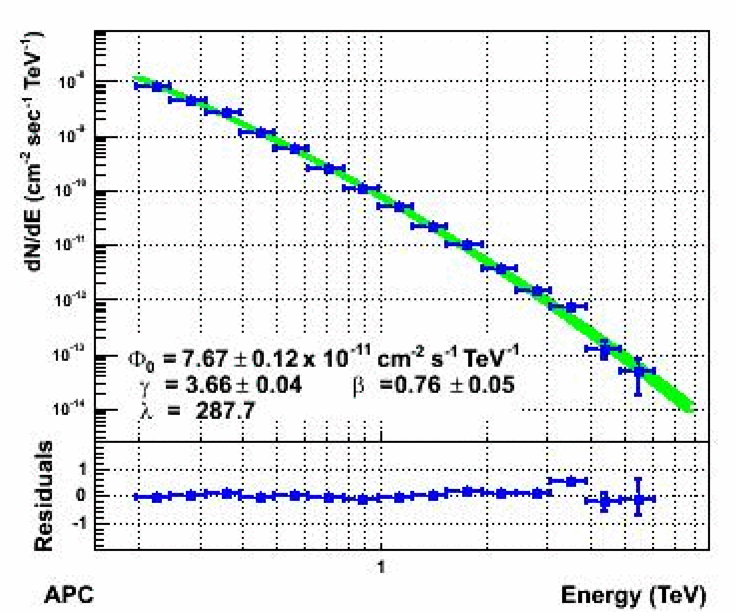
Maximum likelihood approach

$$L(\{\Lambda\}) = \Pi_{i_x, i_e} P(\bar{n}_{\text{on}}, n_{\text{on}}) . P(\bar{n}_{\text{off}}, n_{\text{off}})$$

- Data is separated in sets with similar A(E) and R(E,Ē) ⇒ i_x
  - Ex : similar zenith angle, offset in the camera, telescopes efficiency, etc.
- i_e - $$S^{theo}_{i_x, i_e} = T \int_{min(i_e)}^{max(i_e)} \int_0^{\infty} \phi_o \mathrm{E}^{-\Gamma} A(\mathrm{E}) R(\mathrm{E}, \bar{\mathrm{E}})\, d\mathrm{E}\, d\bar{\mathrm{E}}$$
- n_{on} and n_{off} are measured
- $\bar{n}_{\text{on}}$ and $\bar{n}_{\text{off}}$ are unknown
  - $\bar{n}_{\text{on}}$ and $\bar{n}_{\text{off}}$ $\bar{n}_{\text{on}} = S^{theo}_{i_x, i_e} + \bar{n}_{\text{off}}$
  - Minimisation of $\chi^2 = -2\log(L(\{\Lambda\}))$
  - Likelihood ratio

Example : an AGN flare

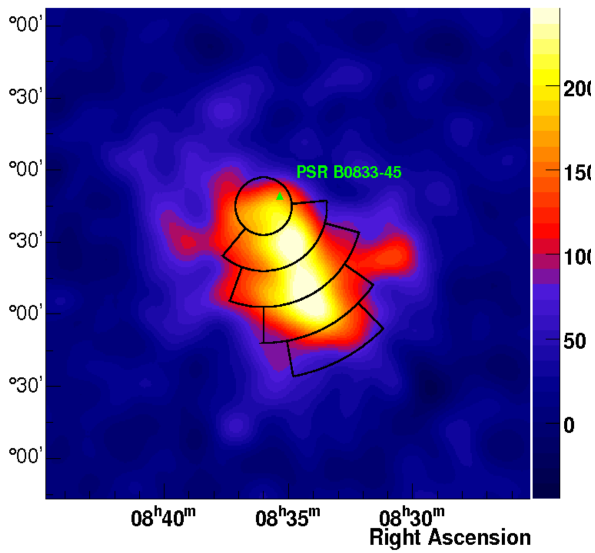


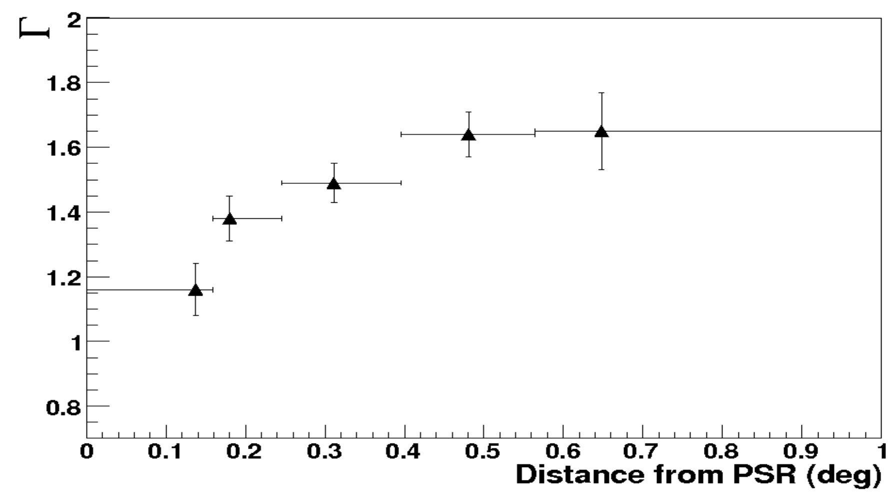


$$\phi_o = 7.67 \pm 0.12 \ [10^{-11} \mathrm{cm}^{-2} \mathrm{s}^{-1} \mathrm{TeV}^{-1}]$$
$$\Gamma = 3.66 \pm 0.04$$
$$\beta = 0.76 \pm 0.05$$
$$+ \text{covariance matrix}$$

Space-dependent spectra

- Some extended sources show space-dependent spectra
  - Ex: PWN (connected to the cooling of particles when they flow out)

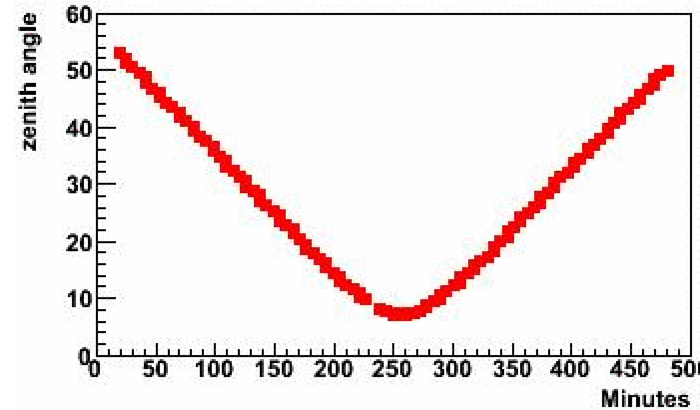


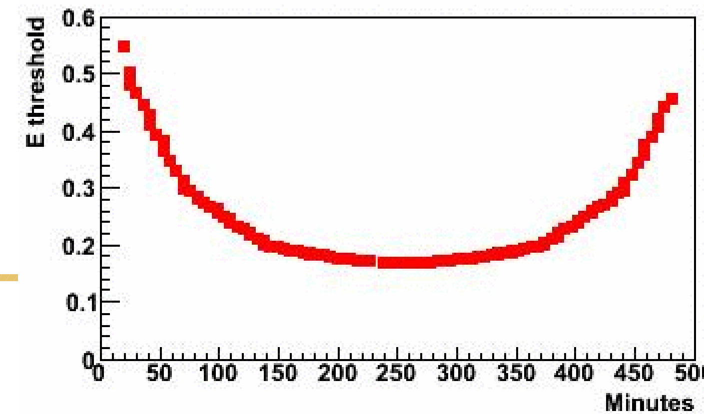


- Effective area : full containement
- Wedge to wedge contamination (due to PSF) could imply :
  - Systematics on the flux level determination
  - Systematics on the spectrum shape (usually pwl index) determination

Time-dependent spectra (1)

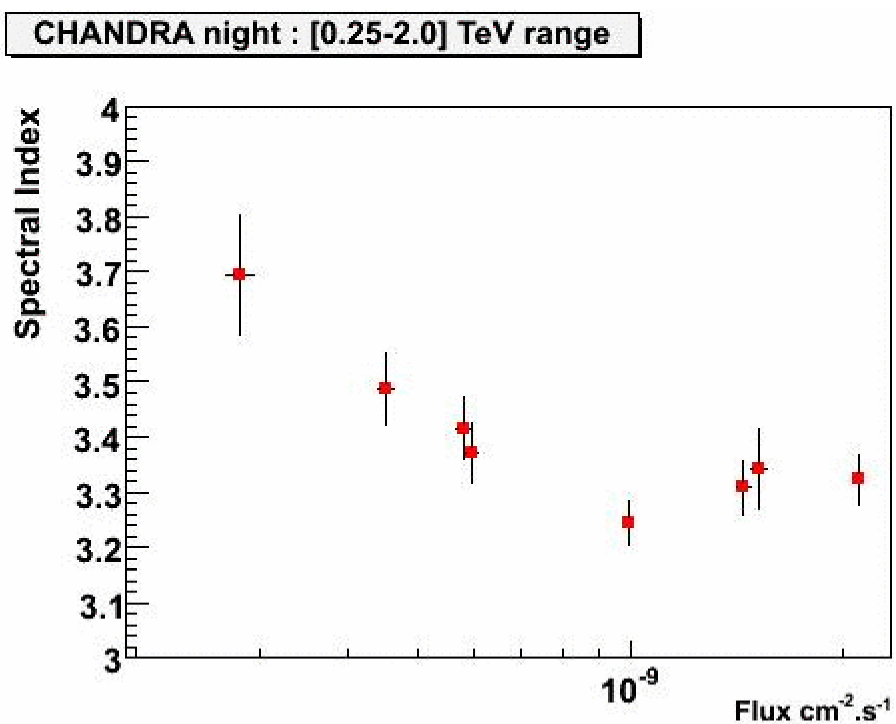
- Typical case of variable sources
  - Try to determine spectral shape variations with time (or flux)
  - Try to determine how integral flux varies with time (light-curves)
- Example of the second flare of PKS 2155-304 in July 2006
  - The "Chandra night"
  - 15 consecutive runs
    - Strong zenith angle variation
    - Strong energy threshold variation
    - Indication of energy cut-off at ~ 2 TeV

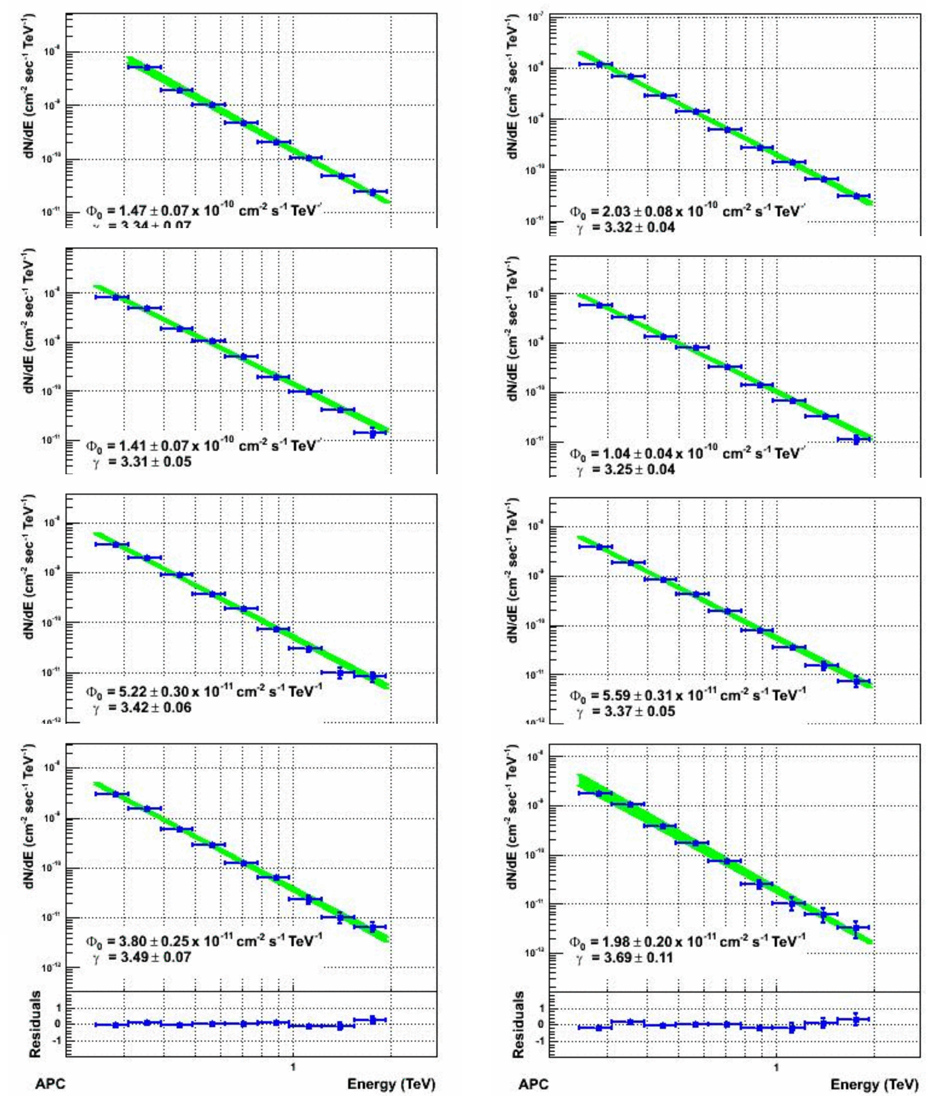




Time-dependent spectra (2)

- Evidence of hardening





Time-dependent spectra (3)

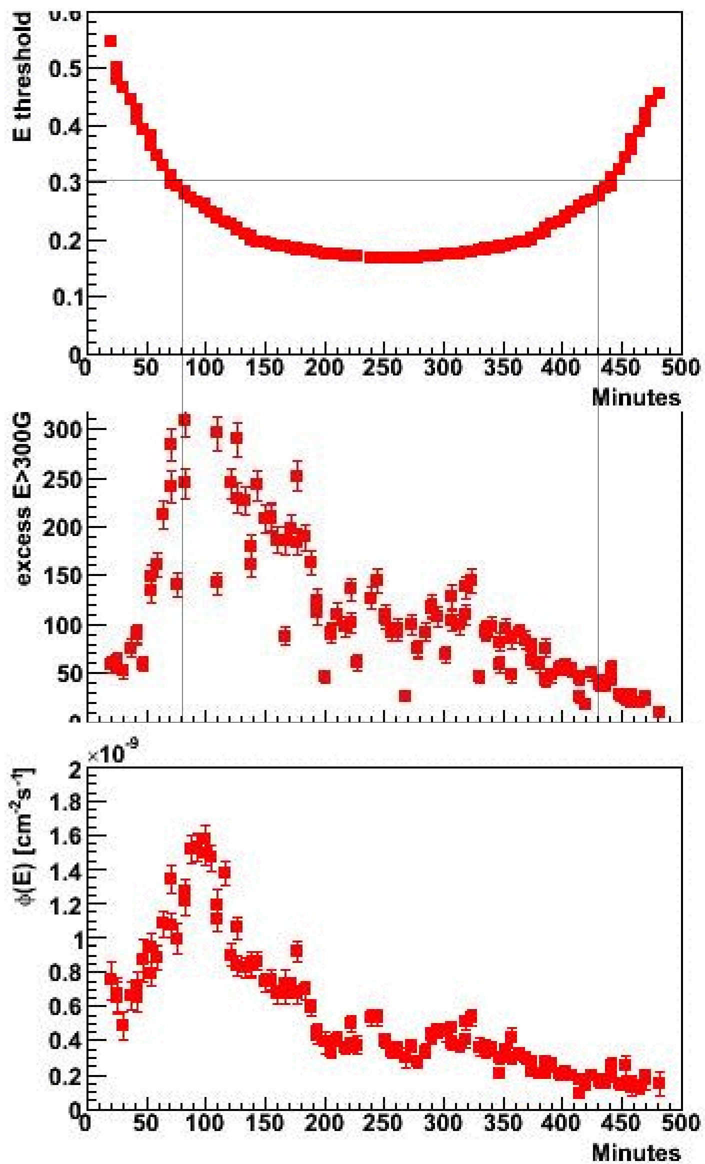
- Integral flux variability : light-curves
  - Allow the use of statistics much lower than those necessary for a spectrum fit
  - Based on a spectrum shape hypothesis

$$\int_{E_{\min}}^{\infty} \phi(E)\,dE = \phi_o \int_{E_{\min}}^{\infty} E^{-\Gamma}\,dE$$

Unknown

$$n_{\exp}(\bar{E} > E_{min}) = T \int_{E_{min}}^{\infty} \int_{0}^{\infty} \phi_o E^{-\Gamma} A(E) R(E, \bar{E})\, dE\, d\bar{E}$$

Spectral variations with time

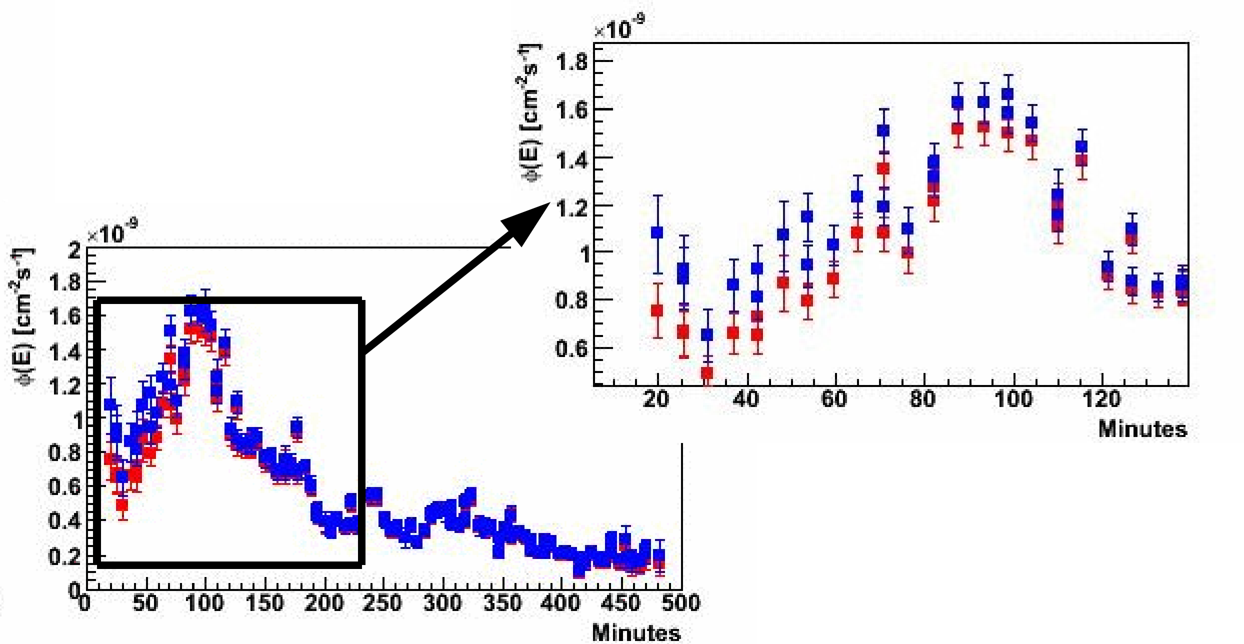


$$\int_{E_{\min}}^{\infty} \phi(E)\,dE = \phi_o \int_{E_{\min}}^{\infty} E^{-\Gamma}\,dE$$

Unknown

$$n_{\exp}(\bar{E} > E_{min}) = T \int_{E_{min}}^{\infty} \int_{0}^{\infty} \phi_o E^{-\Gamma} A(E) R(E,\bar{E})\; d\,E\; d\,\bar{E}$$

$$n_{\exp}(\bar{E} > E_{th}) = T \int_{E_{th}}^{\infty} \int_{0}^{\infty} \phi_o E^{-\Gamma} A(E) R(E,\bar{E})\; d\,E\; d\,\bar{E}$$



Conclusions

- Spectra strongly decrease with energy
- Necessary to take into account :
  - The energy resolution function
  - Effective areas
  - PSF effects (for extended sources)
- Two approaches available
  - Classical
  - Maximum likelihood
- Maximum likelihood is used by all french groups
  - Available in the parisanalysis and HAP frameworks