Reconstruction Techniques for IACTs

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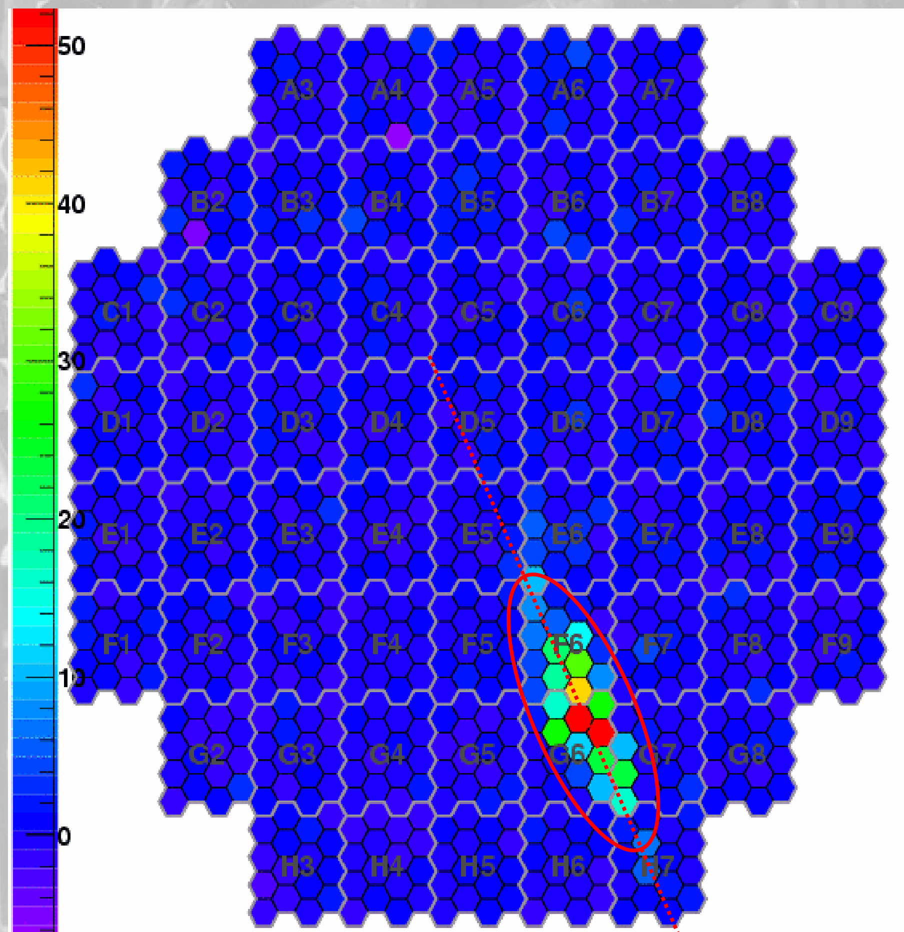
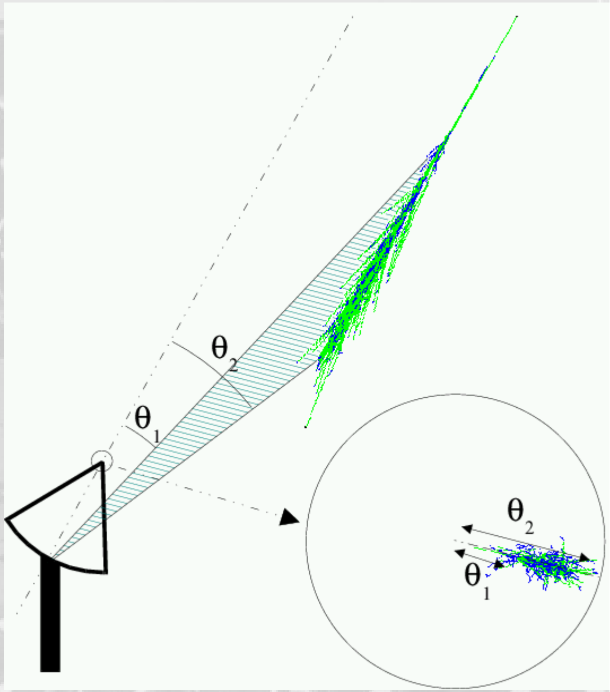
- Standard Hillas Parameters based analysis techniques
- Model Analysis
- 3D Model Analysis
- Some comparison elements

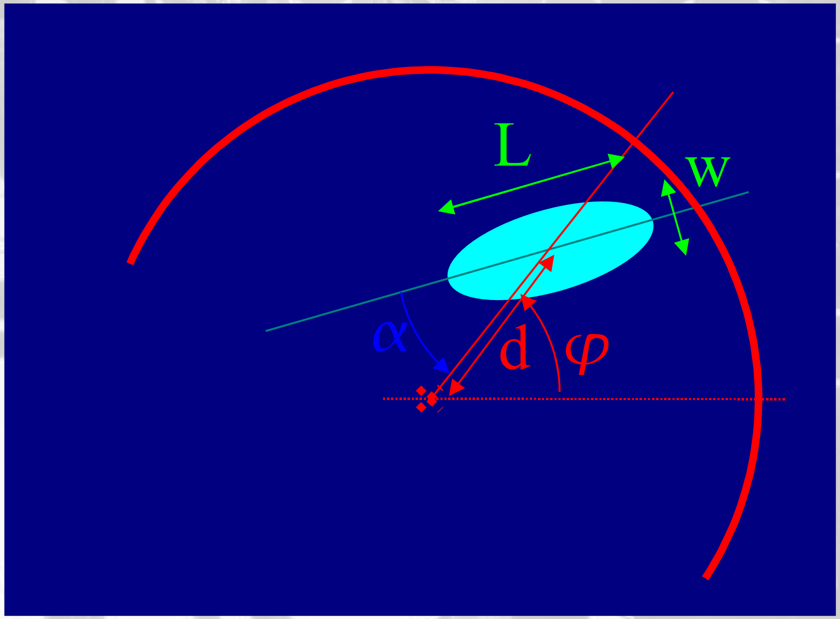
I - "Hillas Parameters based" analyses

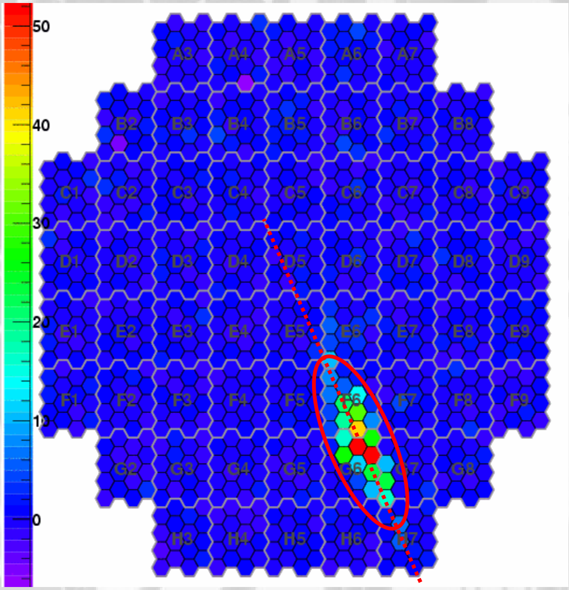


Standard reconstruction

- *Hillas Parameters (1984):*
  γ images are elliptical
  ⇨ reduce image properties to a few numbers:
  - *Length (L) & Width (w)*
  - *Amplitude (size)*
  - *Nominal Distance (d)*
  - *Azimuthal angle (φ) and orientation angle (α)*
  - *Additional parameters: asymmetry, ...*

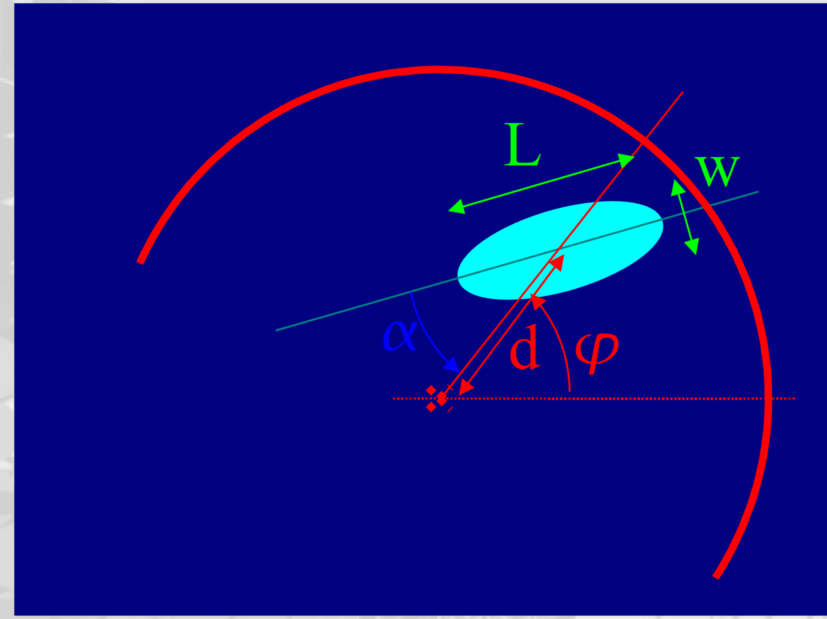


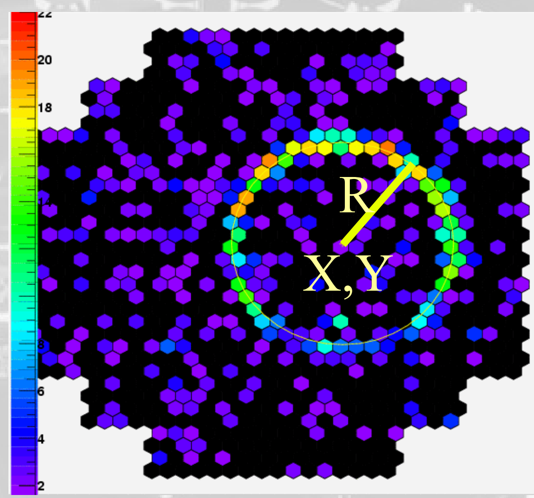


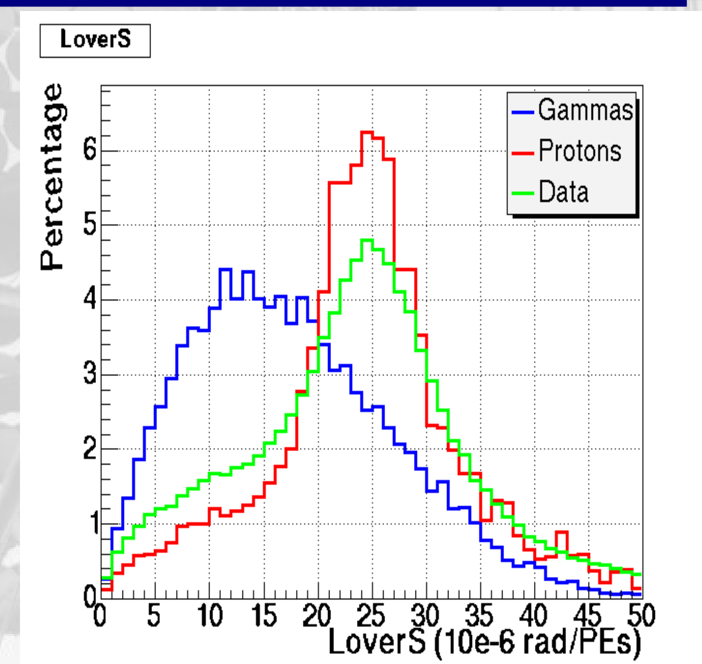


Single Telescope Analysis

- *α plot used to show signal*
- *Shower parameters derived from Hillas parameters (function or lookup table):*
  - *(size, length) “‘ core distance & direction*
  - *(size, nominal distance) “‘ energy*
- *Event selection based on standard cut techniques*
  - *Min size*
  - *Length/Size (muons killer)*
  - *width & length compared to MC*



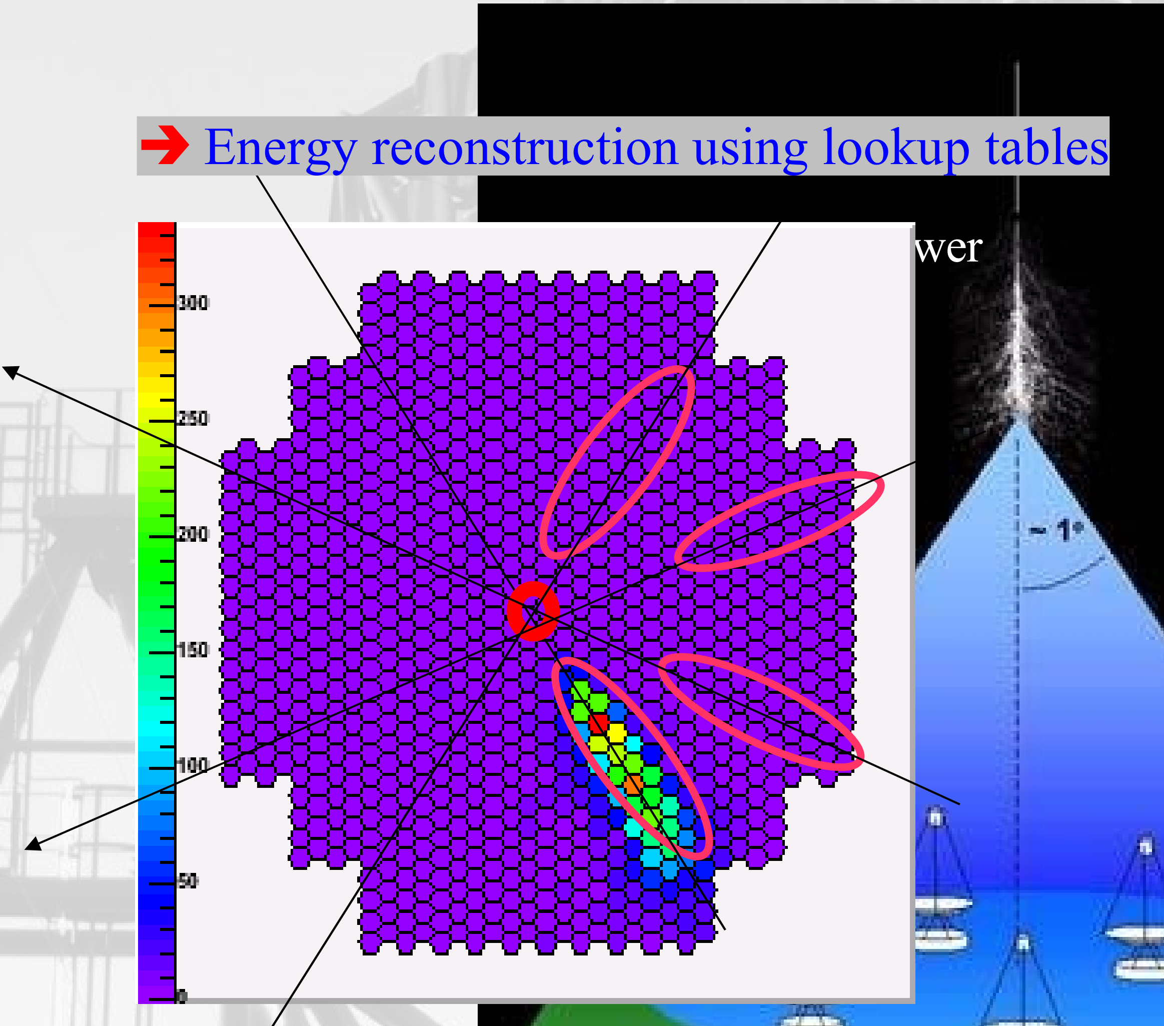


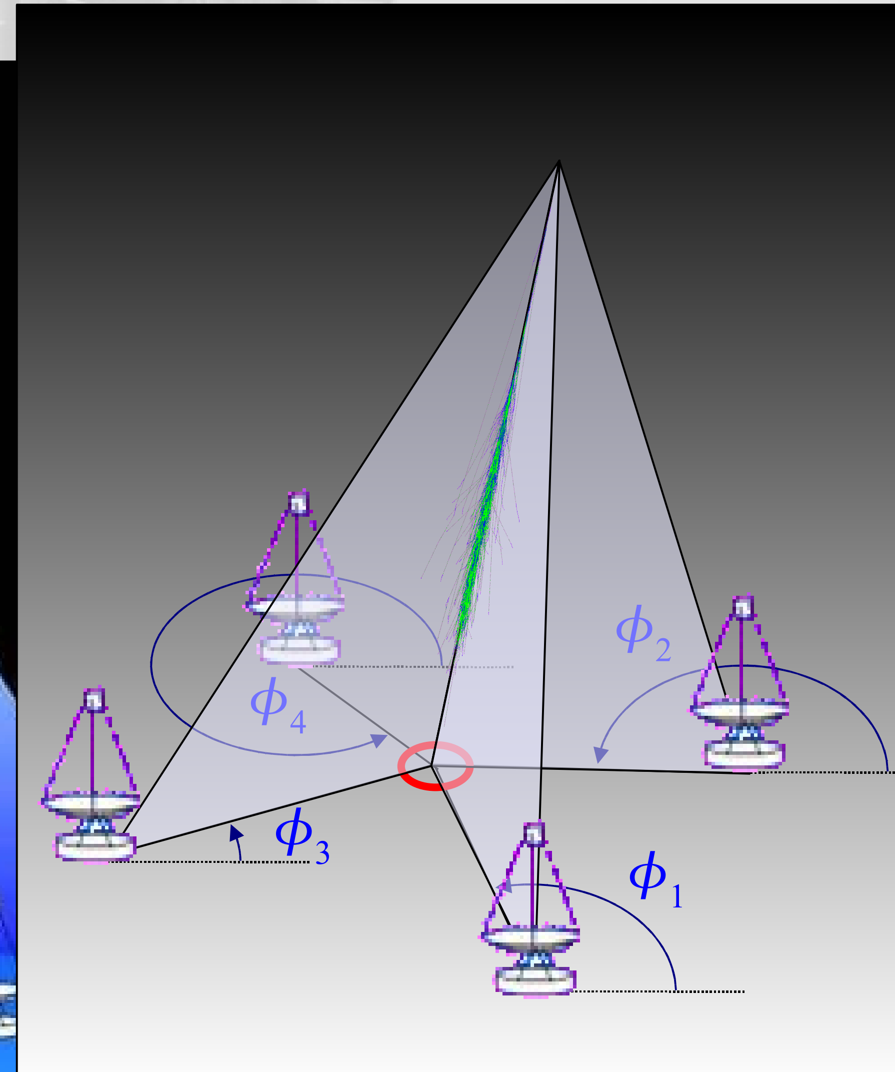


Stereoscopic reconstruction

➔ Improve reconstruction

➔ Energy reconstruction using lookup tables



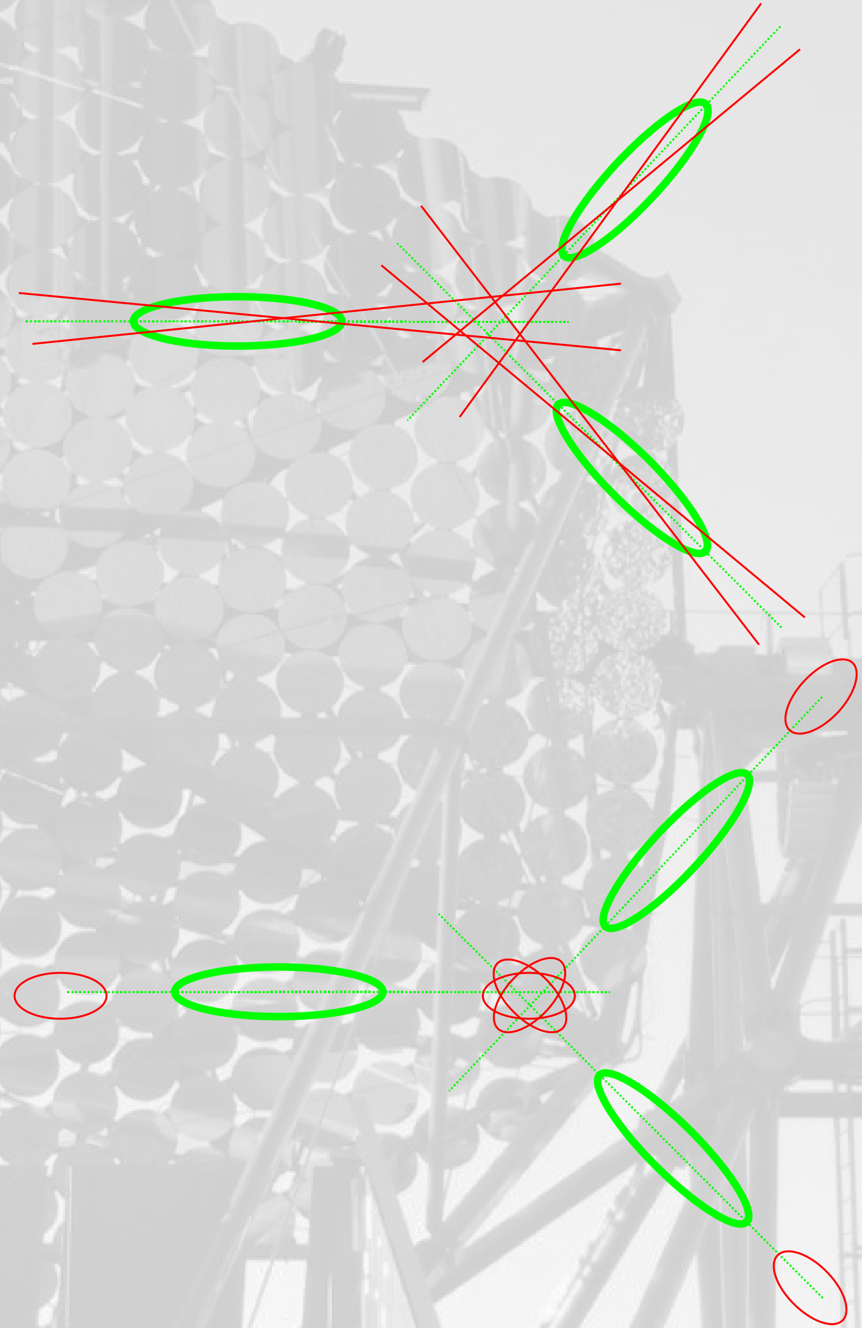


➔ Shower direction given by ellipse main axis intersections in camera frame

➔ Impact given by intersection of (Telescope, ϕ_i) lines on ground

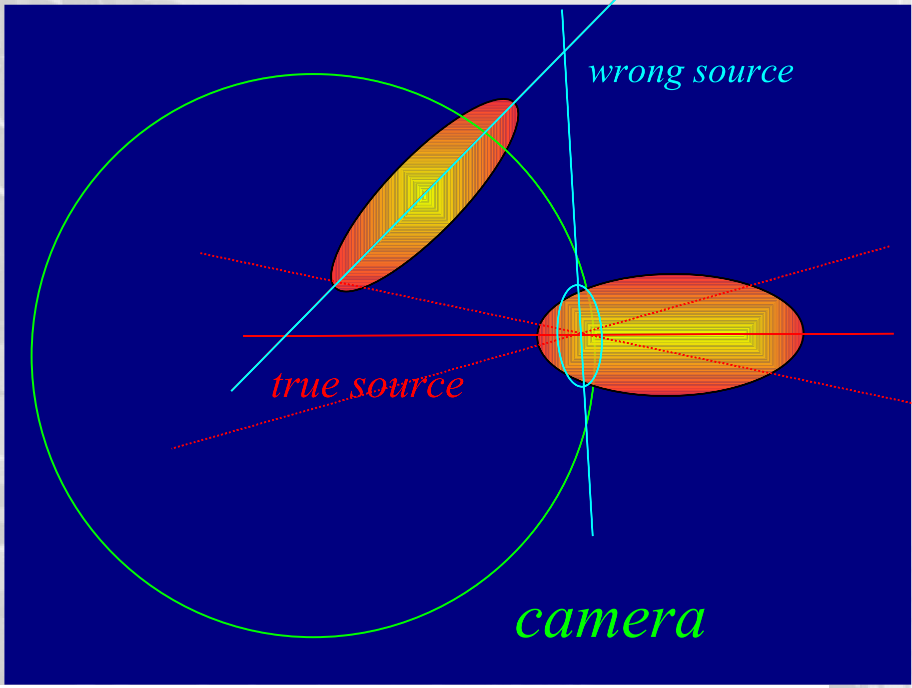
Possible improvements

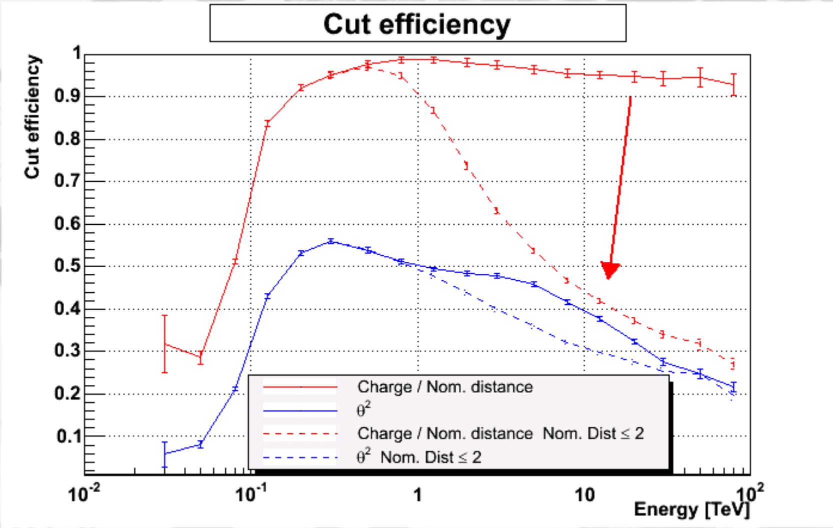
- *Some algorithm* *(Hofmann et al, 1999)* *can improve a little bit the resolution using:*
  - *Errors on Hillas parameters*
  - *Shape information ⇒ constraints on nominal distance*

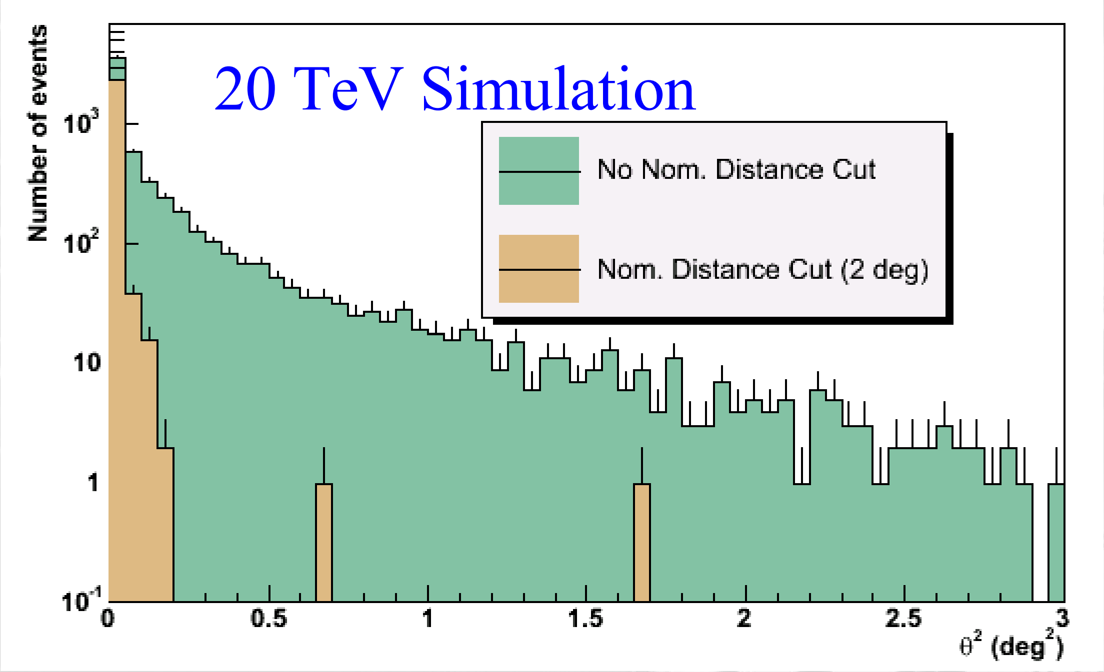


The degeneracy problem

- *Image close to the camera edge often badly reconstructed (also affects other analyses)*
- *Solution: cut images close to the edge (Nominal Distance cut)*
- *Drawback: reduces effective FOV & efficiency (@ high E)*







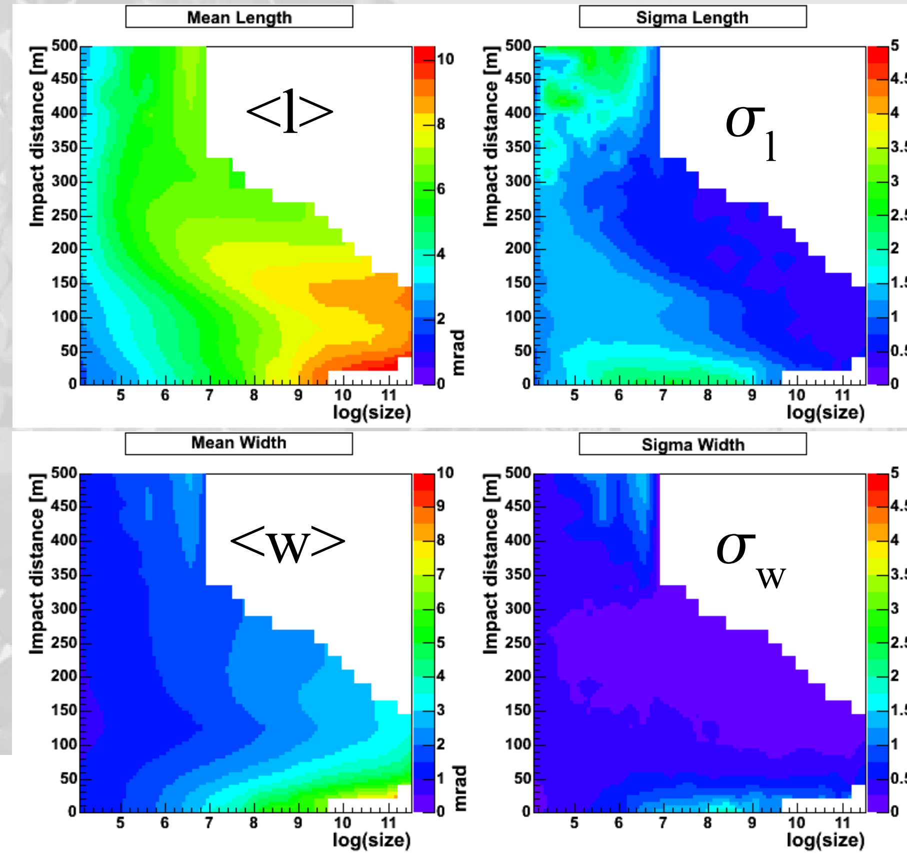
Mean Scaled Parameters (Hegra)

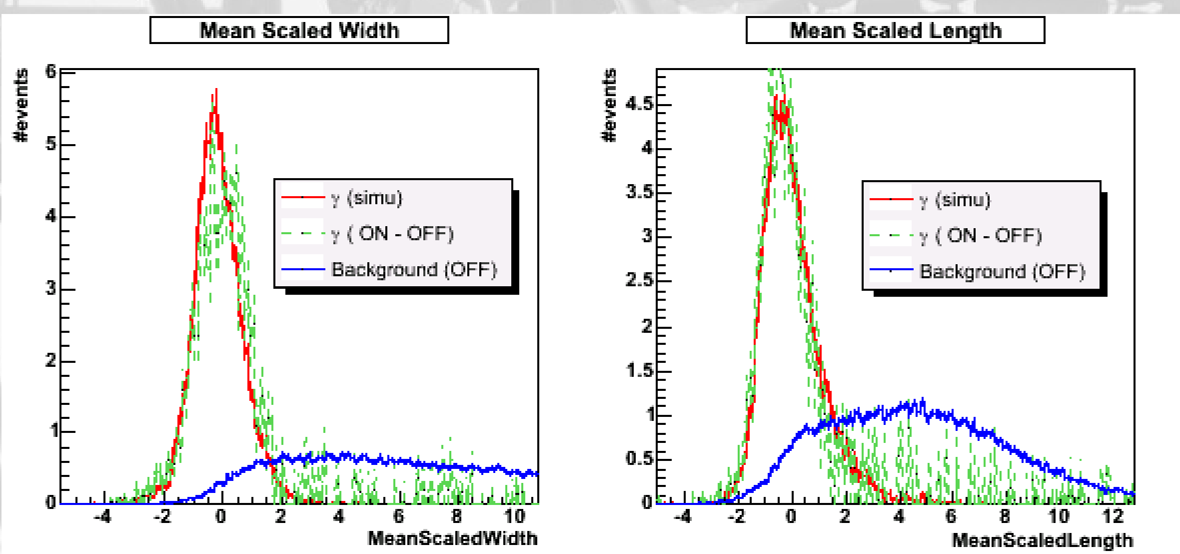
- *Tabulated relation of width, length of images and their RMS as function of impact and size* $\langle w\rangle(\rho,I)$, $\sigma_w(\rho,I)$...

$$S_w = \frac{\left(w - \langle w\rangle\right)}{\sigma_w}$$

- *Scaled parameter, Averaged over all telescopes*
- *Use MeanScaledWidth and MeanScaledLength as selection parameters*

$$MSW = \sum_{tels} S_w / \sqrt{ntels}$$

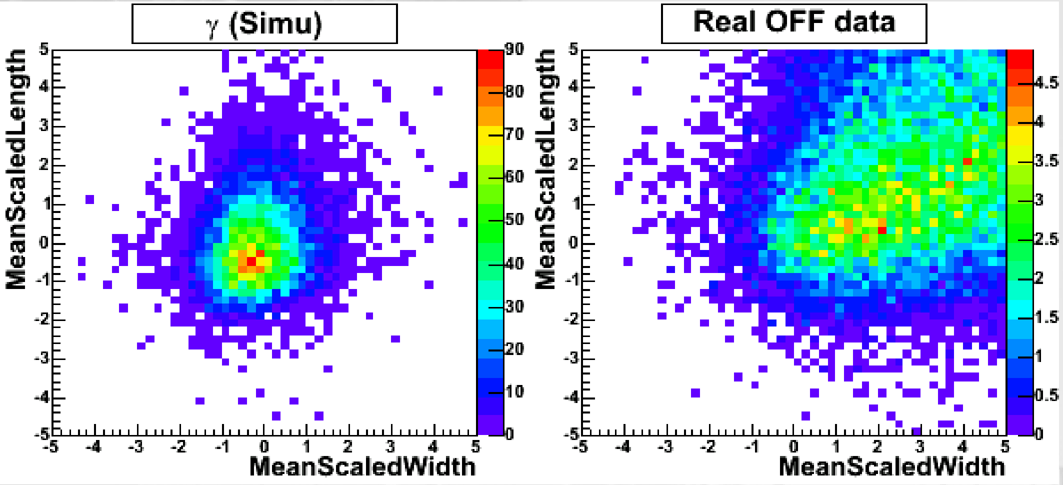




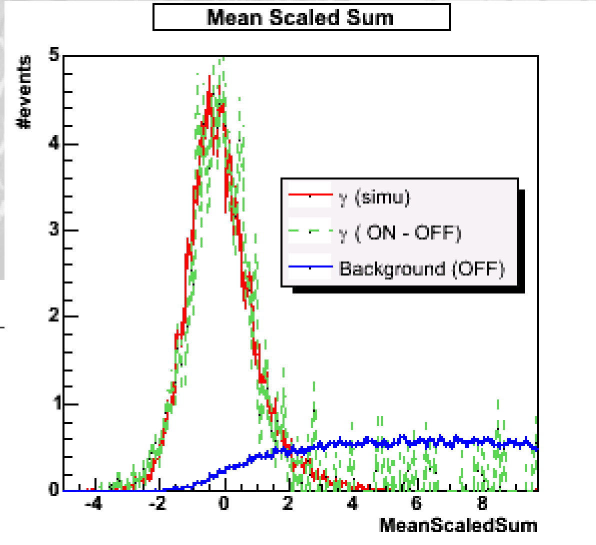
- *Normal variables, well understood*

Mean Scaled Sum

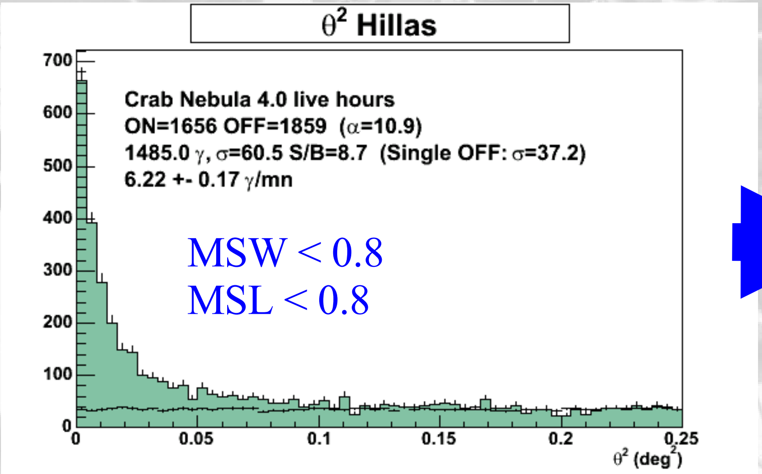
- *Almost no correlation between Mean Scaled Width and Mean Scaled Length for γ*
- *Equivalent new parameter Mean Scaled Sum*

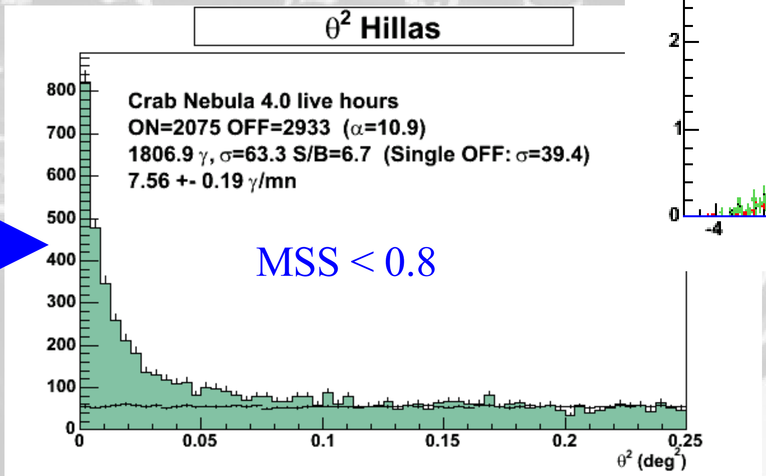


$$MSS = \frac{MSW + MSL}{\sqrt{2}}$$



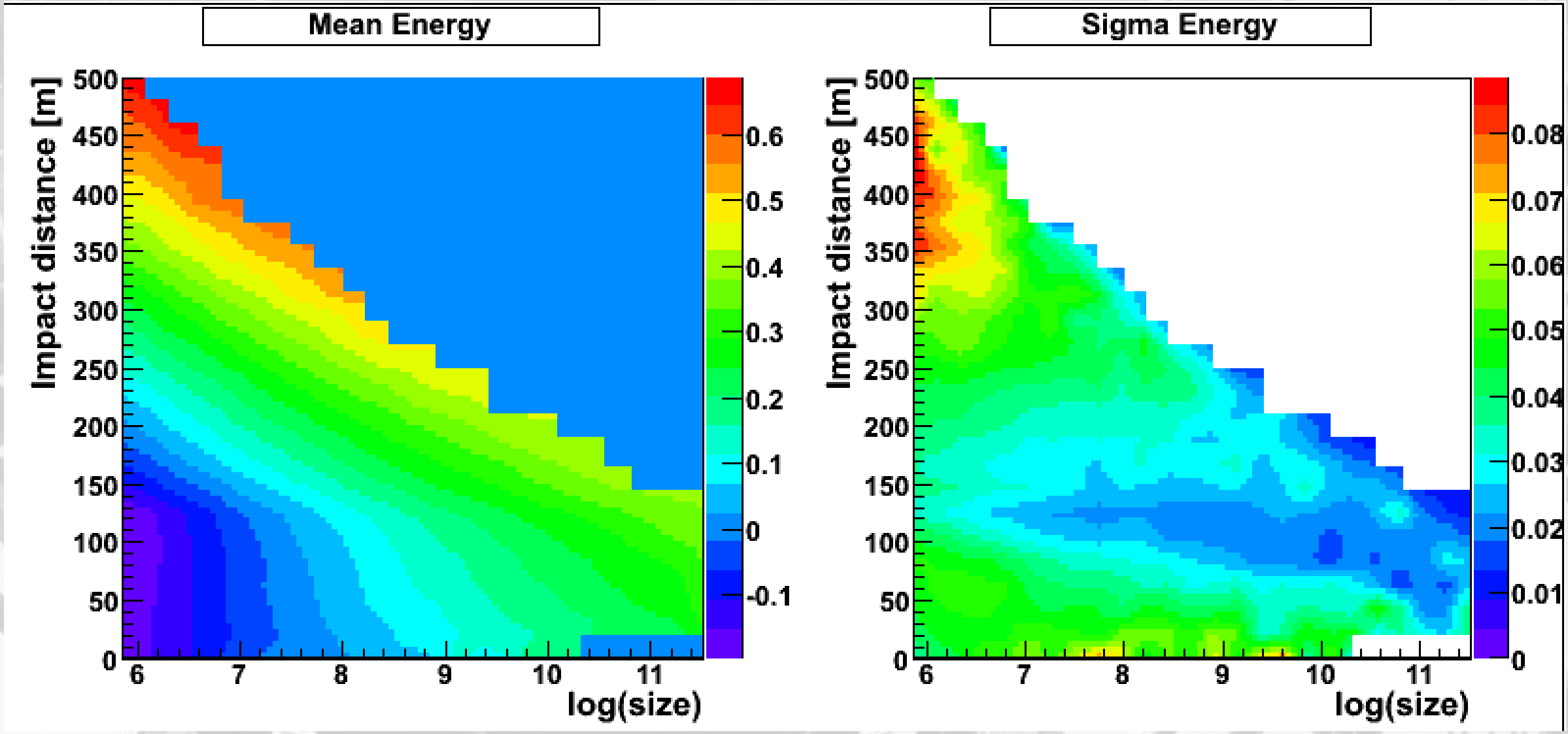
4.0 hours Crab Data





Energy Reconstruction

- *Lookup tables (Image Size, Impact distance) ⇨ Energy and σ_{Energy}*
  - *Generated for each zenith angle, off-axis angle, optical efficiency,...*
  - *Typical energy resolution 10 - 15%*
- *Additional variable help improving resolution and reducing biases*
  - *e.g. Depth of Shower Maximum (See Santiago's talk)*

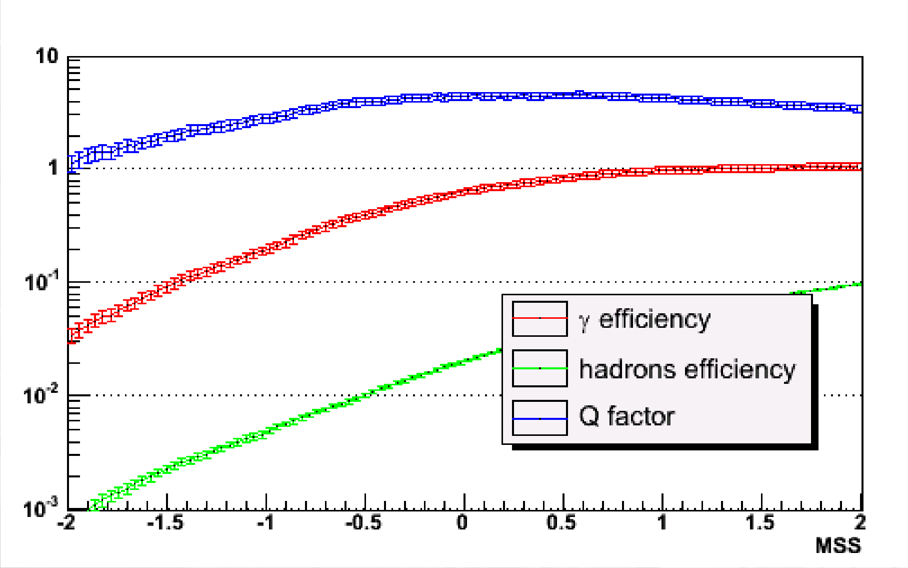


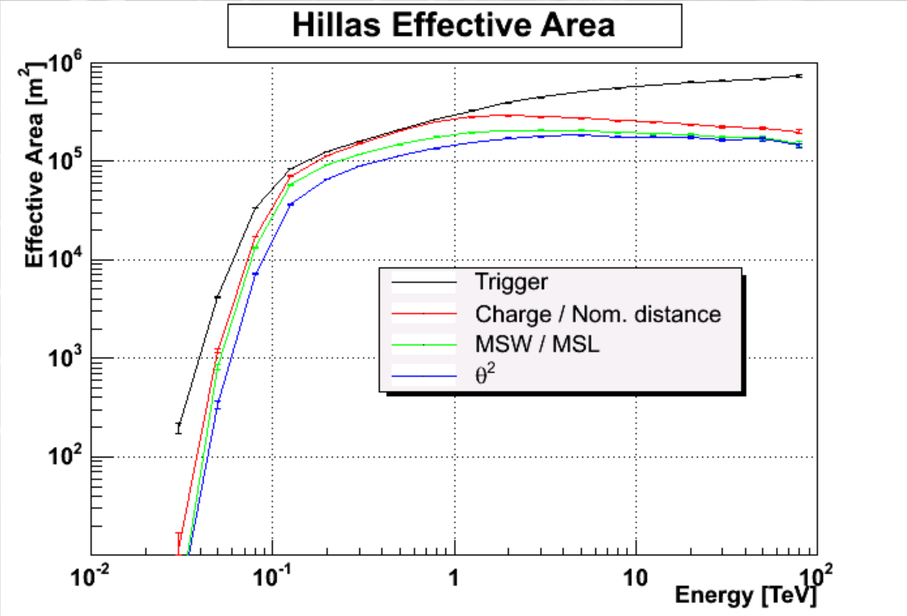
Efficiency

- *After 60 pe image cut and 2 degrees Nom. Distance cut*
- *Quality Factor of MSS > 4 at maximum, with 90% γ efficiency and > 95% hadron rejection (without direction selection)*

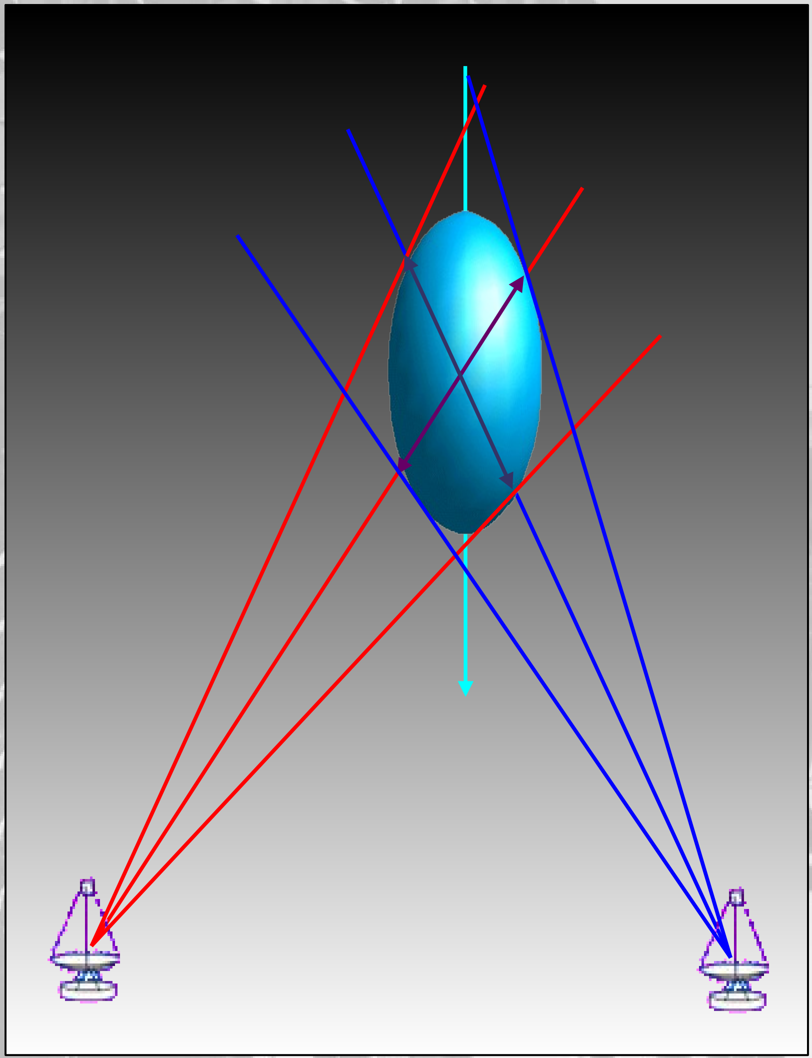
$$Q = \frac{\varepsilon_{\gamma}}{\sqrt{\varepsilon_{hadrons}}}$$

- *Effective area limited at high energy by Nom. distance cut*
- *Energy resolution from 20% at 100 GeV to ~10% at high energy*





II - 3D Model Analysis



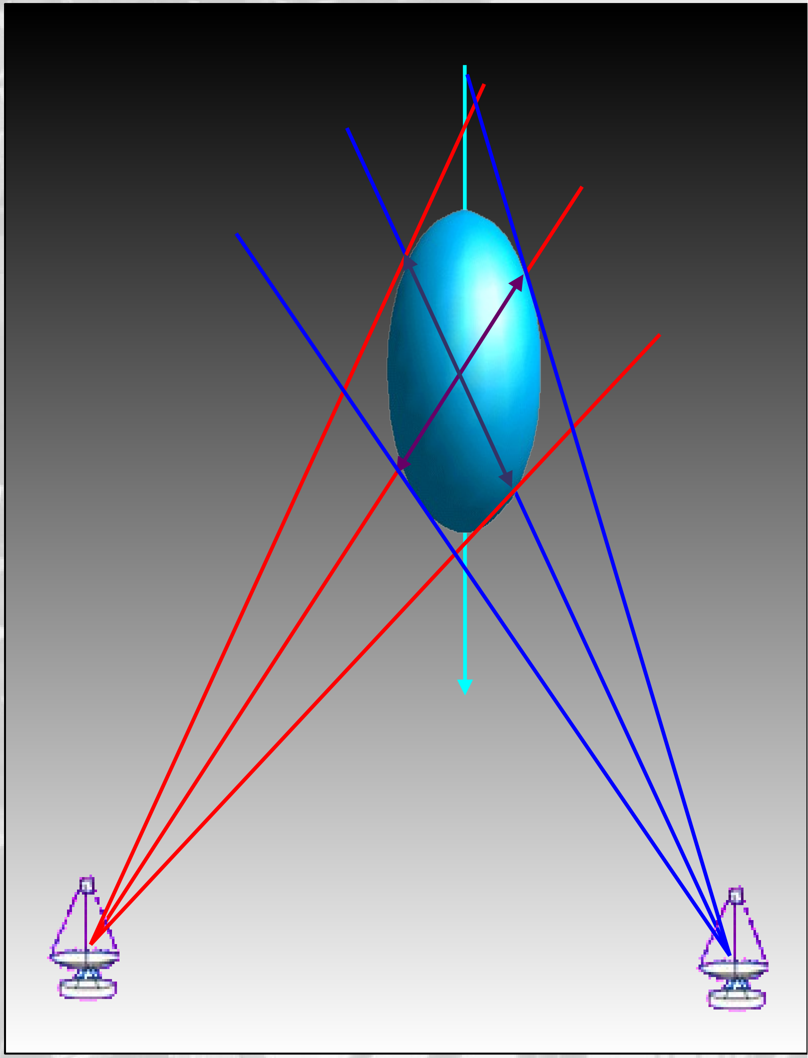
Lemoine-Goumard, Degrange, Tluczykont, Astropart Phy 25 (2005)

3D Model analysis

- *Idea: extend Hillas parameters taking into account correlations between images*
- *Shower modelled as a 3D Gaussian photosphere,* with anisotropic angular distribution
- Path integral along the line of sight gives collected light in each pixel
- Comparison of images with model using a log-likelihood

8 parameters fit

Altitude, impact, direction, width, length, light

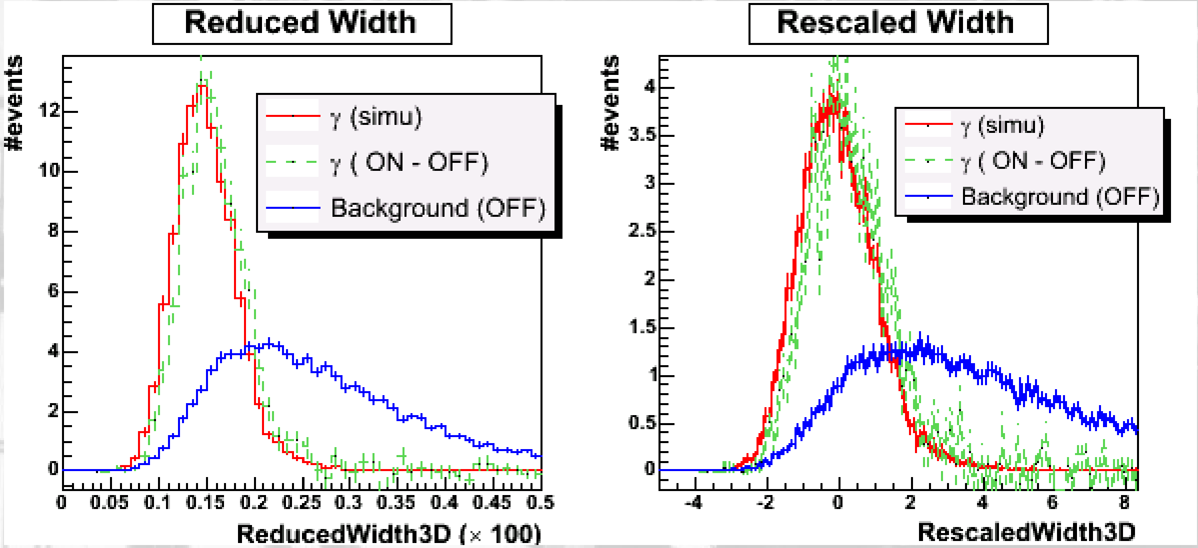


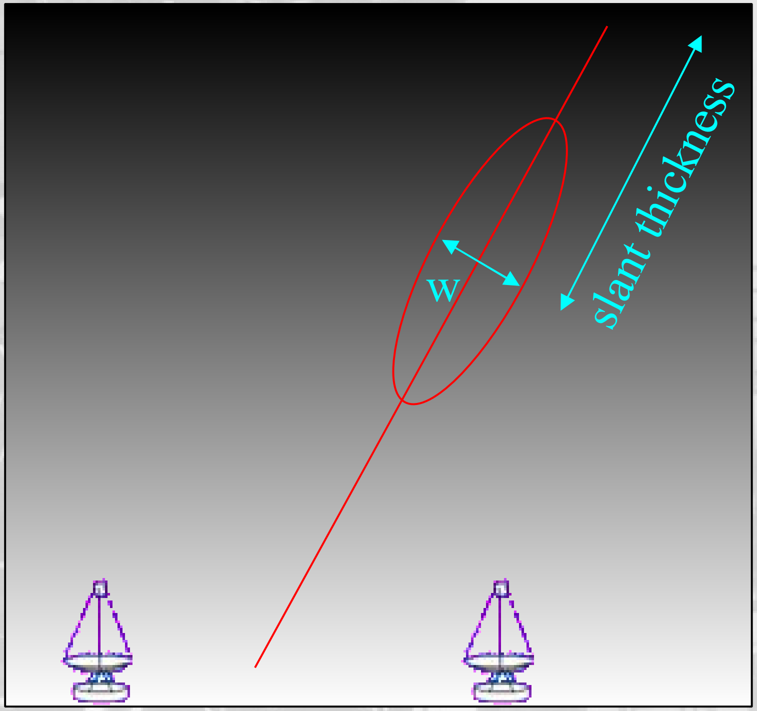
Selection parameters

- *Shower width (in unit of rad. length) found to be proportionnal to slant thickness*
- *Reduced width:*

$$\omega = \frac{w \times \rho\left(z_{max}\right)}{thickness}$$

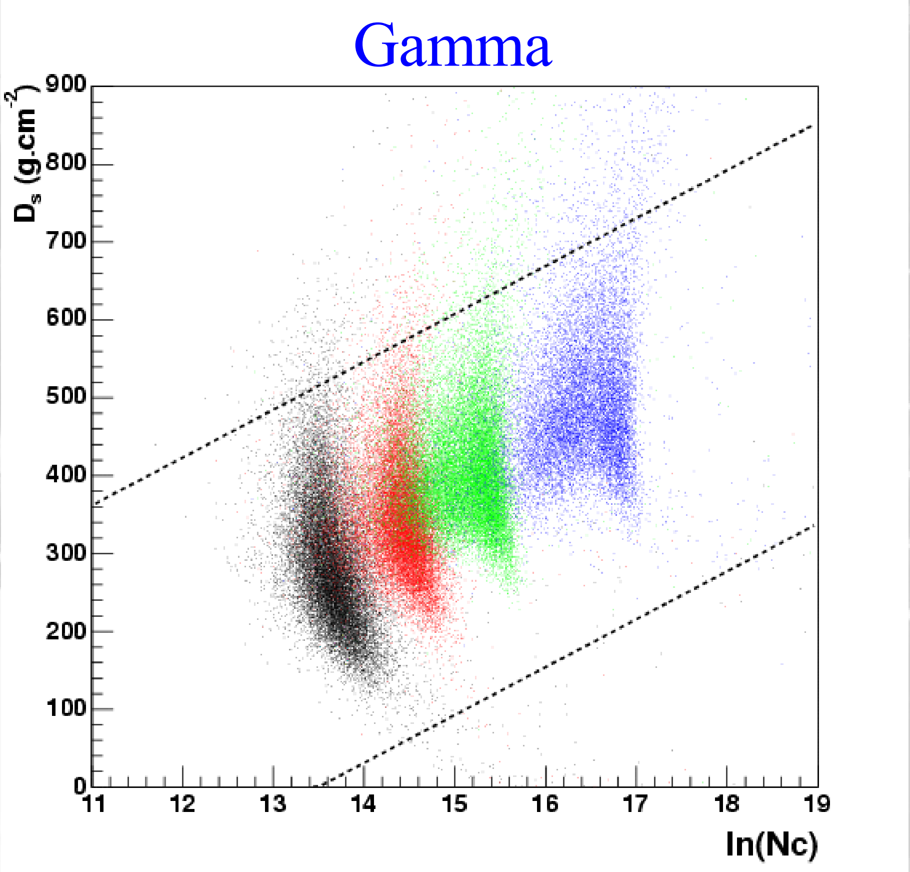
- *Rescaled width Wr_{3D} used here for simplicity (~Normal variable)*

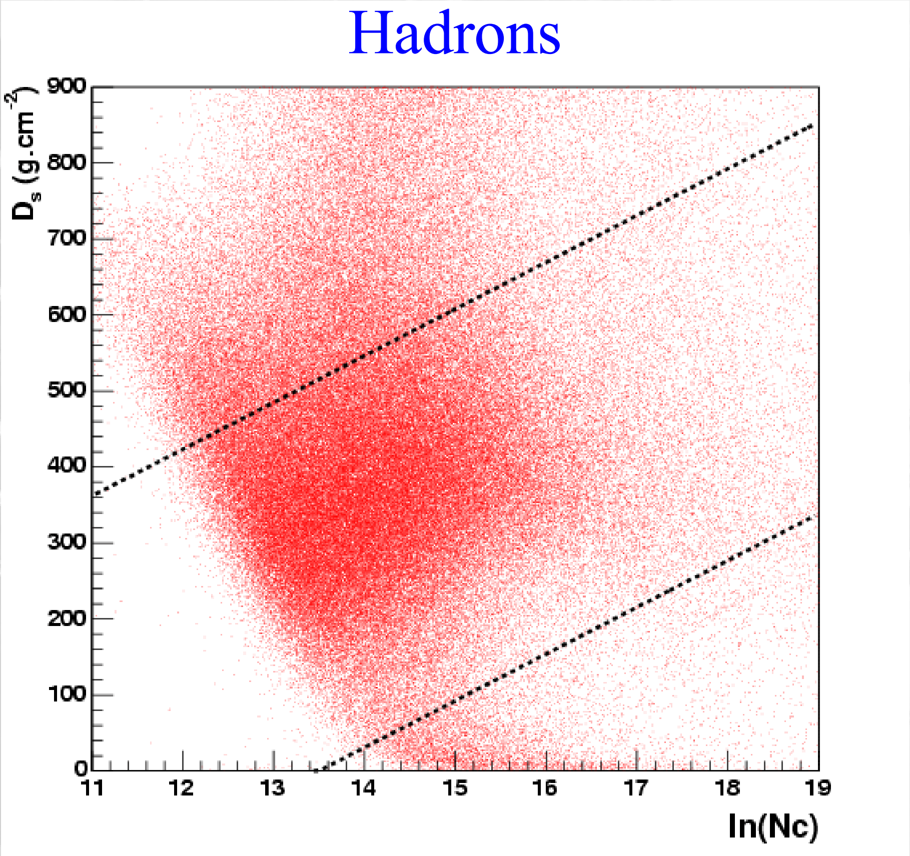




Photon yield

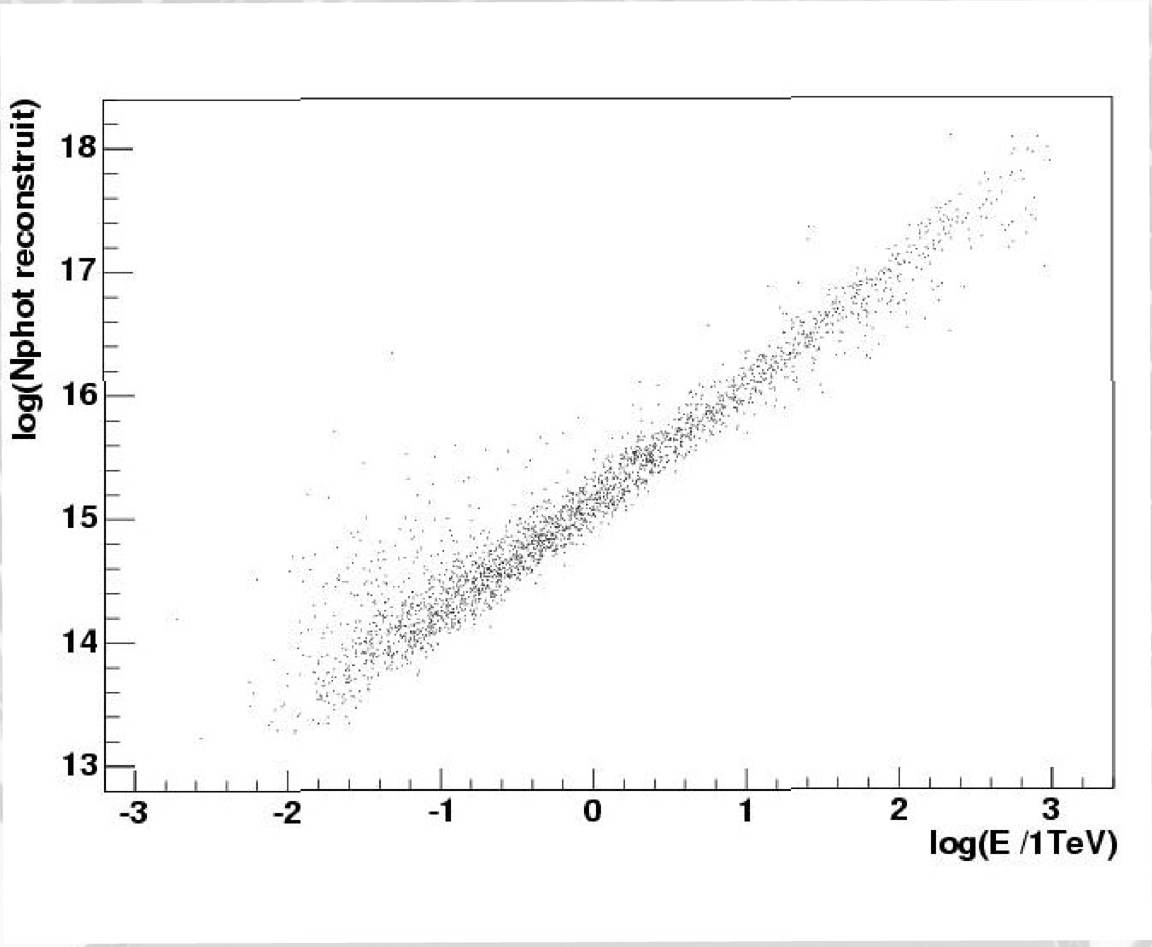
- *Number of Cerenkov photons versus slant thickness help discriminating γ's and hadrons (non physical region for γ's):*
  - *Deep showers ⇒ high energy ⇒ more photons*





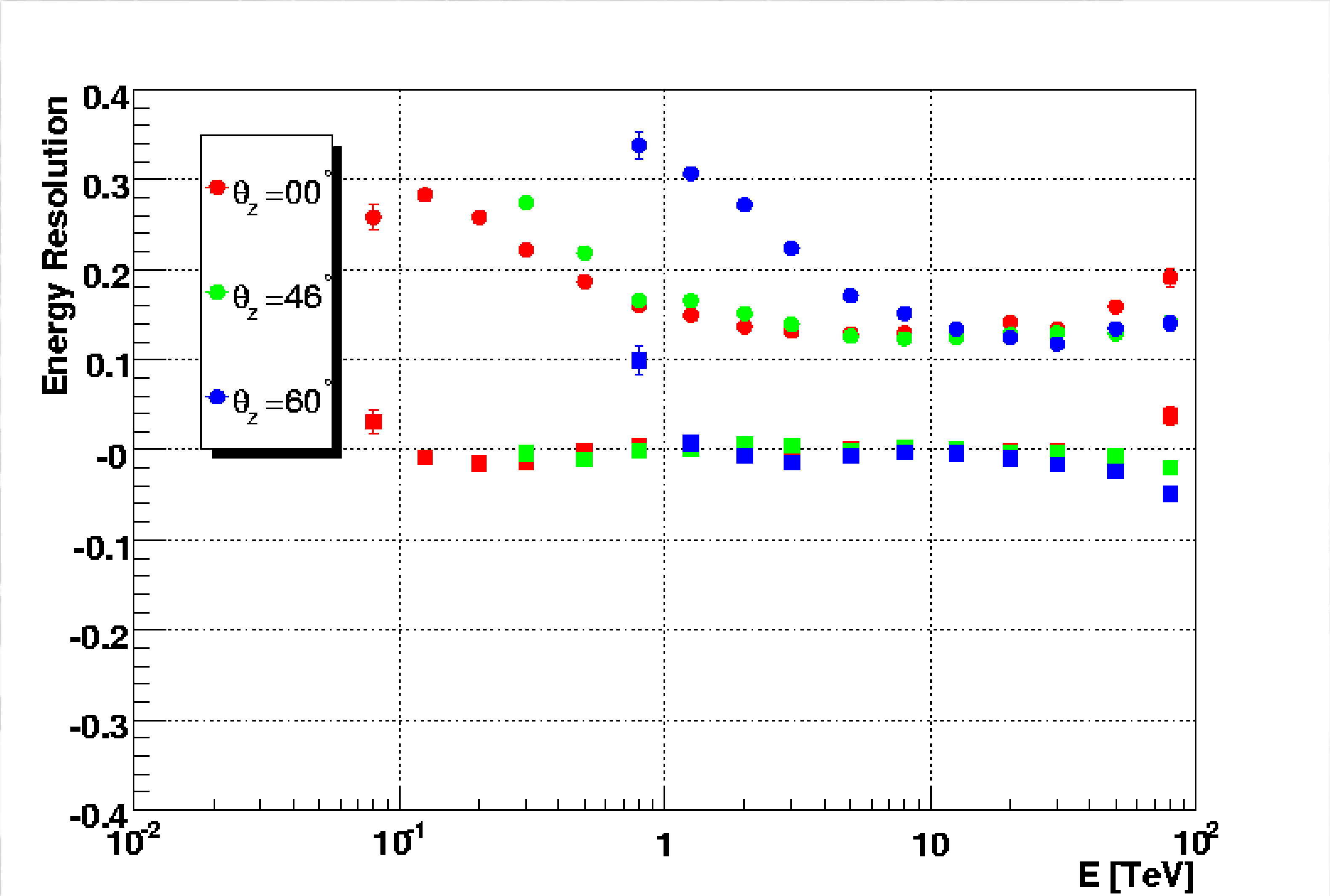
Energy Reconstruction

- *Fitted variable: Number of photons in the whole shower (Nphot)*
- *Calibration from simulation : ln(Eestimated) = a + b * ln(Nphot)*
  a and b depending on the zenith angle and on the number of telescopes



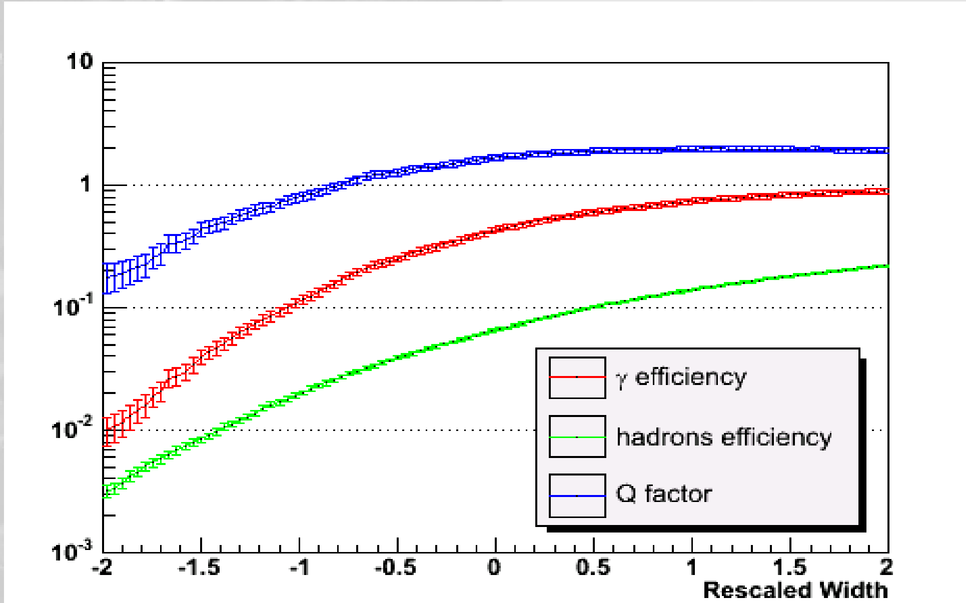
Energy Resolution

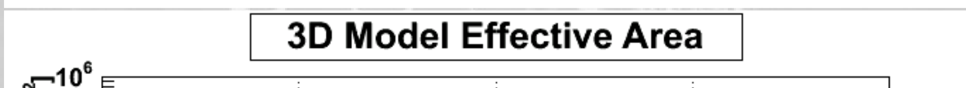
- *Good energy resolution at high energies*
  - *Energy resolution < 15% in [800 GeV – 50 TeV], almost not bias*
  - *But Resoltion > 20% for E < 300 GeV*

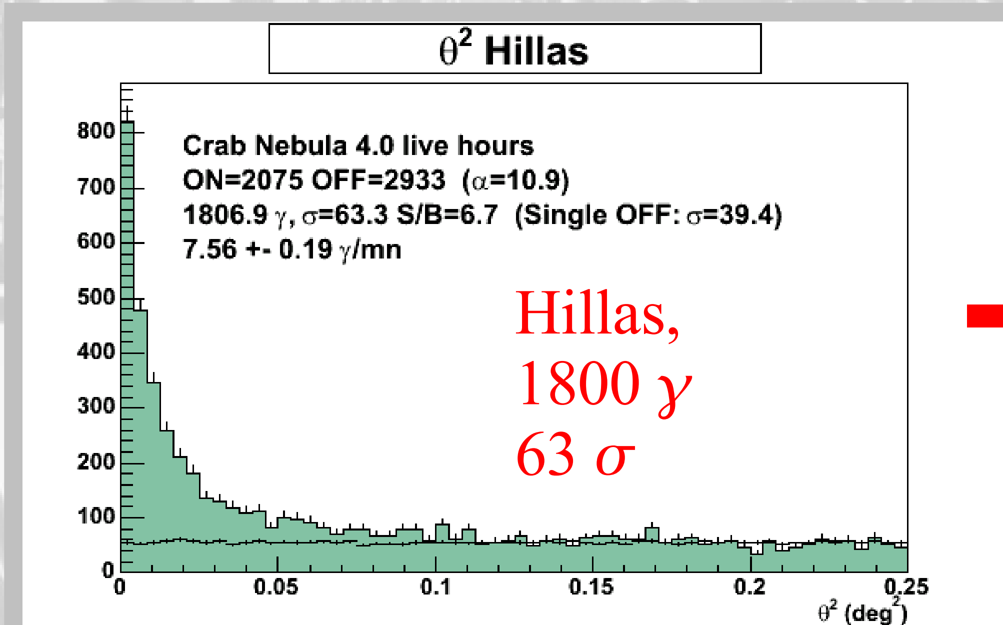


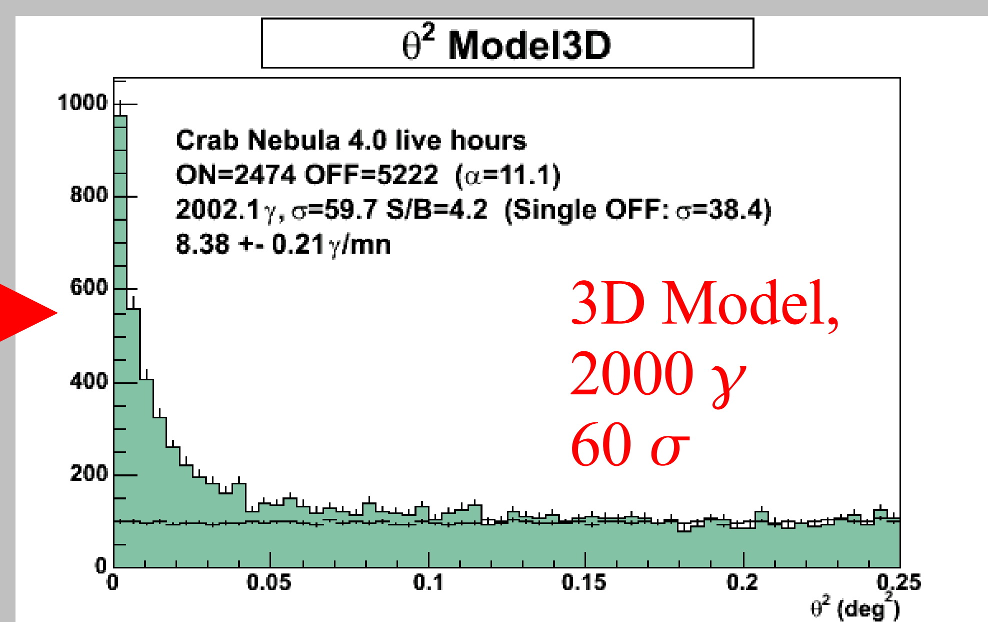
3D Model Performances

- *Q reaches 2 with 80% γ efficiency*
- *Fit convergence already rejects 80% of hadrons with 90% γ efficiency (Q' = 2)*
  ⇨ Global Quality Factor ~ 4
- *Effective area similar to those of Hillas analysis*

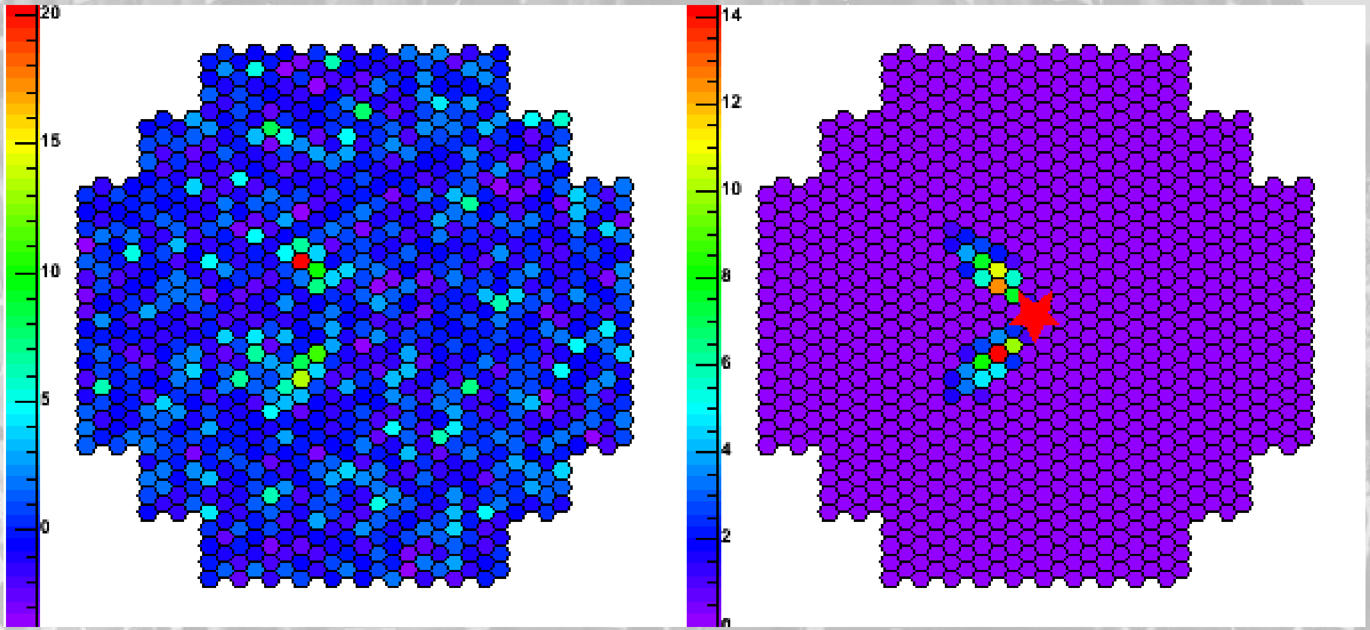


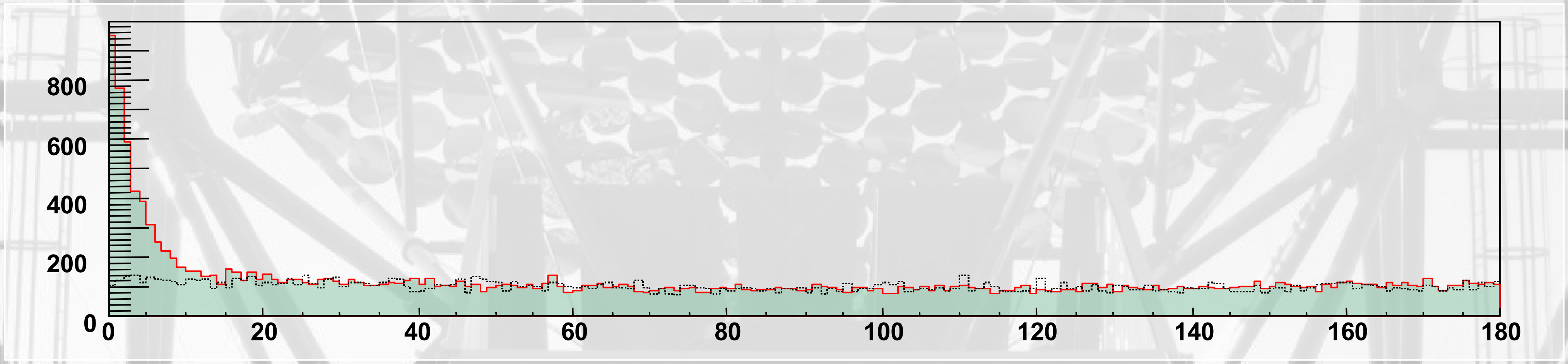






III - Model Analysis

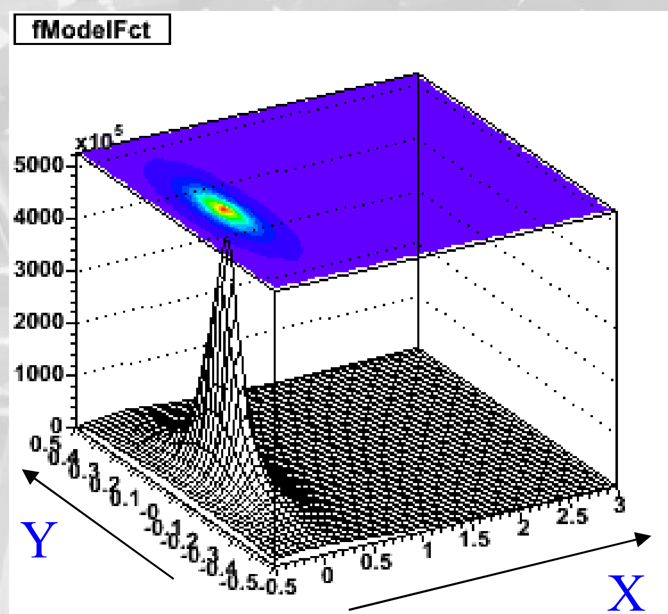


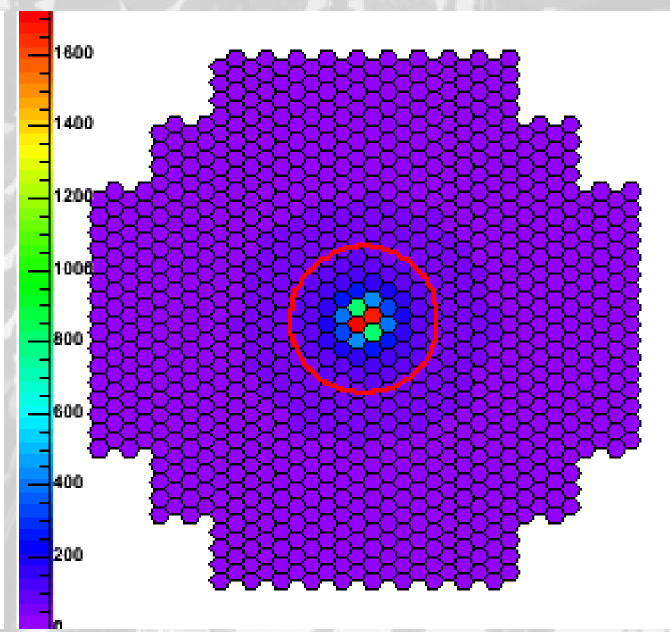


Principles & History

- *Initiated by S. Le Bohec for CAT (1996) and further developed*
  - *Use a model to describe the shower images in the camera as function of shower parameters (Energy, impact, ...)*
  - *Fit the actual raw images (no cleaning) to the model (Log-likelihood fit)*
- *Model can be generated from simulations, or from a dedicated semi-analytical code (long procedure: ~ 500 day × machine)*

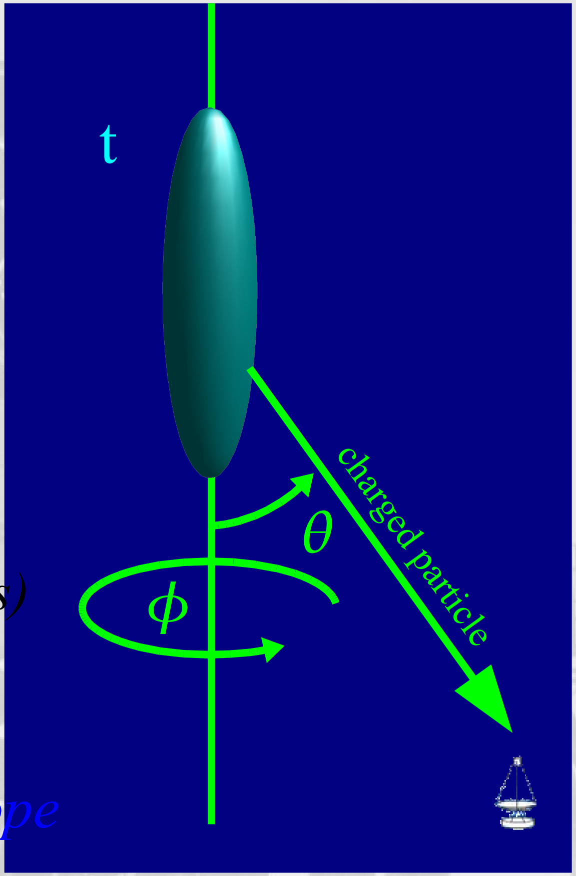
Average shower image for each
- zenith angle
- energy
- impact distance
- primary interaction depth

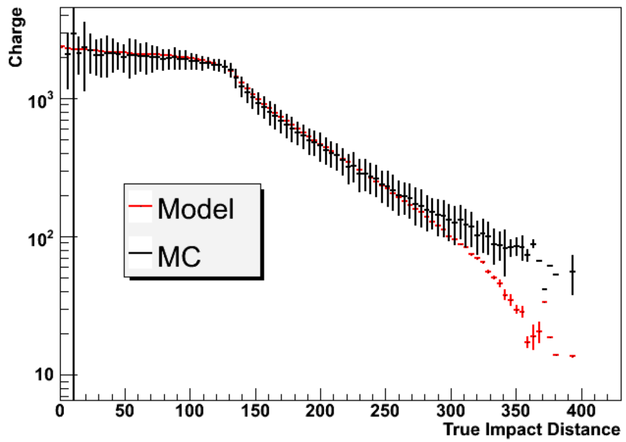


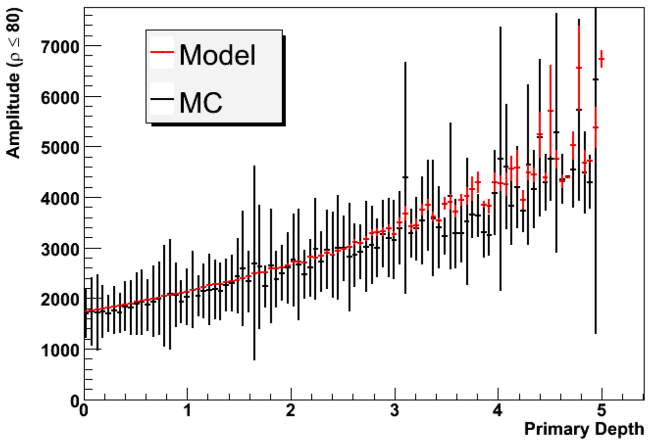


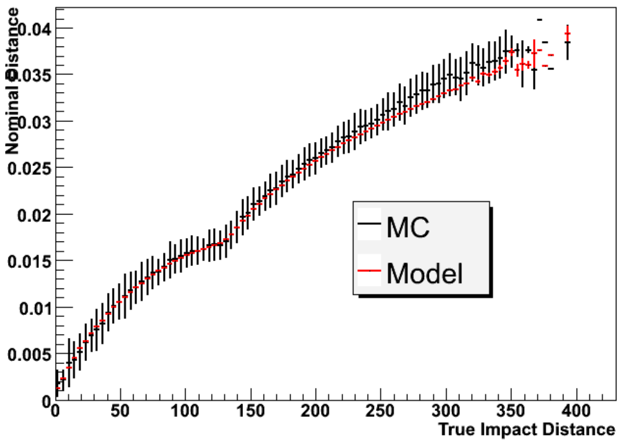
Model generation

- *For each energy (40 bins 50 GeV -> 20 TeV)*
- *For each impact (40 bins 0 -> 400 m)*
- *For each zenith angle (30 values)*
- *For each primary interaction (6 values)*
  - *Integral over shower depth (40 steps)*
  - *Integral over energy distribution (20 steps)*
  - *Integral over angular distribution (10 steps)*
  - *Integral over lateral (x & y) distributions (10× 10 steps)*
  - *Integral over azimutal angle (10 steps)*
- *Cerenkov distribution ⇒ light distribution*
- *Each time limited to parameter space seen by telescope*
- *Base on a paper from M. Hillas (1982)*









Pixel Amplitude PDF

- *Log-likelihood of model to the raw images*
- *Complete analytical expression of pixel PDF (Not gaussian) (convolution of Poisson distribution with increasing size gaussians)*

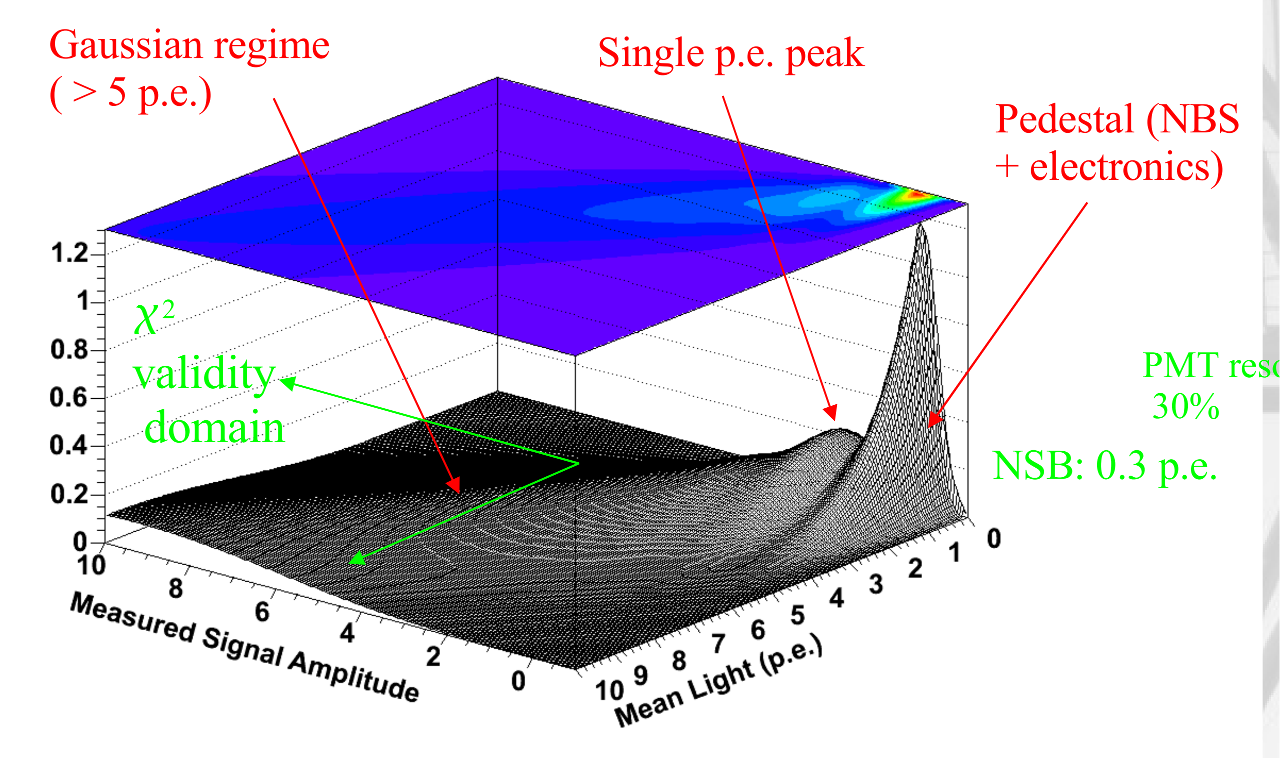
Prob. of observing x when model predicts μ (p.e.)

$$P_{df}(x,\mu,\sigma_p)=\sum_n \frac{\mu^n e^{-\mu}}{n!\sqrt{2\pi\left(\sigma_p^2+n^2\sigma_\gamma^2\right)}}\exp\left(\frac{-(x-n)^2}{2\left(\sigma_p^2+n^2\sigma_\gamma^2\right)}\right)$$

$$\sigma_p = Pedestal\ width\,(NSB+electronic\ noise)$$

$$\sigma_\gamma = PMT\ resolution$$

- *Sensitive to very small signals (shower tails)*
- *Strongly depends on actual NSB level in each pixel*



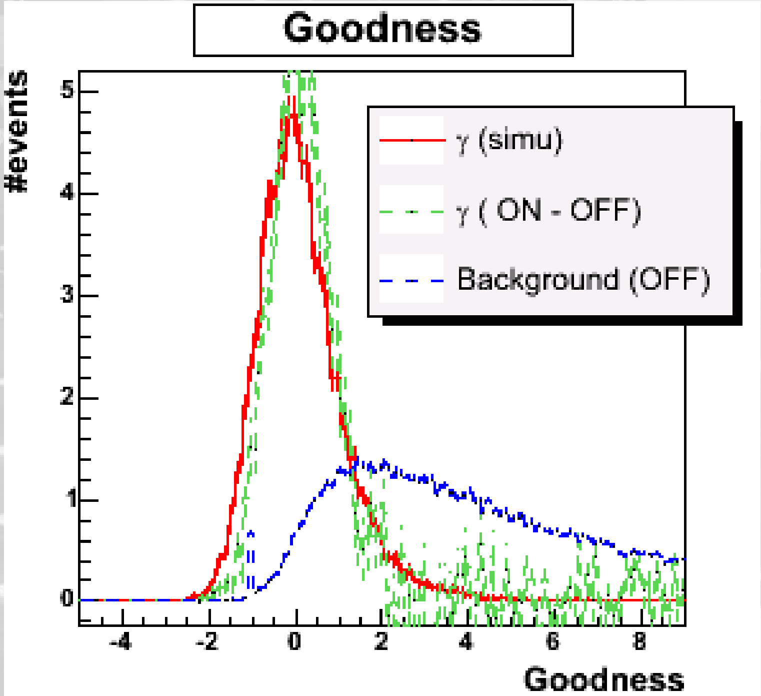
Goodness-of-fit

- *Not a likelihood ratio (no alternate hypothesis to compare with)*
- *Analytical calculation of likelihood value expectation*

$$\langle \ln L \rangle = \int_x \ln\left(P_{df}\left(x,\mu,\sigma_p\right)\right) \times P_{df}\left(x,\mu,\sigma_p\right) \times dx$$

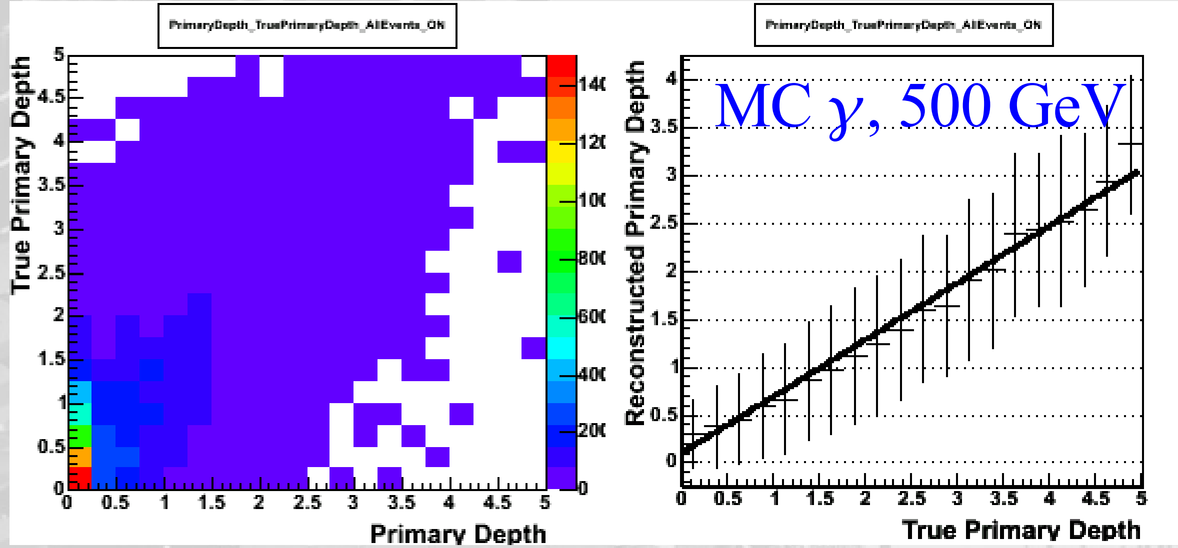
- *RMS ~ √2 per degree of freedom*
- *Goodness-of-fit*

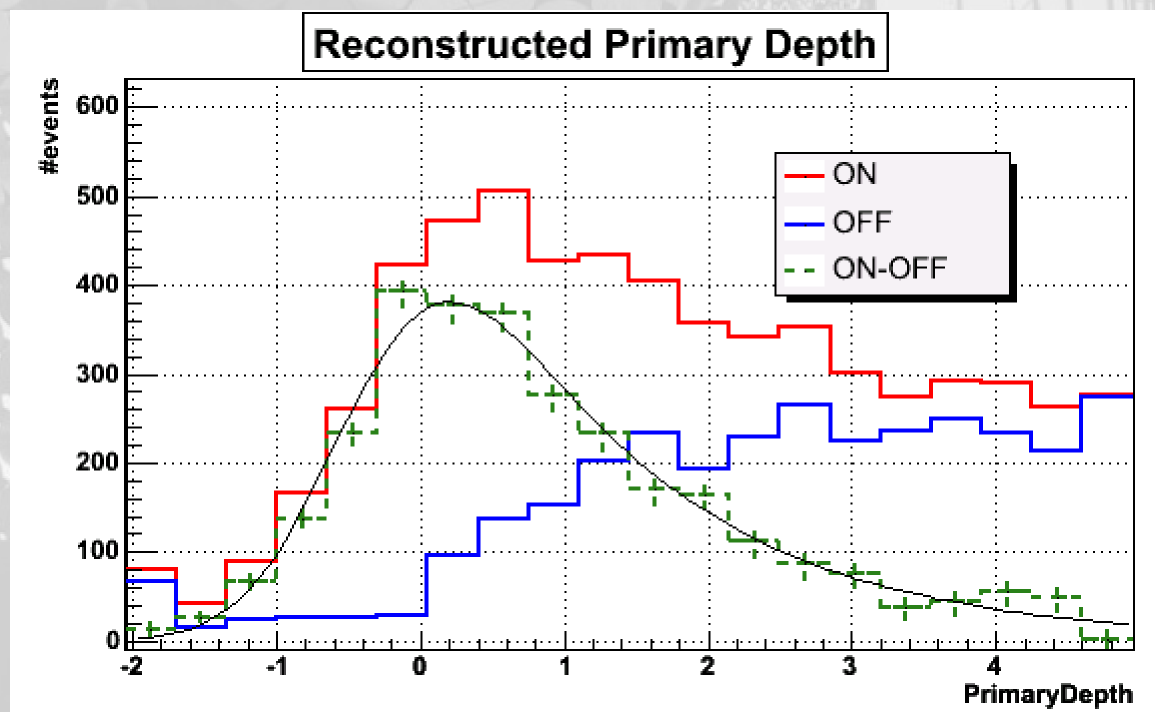
$$g = -\left(\frac{\ln L - \langle \ln L \rangle}{\sqrt{2 \times NDof}}\right)$$



Primary interaction reconstruction

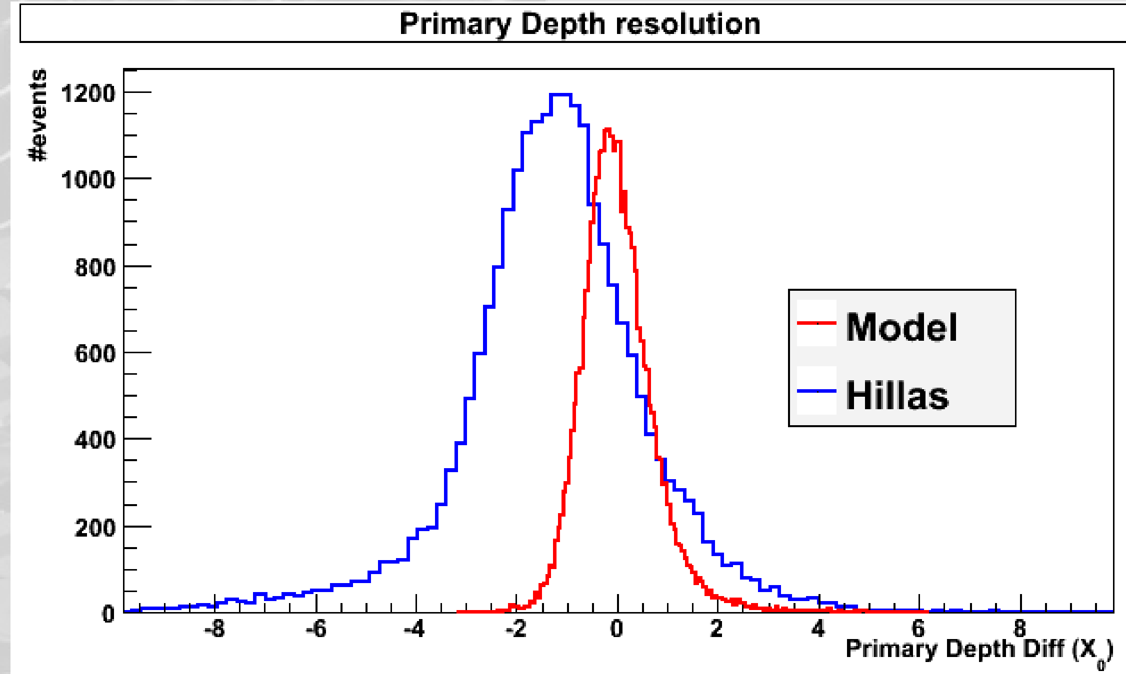
- *Reconstructs primary interaction depth with ~0.5 X_0 resolution*
- *Real data (Crab) compatible with exponential distribution (slope 0.7 ± 0.05) convoluted with gaussian resolution (σ = 0.6) + 0.6 X_0 bias*
- *Expected slope : 7/9 = 0.77*
- *Improves energy resolution*
- *Additional rejection power*



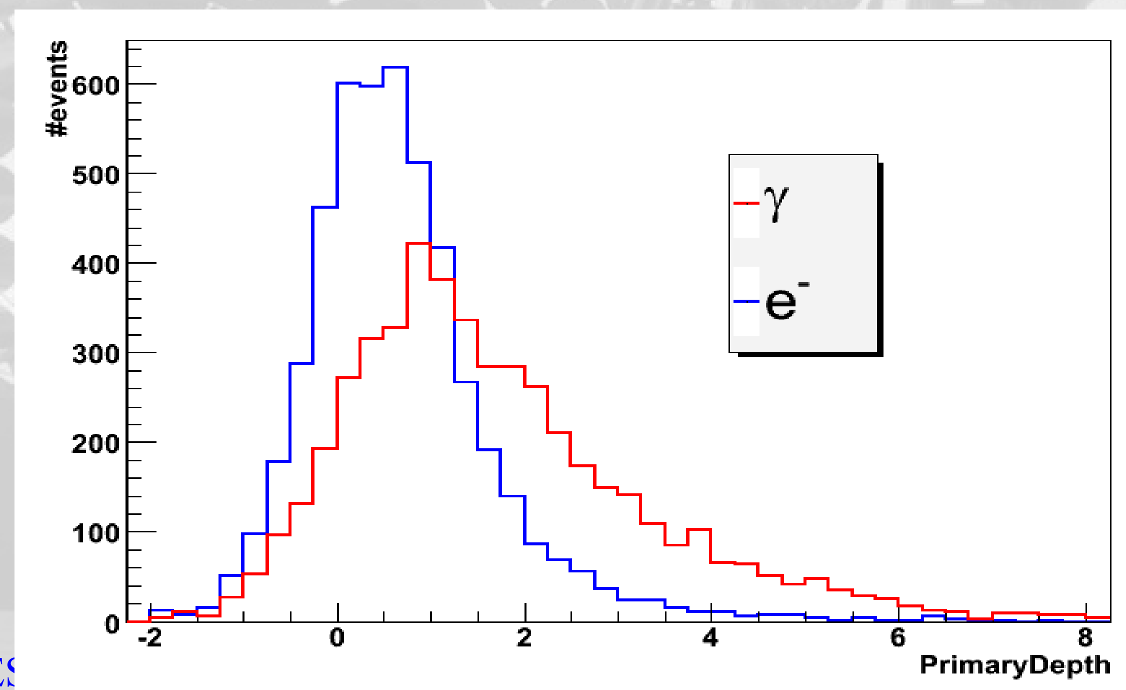


Primary Interaction Depth

- *Model resolution (0.6 X_0) much better than Hillas using shower maximum (~ 1.4 X_0)*

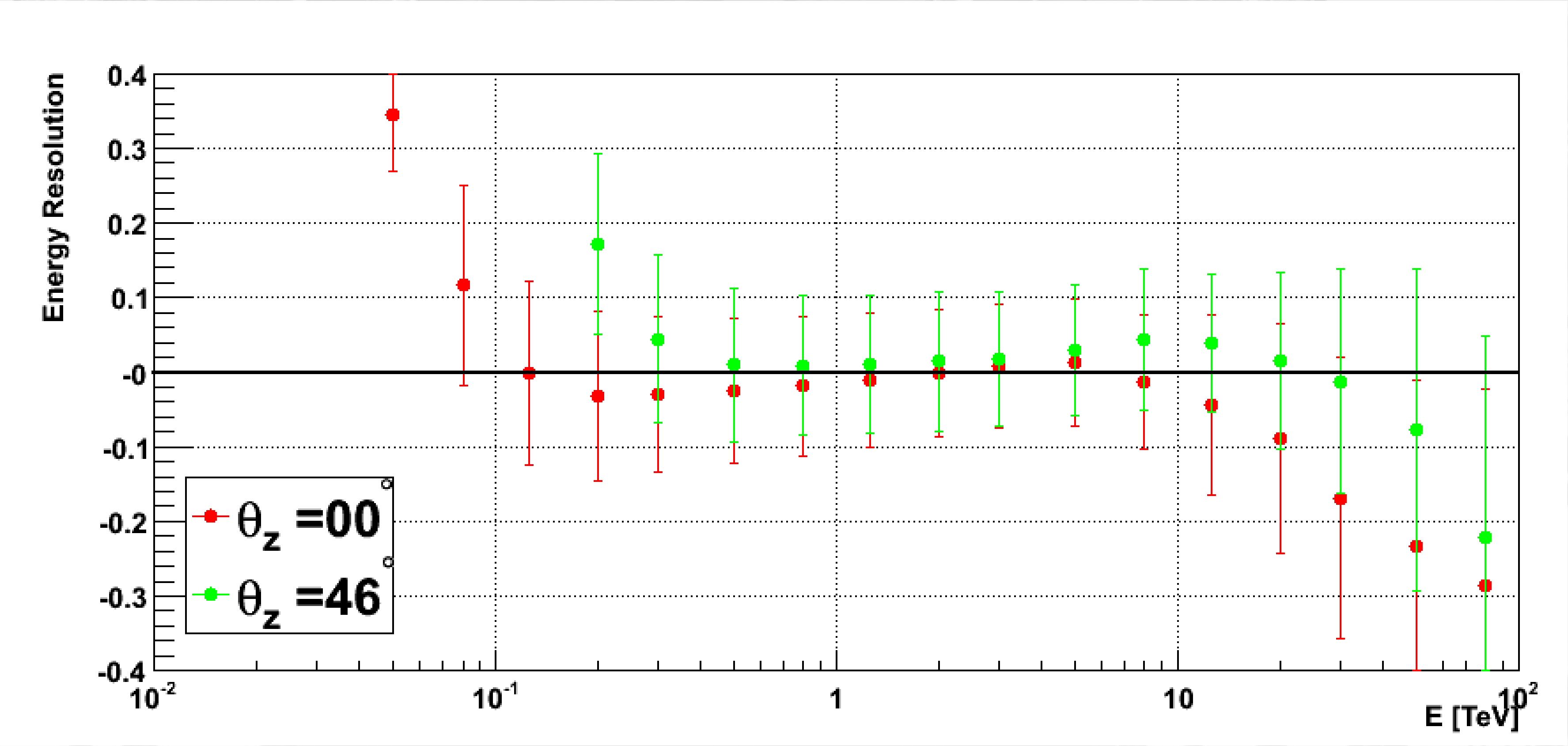


- *Gives some e^-/γ separation capabilities*
  - *e^- spectrum*



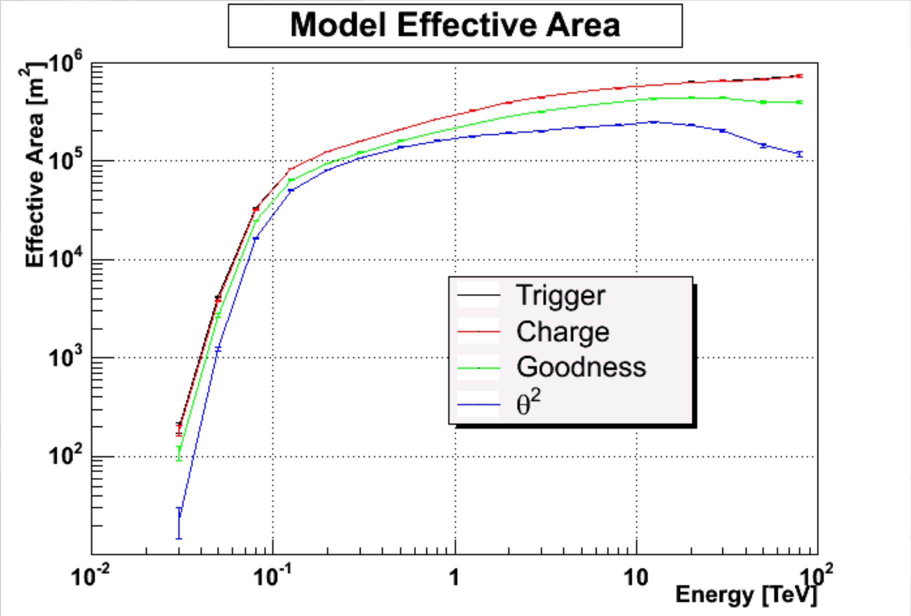
Energy resolution

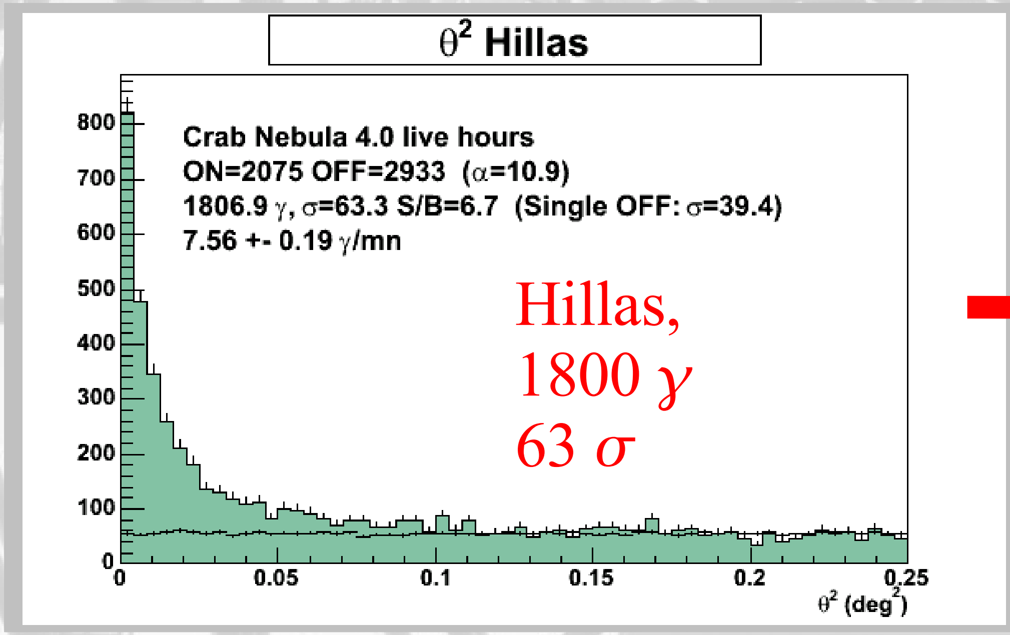
- *Very good energy resolution in [80 GeV – 20 TeV]*
  - *15% energy resolution @ 80 GeV, 8% @ 2 TeV*
  - *Bias < 3% in [100 GeV – 10 TeV], 10% @ 80 GeV and 20 TeV*
  - *Not very good at very high energies (very distant showers)*

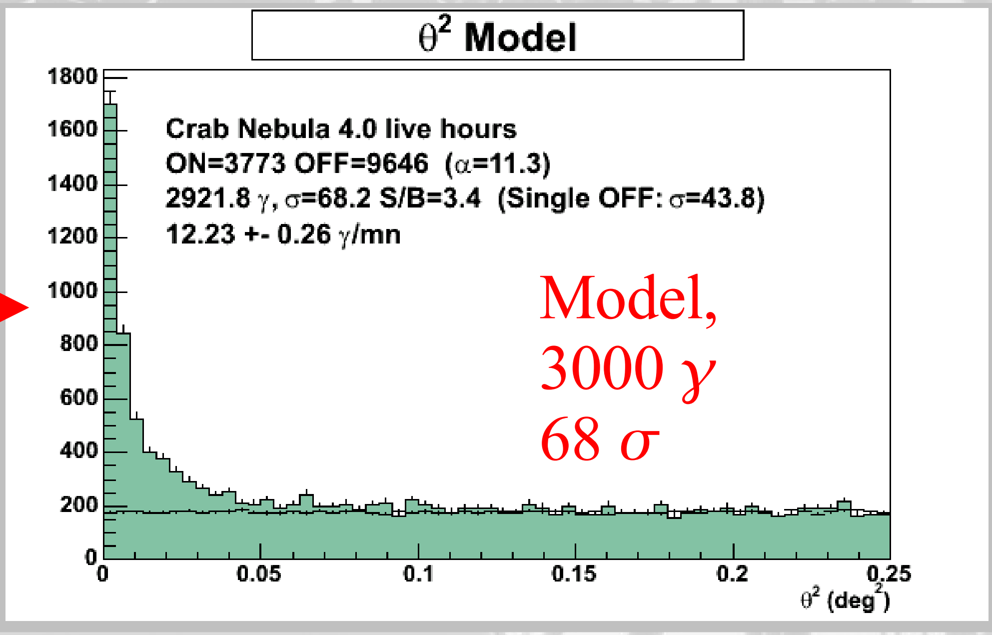


Performances

- *Lower Q factor (~3) but higher reconstruction efficiency (~ 100% of triggered events)*
- *Similar or better significances*
- *Better efficiency at low energy (30% @ 50 GeV, 50% @ 80 GeV)*
- *Provides uncertainties on each parameter*





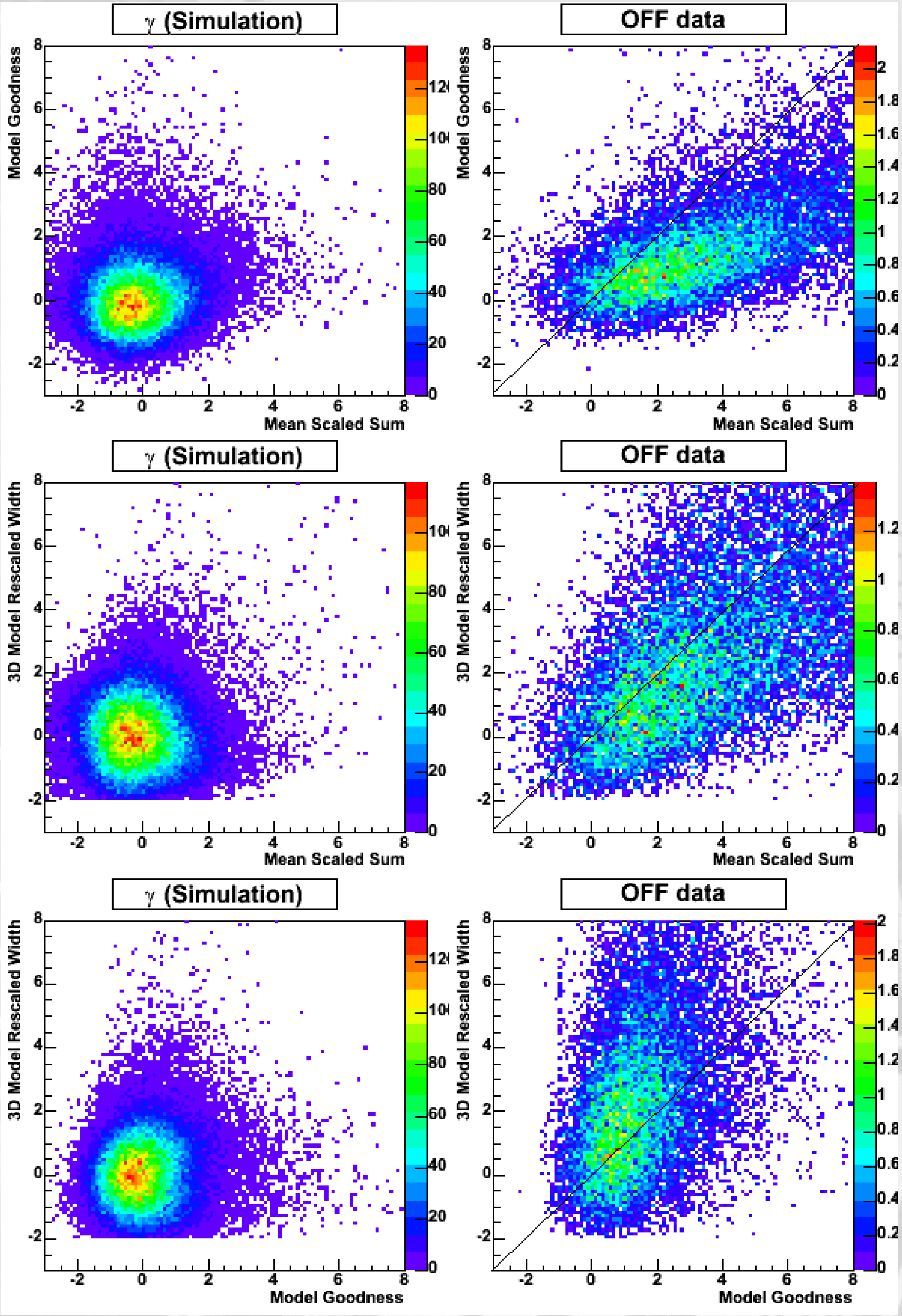


Some comparison elements

- Methodology:
  - Correlate selection variables on same events
  - Use a common base of events to compare angular and energy resolutions
  - Actual number (resolutions,...) not very meaningful:
    - Depends on instrument
    - Depends on selection (Charge cut, Nominal distance,....)
    - Analyses still improving

Selection variables

- *Simulated γ:*
  Almost NO Correlation between
  - *Mean Scaled Sum &*
    Model Goodness
  - *Mean Scaled Sum &*
    3D Model Reduced Width
  - *Model Goodness &*
    3D Model Reduced Width
- *Hadrons (OFF events) more correlated*
- *Analyses sensitive to different shower properties:*
  - *Hillas: most frequent showers, incl. shower fluctuations. No correlation between images*
  - *Model / Model3D : average shower but correlations between images, …*



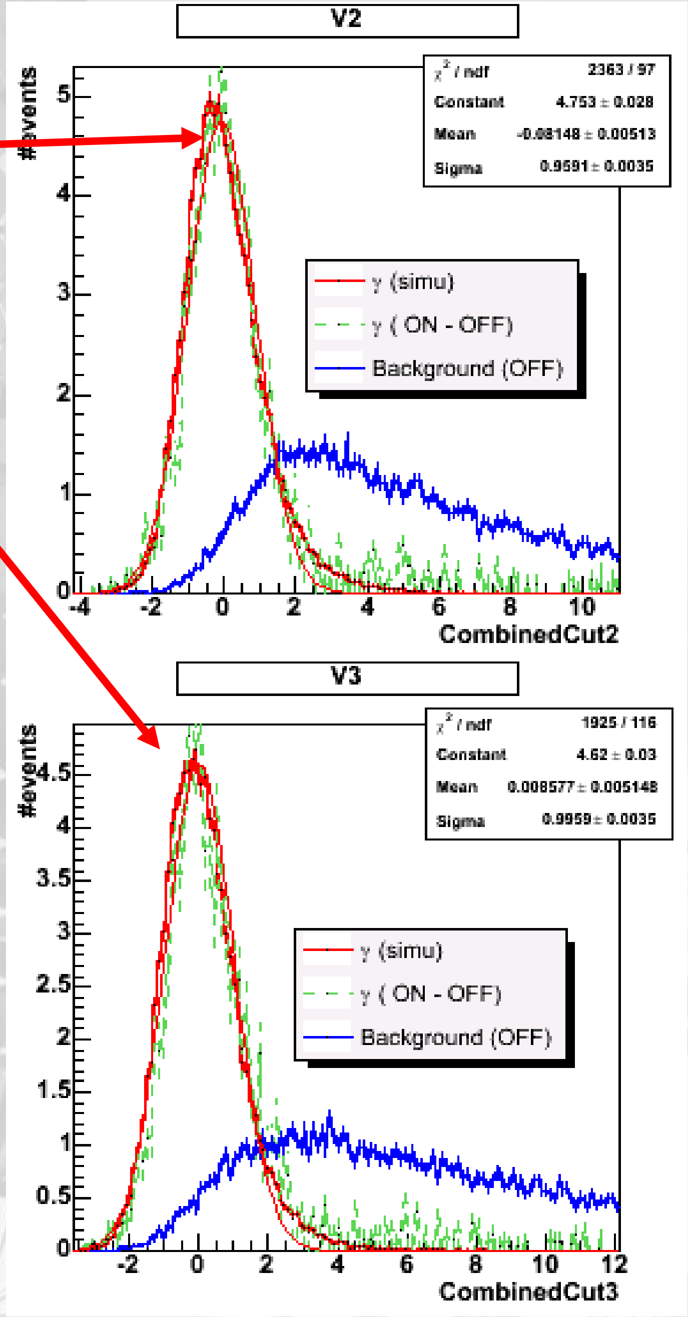
Analyses Combination

- *Combine independent selection variables:*

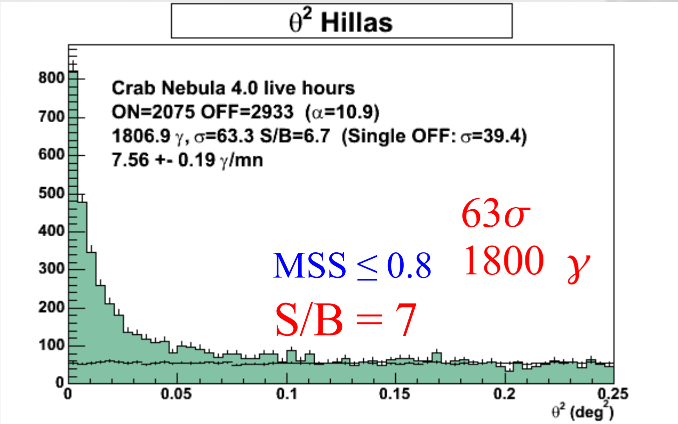
$$V2=\frac{MSS+G}{\sqrt{2}},V3=\frac{MSS+G+Wr_{3D}}{\sqrt{3}}$$

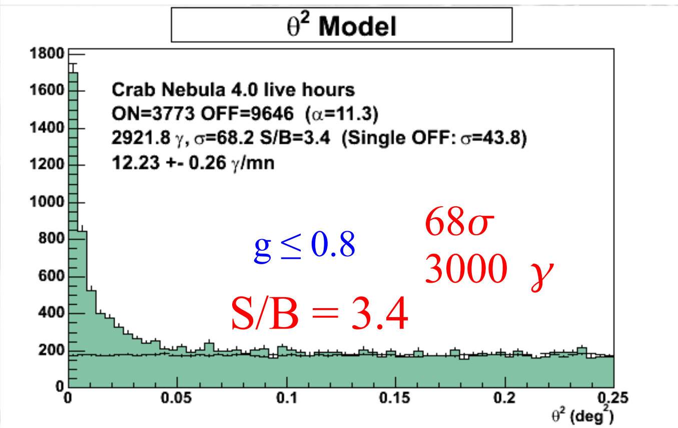
⇨ Still a normal variable

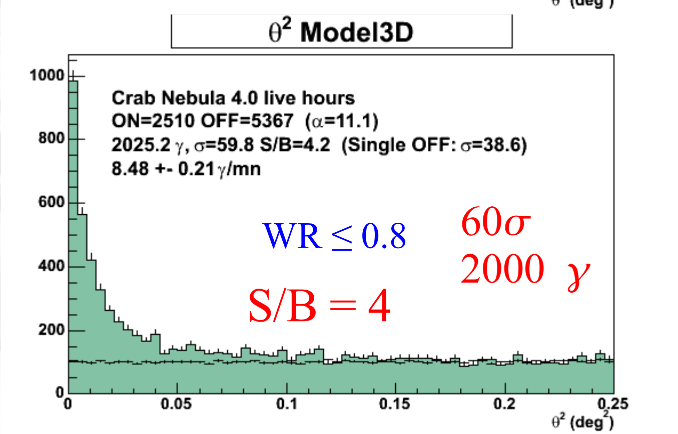
- *Still well reproduced by simulation*
- *Increased rejection capabilities*

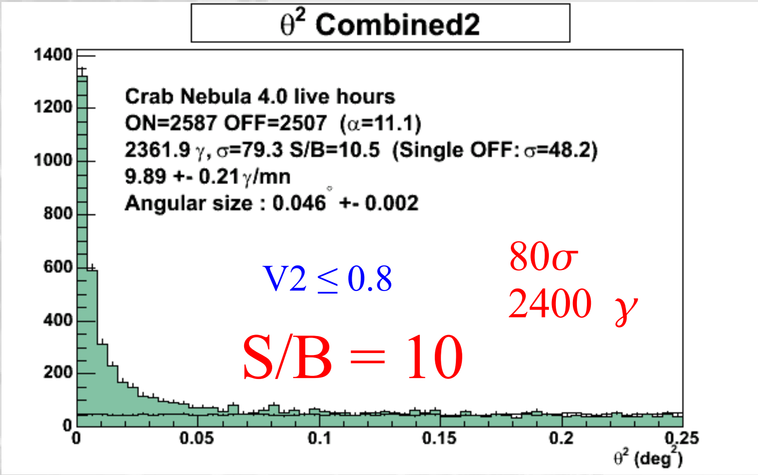


Combination example

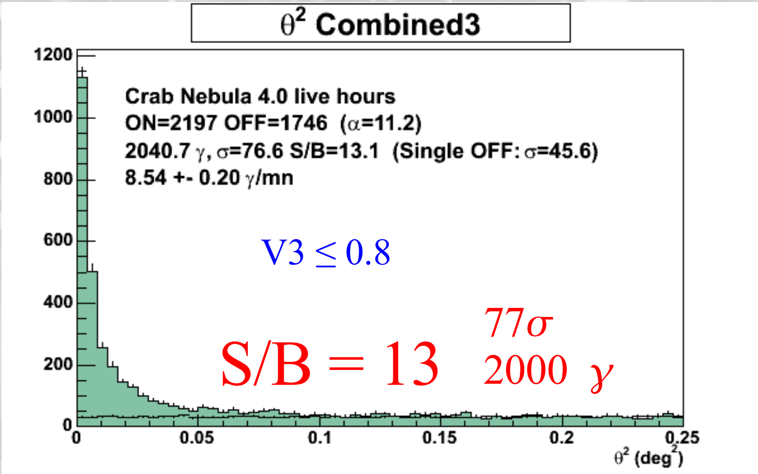






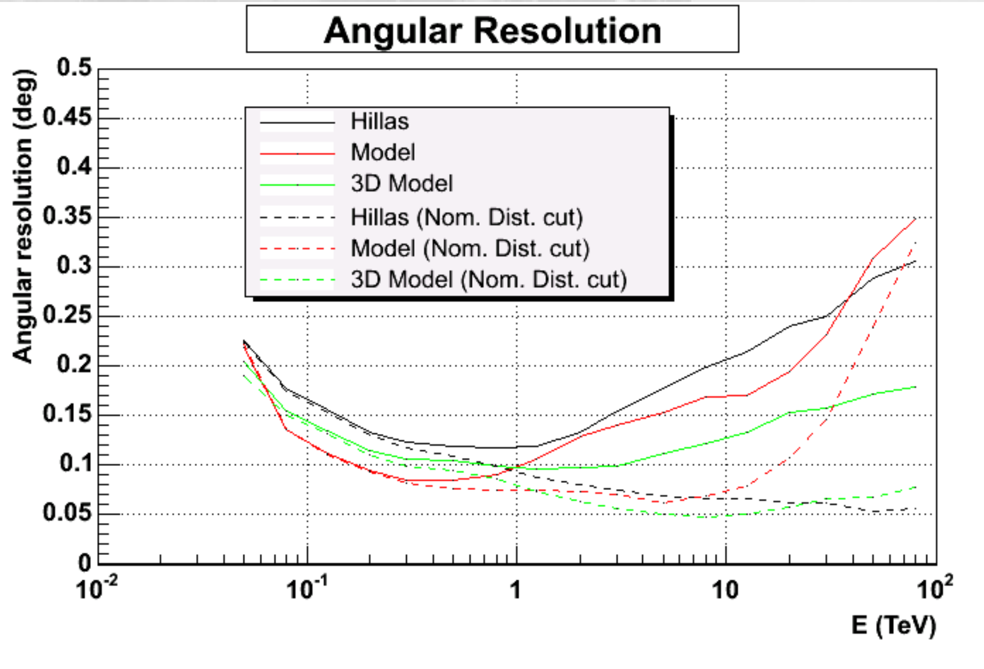


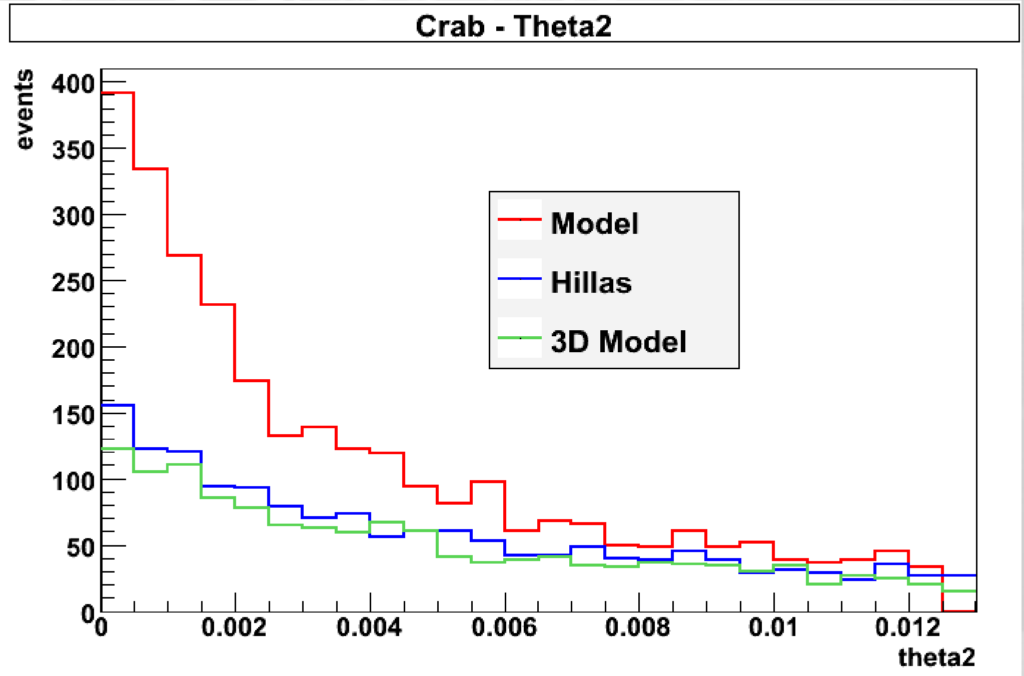
- *Better significance, better S/B*
- *Combines efficiency of Model with rejection power of Hillas*
- *+40% sensitivity for faint sources*



Angular resolution

- *Comparison of angular resolutions (R68) in V3 events (without Nom. distance cut):*
  Model best at low energy, 3D Model best at high energy
- *Nom. distance cut applied:*
  - *Big improvement for Hillas/3D Model at high energy*
  - *Model: some problems at E > 10 TeV (current limit of the generated models)*

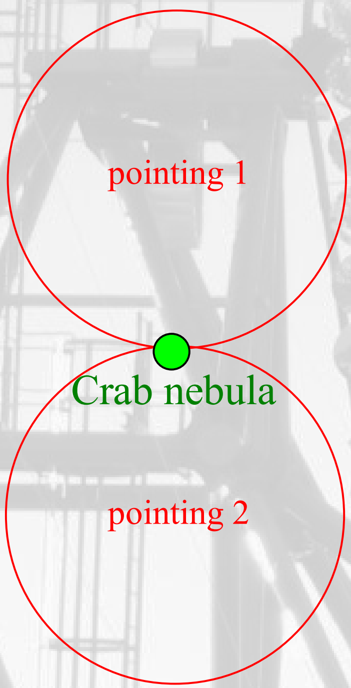




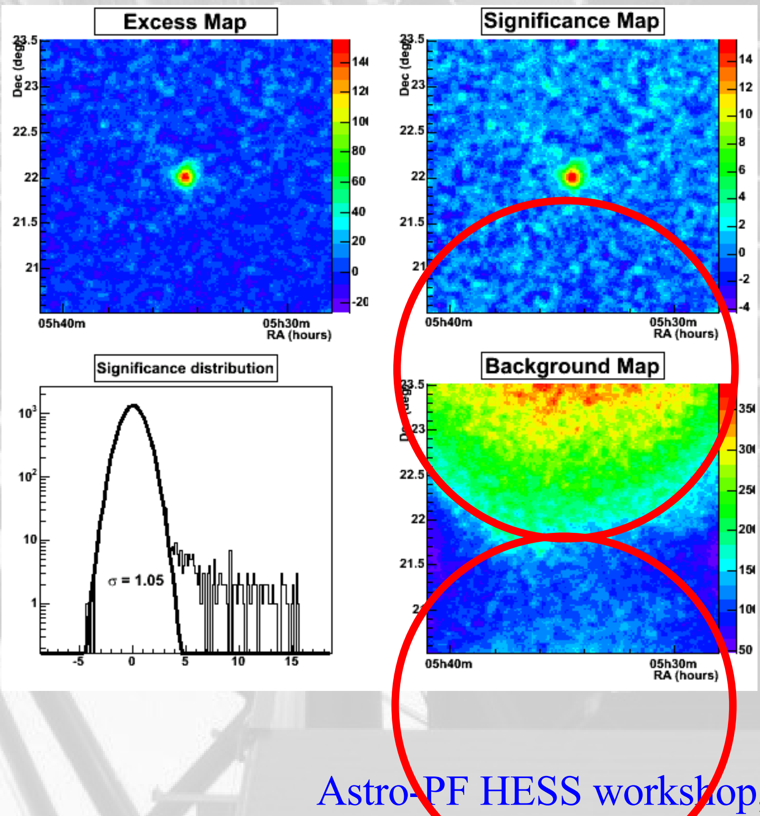
- *Difference more stricking at large zenith angles and low charge cut:*
  - *Crabe (46°), 60 pe cut (same events)*

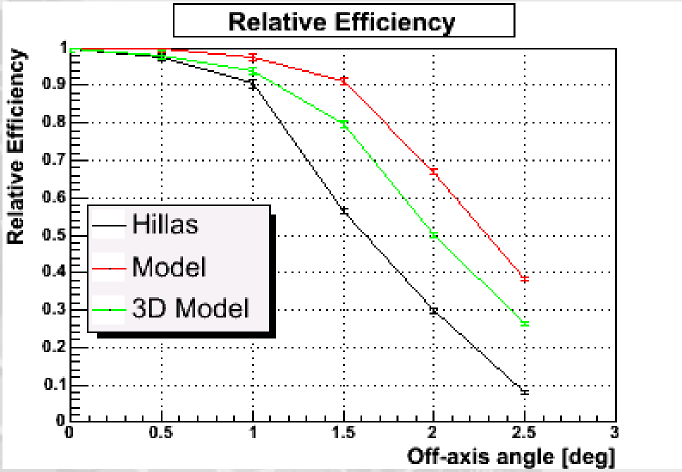
OFF-axis observations

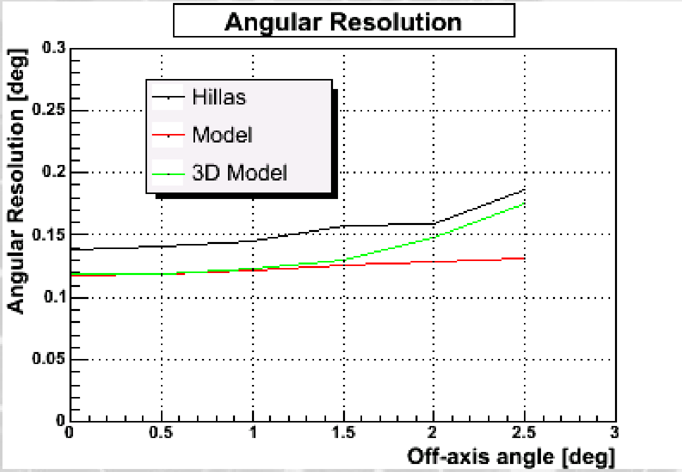
- *Nom. distance cut reduces Hillas efficiency*
- *Model analysis performs better off-axis (good for small FOV cameras)*

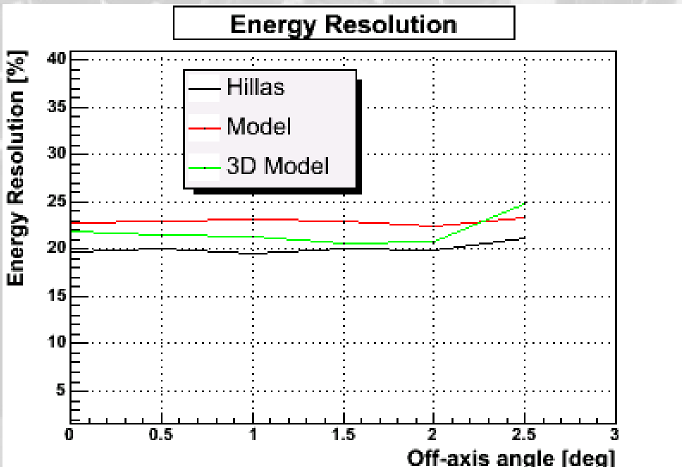


2.5 deg offset crab observation



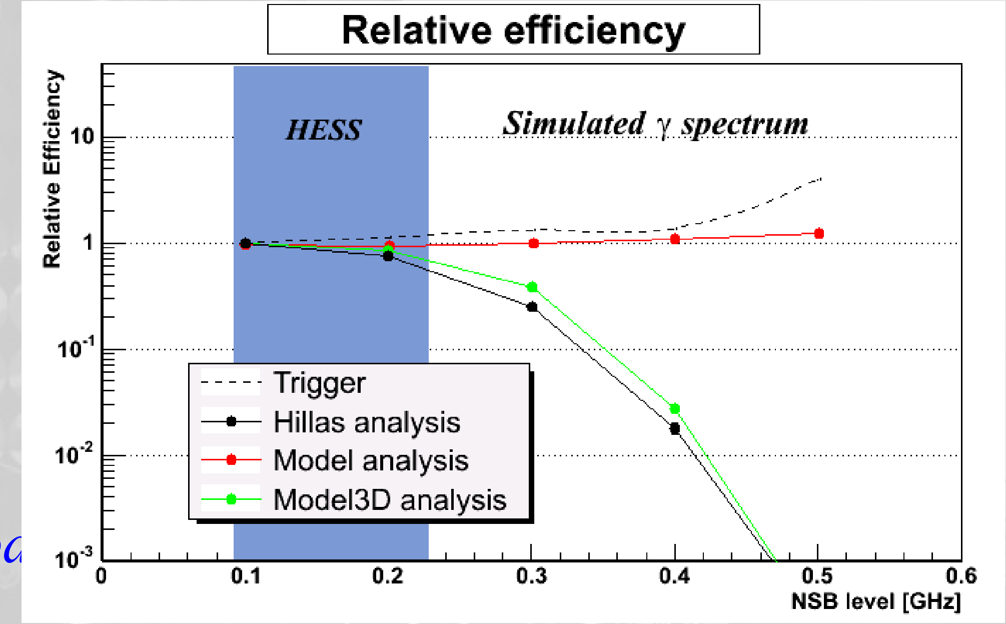


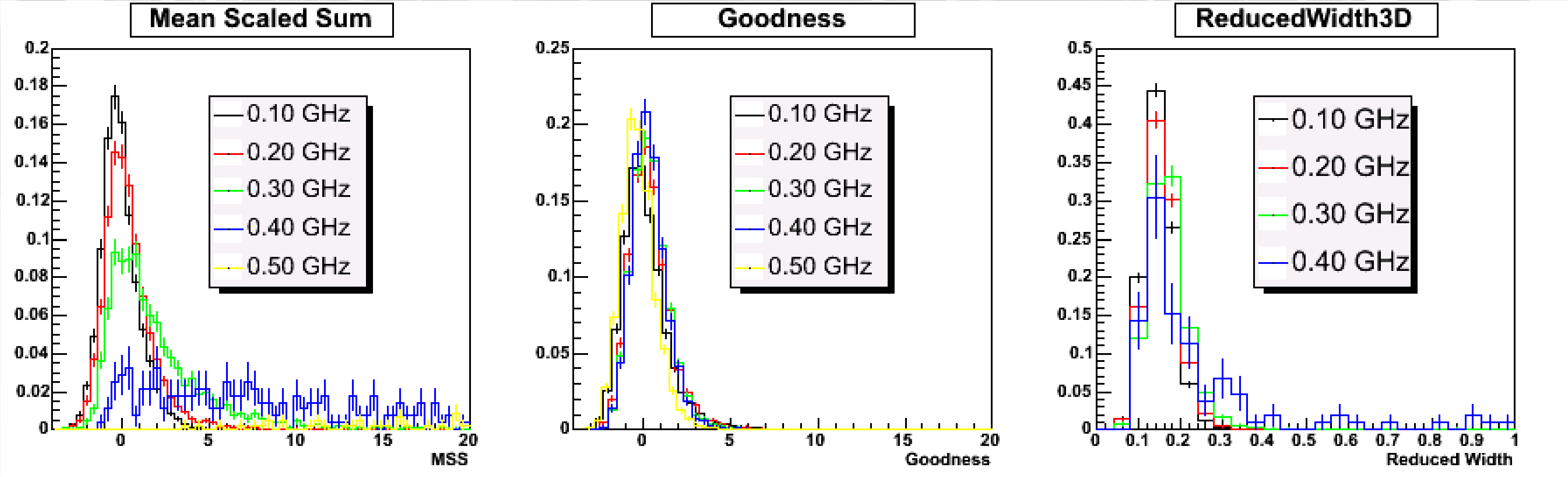




NSB Influence

- *Different γ efficiency vs NSB behaviour*
  - *Hillas: Scaled parameters degraded (break position depends on cleaning)*
  - *Model : goodness is stable*
  - *Model 3D : Fit convergence problem (can be solved?)*
- *Protons behave in a similar way (good*





Conclusions

- *3 different reconstruction with different properties:*
  - *Hillas / Scaled cuts : good rejection power, robust, stable*
  - *3D Model : Similar efficiency/rejection, better resolution at high energy*
  - *Model : higher efficiency (at low energy),*
    more robust to NSB variation, optimal use of camera FOV
    (⇨ good for small FOV camera, small energy studies)
- *Analyses are sensitive to different shower properties:*
  - *Hillas : Compatibility to most frequent shower (inc. fluctuations)*
  - *Model : Compatibility with an average shower (no fluctuation, but correlations)*
  - *3D Model : in between*
- *Timing information can be (easily) incorporated in Model / 3D Model*
- *Combination of analyses improves results*
  ⇨Definitive analysis still to be invented....

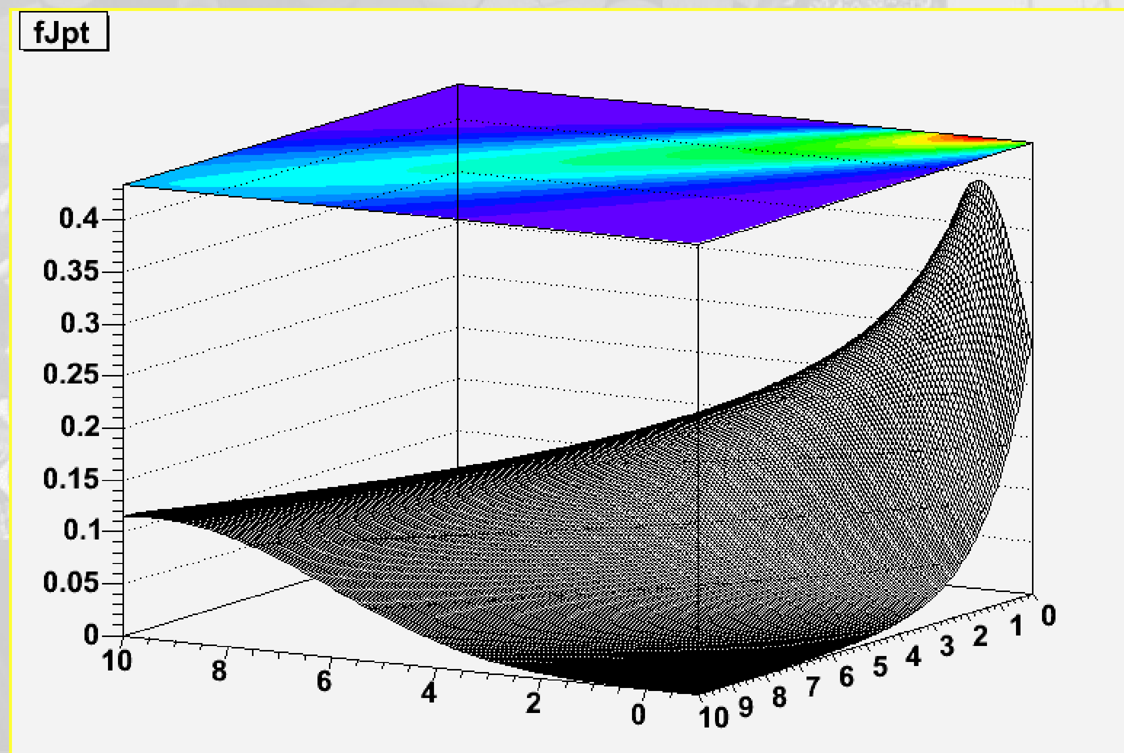
Probability density function

- *Complete analytical expression*
  (convolution of Poisson distribution with increasing size Gaussians)

$$P_{df}(x,\mu,\sigma_p)=\sum_n \frac{\mu^n e^{-\mu}}{n!\sqrt{2\pi\left(\sigma_p^2+n^2\sigma_\gamma^2\right)}}\exp\left(\frac{-(x-n)^2}{2\left(\sigma_p^2+n^2\sigma_\gamma^2\right)}\right)$$

$\sigma_p = \textit{Pedestal width}\,(\textit{NSB}+\textit{electronic noise})$

$\sigma_\gamma = \textit{PMT resolution}$



- *Strongly depends on*
- *NSB level*