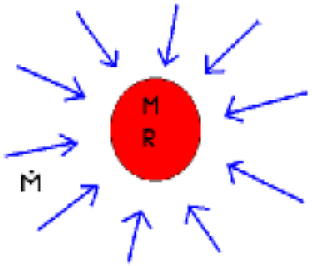
Spherical accretion

In this lecture we will consider the dynamics of the accretion onto a central body. In general we do not have any symmetry, for example when the stream hits the surface of the star. So will start with the case of the highest symmetry – spherical symmetry – since it might seem the simplest example.

It is not.



But the spherical flow assumption is a good approximation in several realistic flows:

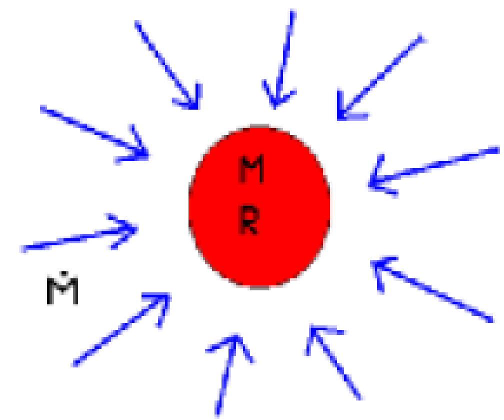
- Hot accretion flow in Sgr A*
- Hot accretion in elliptical galaxies
- Accretion from the Keplerian disk BELOW the Innermost Stable Circular Orbit (ISCO)
- Advection-Dominated Accretion Flow (ADAF)

1. Hydrodynamical description of the flow – assumptions

Accreting mater is frequently not dense and highly ionized, being a mixture of the ions and electrons. The particles exchenge energy between themselved by scattering (Coulomb interations). If interations are frequent, the matter can be characterized by the density, pressure and temperature, and the hydrodynamical approach is a good approximation. However, if interactions are not frequent we have to apply the microscopic approach (complex distribution of particle velocities etc.). All becomes complex, and we prefare to avoid this necessity. Thus is is important to have a criterium when the hydronamincal approximation applies.

1. Hydrodynamical description of the flow – assumptions

We have to estimate the mean free path λ of the particle and the typical distance L at which the macroscopic conditions in the flow change.



Condition for the hydrodynamical approximation to apply:

Mean free path λ $\ll$ Characteristic distance L

The mean free path in many situations can be estimated from the formula

$$\lambda \approx \frac{7\times 10^5 T^2}{N \ \ln \Lambda}$$

where T is the plasma temperature, N is the number density of the particles in [cm^{-3}], and ln Λ is 10-20
We can consider the simple case of the spherically symmetric accretion flow, guessing the equations governing the flow (we will talk in detail later):

$$\dot{M} = 4\pi r^2 \rho v; \qquad v \approx v_{ff} = \sqrt{\frac{GM}{r}}; \qquad \rho = N m_p$$

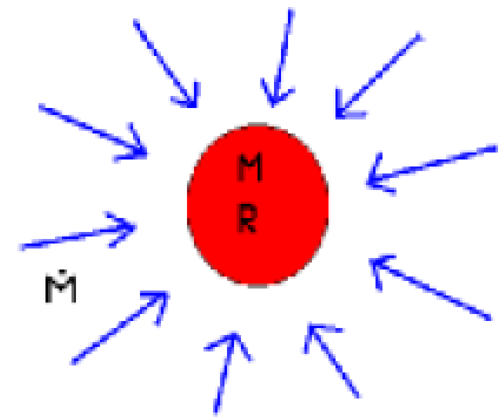
We can use now the dimensionless quantities introduced in Lecture 1:

$$\dot{m} = \frac{\dot{M}}{\dot{M}_{Edd}}; \qquad x = \frac{r}{R_{Schw}}; \qquad kT = \frac{GM m_p}{r}$$

1. Hydrodynamical description of the flow – assumptions

Mean free path λ = Characteristic distance r

r



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$$\frac{\lambda}{r} = \frac{10^6}{x^{3/2}\dot{m}}$$

if T = T_{vir}

$$\lambda \approx \frac{7\times 10^5 T^2}{N \ln \Lambda}$$

For small x and/or small mdot we should not use the hydrodynamical approach! Fortunately, the situation changes dramatically, by many orders of magnitude, if there is any magnetic field in the plasma. And in the astrophysical plasma usually there is some magnetic field.

In that case the mean free path is actually set by the **Larmor radius.** It shows how important is magnetic field in accretion flow.

Further we will assume we can use the hydrodynamical approach.

2. Hydrodynamical equations

We know from physics courses that in general hydrodynamical equations have the following form

- Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0$$

- Equations of motion

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \nabla \vec{v} = -\nabla P + \vec{f}$$

- Energy equation

$$\rho T \frac{\partial S}{\partial t} + \rho T \vec{v} \nabla S = -\nabla F_{rad} - \nabla q + \Pi$$

- Plus thermodynamic relations (equation of state - EOS, enthropy S), and other supplementary equations determining the radiation flux, F_{rad}, thermal conduction flux, q, viscous stresses, Π, etc.

We do not always need all that complexity. But sometimes we do. Even EOS is not always perfect fluid, in the region of partial ionization mean molecular weight changes, in white dwarfs or netutron stars the matter is degenerate, etc.

$$\boxed{P = \frac{kT\rho}{\mu m_p}}$$

3. Solutions stationary, adiabatic, isothermal, politropic

Stationarity: partial derivatives (explicit dependence on time) is zero, for example the continuity equations changes in the following way:

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{v}) = 0 \quad \Longrightarrow \quad \nabla (\rho \vec{v}) = 0$$

Now we will simplify the third equation assuming that there is no energy losses in the system, i.e. the entropy is conserved (**adiabatic case**):

$$\rho T \frac{\partial S}{\partial t} + \rho T \vec{v} \nabla S = -\nabla F_{rad} - \nabla q + \Pi \quad \Longrightarrow \quad S = \text{const}$$

$$\frac{3}{2} kdT$$

To use this info, we need thermodynamics ans EOS. From thermodynamics we can get

dE = TdS – pdV (*)

where E is the energy density, p is pressure and V = 1/ρ is the proper volume.
For the perfect fluid monoatomic non-relativistic gas

$$E = \frac{3}{2} \frac{kT}{\mu m_p} \qquad P = \frac{k T \rho}{\mu m_p}$$

Under the condition S = const the equation (*) translates into

$$\frac{3}{2} \frac{k\, dT}{\mu m_p} - \frac{k T \rho\, d\rho}{\mu m_p \rho^2} = 0 \quad \Longrightarrow \quad T^{3/2} \rho^{-1} = const \quad \Longrightarrow \quad \boxed{P \rho^{-5/3} = const}$$

3. Solutions stationary, adiabatic, isothermal, politropic

Adiabatic non-relativistic case: $P\rho^{-5/3}=const$

Adiabatic relativistic case: $P\rho^{-4/3}=const$

Politropic case (more general): $P\rho^{-\gamma}=const$

Isothermal case: $\gamma=1$

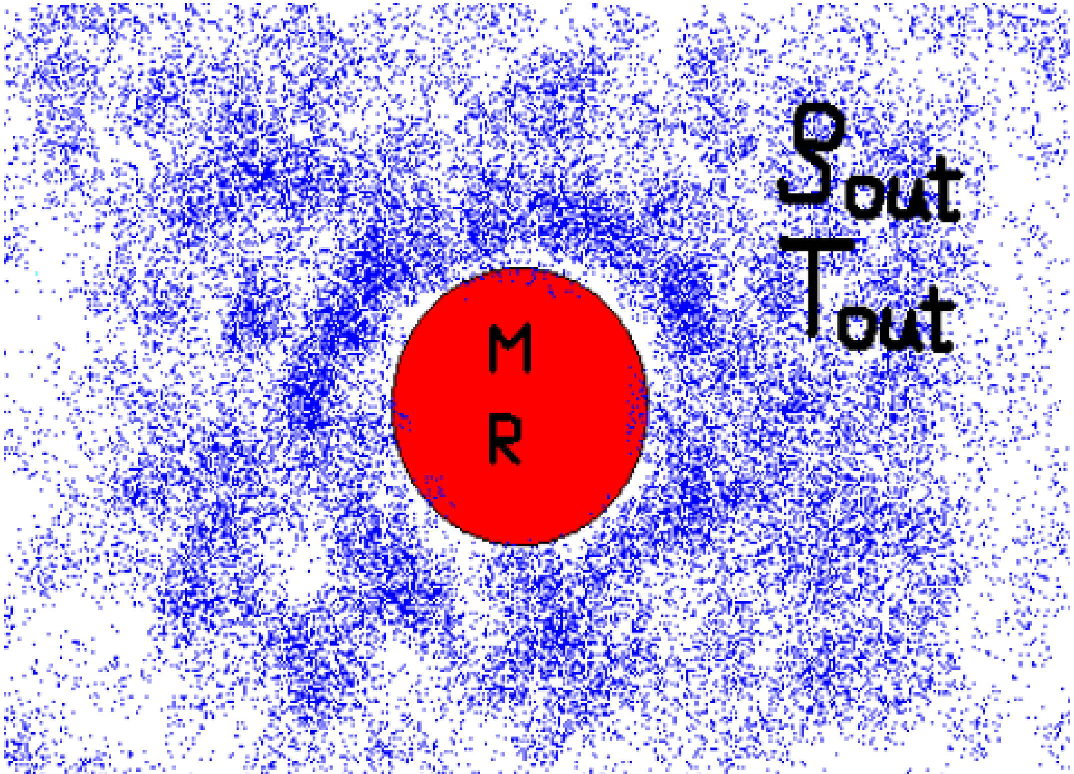
In any of these approximations we do not have to solve the equation of energy any more. However, we will never use the approximation of the incompressible fluid since we consider gases, not liquids, in astronomy most of the time. Now we are ready to address the real astronomical problem.

4. Bondi flow

The problem: a star of the mass M and the radius R is embeded in the interstellar medium which asymptiotically has the density ρ_{out} and the temperature T_{out}.

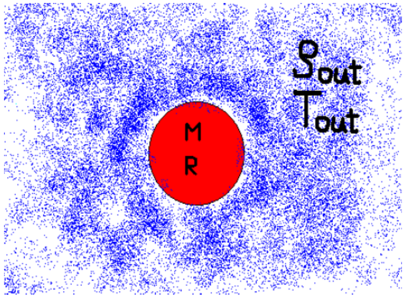
What is the accretion rate of the interstellar matter onto the star?

The problem has been solved in 1952 by Herman Bondi.



4. Bondi flow

It might seem that the whole medium will fall onto the star immediately under the action of the gravity force from the star, but we assumed that the medium is infinite, so the mass reservoire is large. As Bondi (1952) showed, the inflow is not that rapid, and an approximate stationary equilibrium forms.



So we will assume a stationary flow, spherical symmetry, inviscid fluid, and a politropic approximation, so we do not need energy equation. Then the Euler equations in spherical coordinated reduce to:

In spherical coordinates, (r, θ, ϕ), Euler's equations of motion for an inviscid fluid become:

$$\rho \left\{ \frac{Du_r}{Dt} - \frac{u_\theta^2 + u_\phi^2}{r} \right\} = -\frac{\partial p}{\partial r} + f_r \quad \text{(Bdc6)}$$

$$\rho \left\{ \frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r} - \frac{u_\phi^2 \cot\theta}{r} \right\} = -\frac{1}{r}\frac{\partial p}{\partial \theta} + f_\theta \quad \text{(Bdc7)}$$

$$\rho \left\{ \frac{Du_\phi}{Dt} + \frac{u_\phi u_r}{r} + \frac{u_\theta u_\phi \cot\theta}{r} \right\} = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} + f_\phi \quad \text{(Bdc8)}$$

where u_r, u_θ, u_ϕ are the velocities in the r, θ, ϕ directions, p is the pressure, ρ is the fluid density and f_r, f_θ, f_ϕ are the body force components. The Lagrangian or material derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial}{\partial \phi} \quad \text{(Bdc9)}$$

For completeness the equation of continuity for an incompressible fluid in spherical coordinates is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} = 0 \quad \text{(Bdc10)}$$

For compressible flud the density appears inside the parenthesis, and this will be the version we use.

Taken from http://brennen.caltech.edu/fluidbook/basicfluiddynamics/newtonslaw/eulerothercoords.pdf

4. Bondi flow

From politropic approximation $P = K\rho^{\gamma}$

We assume spherical symmetry. Thus the **continuity equation**, in spherical coordinates, reduces to:

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho v) = 0$$

After multiplication by r^2 and integration over radius we get: $r^2 \rho v = const$

Then we introduce the flow constant in the traditional form: $\dot{M} = const \equiv 4\pi r^2 \rho(-v)$

The accretion rate Mdot is positive (for negative velocity) and represents the total mass flow rate [e.g. in g/s]. We used it already in Slide 2.
The equation of motion has only one interesting component, 'r', since we have only one velocity component, $v_r = v$:

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM}{r^2} = 0$$

The first term represents the stationary (not static) part of the time derivative, the second and third are the pressure gradient and the gravity force. Equation (Bdc6) was divided by density.

We thus have two equations for v(r) and ρ(r), and P(r) can be derived from the politrope, so the system can be solved. Actually, also the second equation can be integrated analytically, but the character of motion is much better recognized **BEFORE** we integrate. But we have to rewrite it to see clearly what is happening.

4. Bondi flow

We have to introduce one more useful concept, sound speed:

$$c_s^2 = \frac{dP}{d\rho}$$

Then the pressure gradient can be expressed through the density gradient (for a politrope)

$$\frac{dP}{dr} = \frac{dP}{d\rho}\frac{d\rho}{dr} = c_s^2 \frac{d\rho}{dr}$$

Finally, we put that into equation od motion

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{d\rho}{dr}c_s^2 + \frac{GM}{r^2} = 0$$

Now we have to use the continuity equation, but before we integrated it, where we have also a derivative of the velocity and derivative of the density.
We have to change vdv/dr into ½ dv^2/dr, and by combining the two equations we can get two separate equations: one for dρ/dr and the second for dv^2/dr. We will concentrate on this second form:

$$\frac{dv^2}{dr} = \frac{\frac{-GM}{r^2}\left[1 - \frac{2c_s^2 r}{GM}\right]}{\frac{1}{2}\left(1 - \frac{c_s^2}{v^2}\right)}$$

This shows the essential point of the Bondi flow. In the denominator we compare the local velocity to the local (!) sound speed.

If the local velocity crosses the local sound speed
$v^2 = c_s^2$
the denominator of the equation becomes zero, and the equation has the critical point.

4. Bondi flow

$$\frac{dv^2}{dr} = \frac{\frac{-GM}{r^2}[1 - \frac{2c_s^2 r}{GM}]}{\frac{1}{2}(1 - \frac{c_s^2}{v^2})}$$

Do we expect the flow crossing $v^2 = c_s^2$?

At inifinity, the matter is almost at rest (assumption) but the temperature (c_s) is positive so v^2 is slowly rising inwards, the dominating terms are:

The two things can happen:

- The star is large, so at some point the numerator becomes zero, the matter stops at the star surface before reaching the sound speed
- The star is small, so we approach the sound speed and the velocity diverges. But we know that the supersonic motion is possible – how to do it?

Denominator = 0 when Numerator equal zero

This allows the flow to cross the spound speed, with the derivative of the velocity being (in principle) continuous, 0/0 = something.

The radius where it happens is called Bondi radius, r_s. Thus two conditions are satisfied at this radius:

$$\boxed{v^2 = c_s^2 \qquad\qquad r_s = \frac{GM}{2c_s^2(r_s)}}$$

This is the radius of a sphere which divides the flow into two parts:

- Outer subsonic flow, $v \ll c_s$
- Inner supersonic flow, $v \gg c_s$

4. Bondi flow

We cannot calculate this radius immediately in a precise way since c_s is also a function of the radius. But in the first approximation:

$$\boxed{v^2 = c_s^2 \qquad r_s = \frac{GM}{2c_s^2(r_s)}}$$

$$r_s \approx \frac{GM}{2c_s^2(\infty)} \qquad c_s^2(\infty) = \frac{kT_{out}}{\mu m_p}$$

This is how the Bondi radius is calculated for elliptical galaxies in many papers.

Or, equivalently,

$$\frac{r_s}{R_{Schw}} \approx 4\frac{c^2}{c_s^2}(\infty)$$

Again, in the approximation that we use the values from infinity, we can calculate the accretion rate using the density and sound speed at infinity, but the radius as above

$$\dot{M}_{Bondi} \approx \frac{4\pi\rho(\infty)G^2M^2}{c_s^3}$$

Thus, knowing the temperature and the density of the interstellar medium we get the estimate of accretion rate.

Actually, we could have guessed that.
Thermal energy of a particle is: $\rightarrow kT_{out}$
Gravitational energy is $\rightarrow \dfrac{GMm_p}{r}$

Condition of escape is: $kT_p > \dfrac{GMm_p}{r}$ condition of accretion: $kT_p < \dfrac{GMm_p}{r}$

This the radius of the marginal accretion is:
And it is pretty similar to the Bondi formula (factor 2 difference).

$$r = \frac{GMm_p}{kT_{out}}$$

4. Bondi flow

Now we can derive the expression for Bondi radius and accretion rate more carefully, using local instea of asymptotic values.
Starting again from the original equation

$$v\frac{dv}{dr}+\frac{1}{\rho}\frac{dP}{dr}+\frac{GM}{r^2}=0$$

$$P = K\rho^{\gamma}$$

we can integrate it and get:

$$\frac{v^2}{2}+\frac{K\gamma}{\gamma-1}\rho^{\gamma-1}-\frac{GM}{r} = const$$

Using the cs concept again, we get th classical for of the **Bernoulli equation:**

$$\frac{v^2}{2}+\frac{c_s^2}{\gamma-1}-\frac{GM}{r} = const$$

We still have the continuity equation, and we have the boundary conditions at infinity. We get:

The Bernoulli constant: $\dfrac{c_s^2(\infty)}{\gamma-1}$

The Bondi radius: $r_s = \dfrac{GM(5-3\gamma)}{4c_s^2(\infty)}$

Bondi accretion rate

$$\dot{M} = \pi G^2 M^2 \frac{\rho_{out}}{c_s^3(\infty)}\left(\frac{2}{5-3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}}$$

Now it depends on the adiabatic index γ, and actually the formulae require

$$\boxed{\gamma < 5/3}$$

4. Bondi flow

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The Bernoulli constant: $\frac{c_s^2(\infty)}{\gamma - 1}$

The Bondi radius: $r_s = \frac{GM(5-3\gamma)}{4c_s^2(\infty)}$

Bondi accretion rate $\dot{M} = \pi G^2 M^2 \frac{\rho_{out}}{c_s^3(\infty)} \left(\frac{2}{5-3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}}$

For γ= 5/3 perfect fluid the Bondi radius is zero, but the accretion rate has some limit. For interstellar partially ionized medium more appropriate is γ= 1.4, problem disappears.

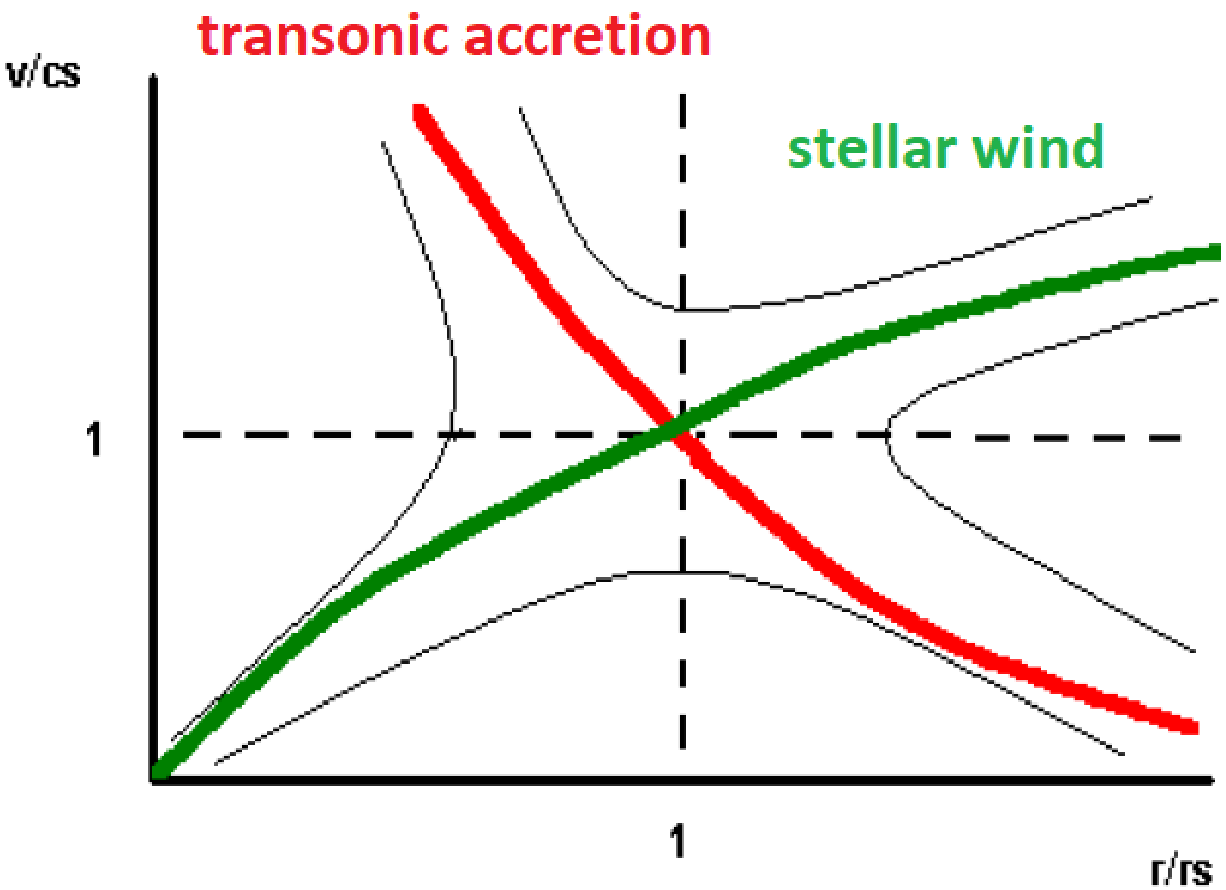
Now using the dimensionless quantities, we can express the Bondi accretion rate as

$$\dot{m} \approx 1.1 \times 10^{-6} \frac{M}{M_s}$$

So accretion from the interstellar medium is not important for stellar-mass objects but it is important for AGN.

5. Topology of general solutions

Mathematically, the solution we constructed is not the only solution to the presented equations.

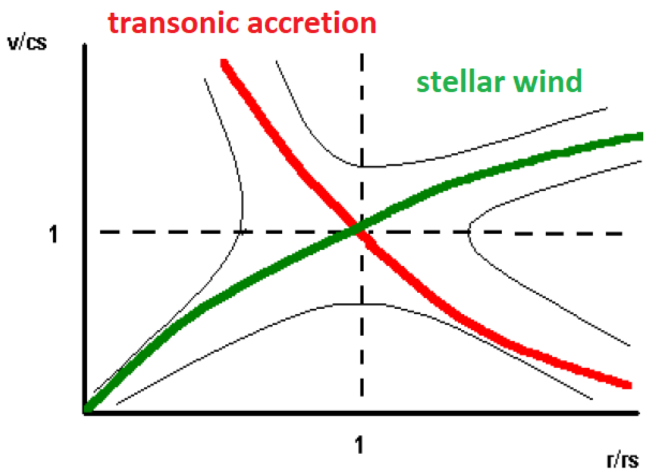


There are two special stationary solutions corresponding to a smooth srossing between the subsonic and supersonic motion:

- Bondi (1952) accretion flow
- Parker (1958) solar wind solution

In the case of a black hole accretion only the transonic solution is appropriate since the flow speed has to reach the light speed when crossing the black hole horizon.For stellar accretion The lower branches can in principle apply, with the matter settling on a star in quasi-hydrostatic equilibrium.

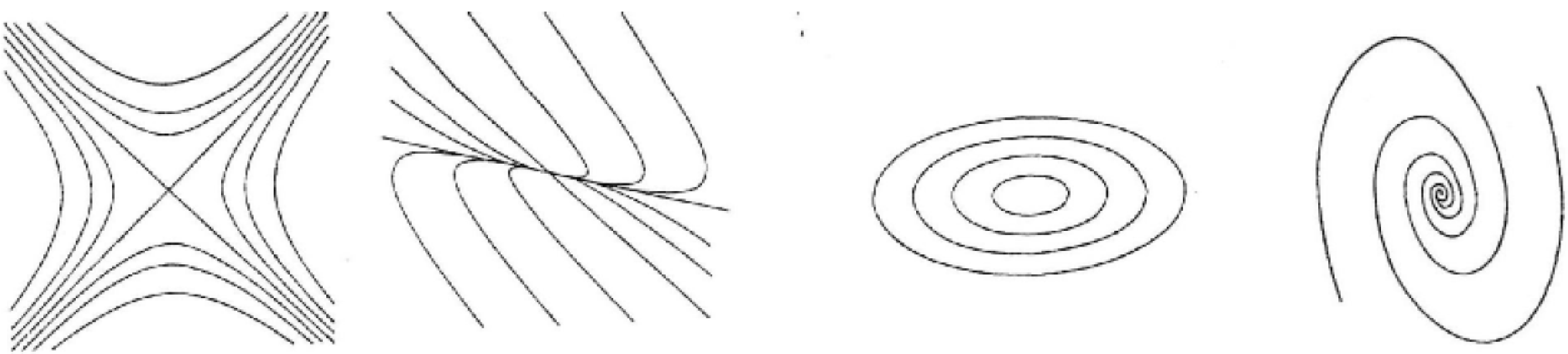
5. Topology of general solutions



This type of the flow appear not only in the spherical case but in many situations when the symmetry is partially broken by the presence of small angular momentum, and additionally some action of viscosity, cooling etc.
In many such cases equations reduce to one singular equation

$$\frac{dV}{dr} = \frac{N}{D} \qquad N - nominator \qquad D - denominator$$

plus several others regular differential equations, where N and D are comples implicite function of velocity and radius. That allows, in general, for appearance of the four main topologies:



saddle **noddal** **focus** **spiral**

Only the first two allow for an inflow without a shock formation (discontinuity).

6. Spherical accretion onto a star – zero velocity condition

The solution with ‘infinite’ velocity in the innermost part of the flow applies to black holes, although we have to include GR corrections properly, but this does not modify the flow qualitatively.
In the case of a star, like main sequence star, white dwarf or a neutron star, we have a firm stellar surface where materia has to stop and accumulate there. Accretion never crossing the sound speed mentioned before does not actually apply in practice.
Most frequently we have a following sequence:

subsonic accretion

↓

sonic point

↓

supersonic accretion

↓

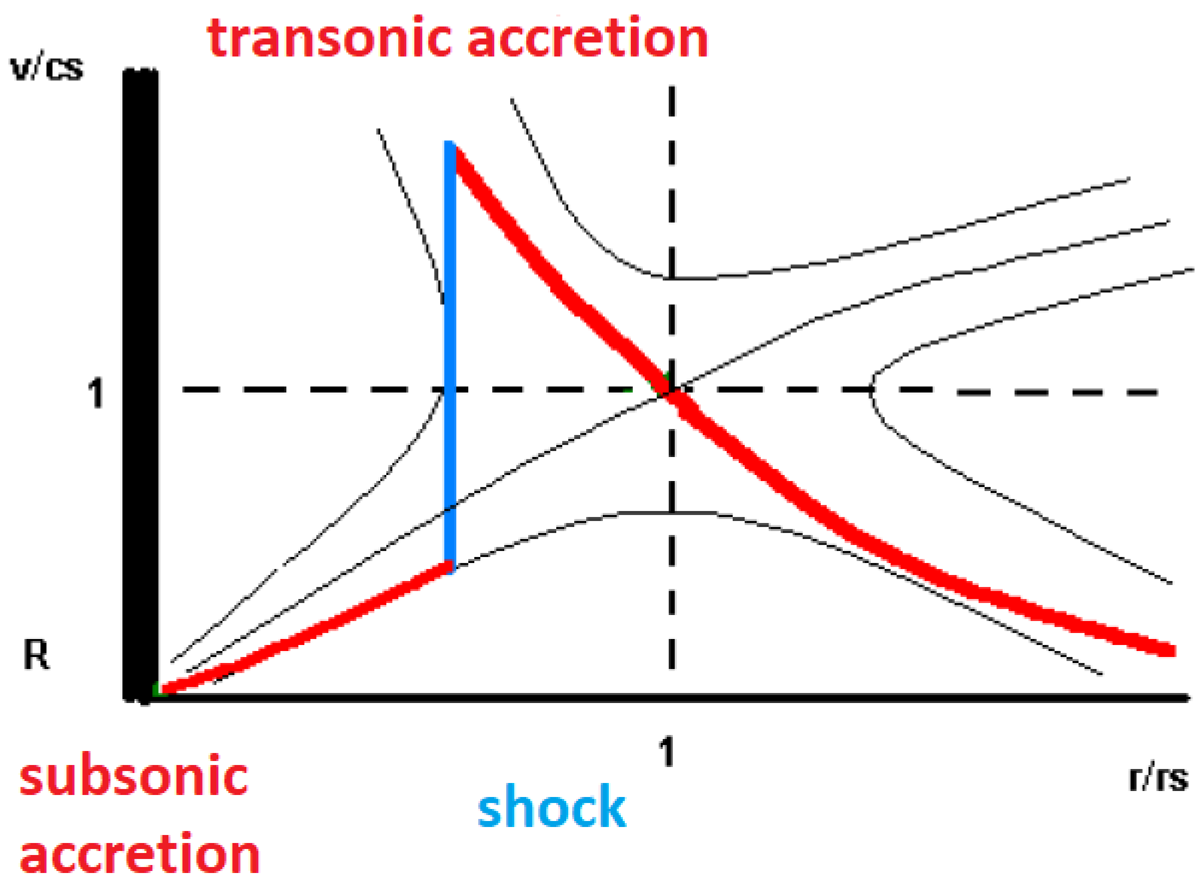
shock

↓

subsonic accretion

↓

Matter at rest.



The development of a shock allows to jump from one branch of solutions to the other.

6. Spherical accretion onto a star – zero velocity condition

However, the transition from supersonic to subsonic flow cannot happen in an arbitrary way. Following conservation laws have to be satisfied:

- Conservation of mass (continuity equation)
- Conservation of momentum
- Conservation of energy (if the shock is adiabatic)

On the other hand, (approximately) discontinuous change is allowed in density, and velocity of the flow, as well as pressure and temperature. Shock changes the kinetic energy of the motion into heat, and this thermodynamical process is irreversible, works always

from supersonic flow to subsonic flow,

never the other way around.

Conituity equation reads like this: $\rho v = const$

Euler equation can be written in the following way: $$v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + F_g = 0$$

Now we multiply it by the density: $$\rho v\frac{dv}{dr} + \frac{dP}{dr} + \rho F_g = 0$$

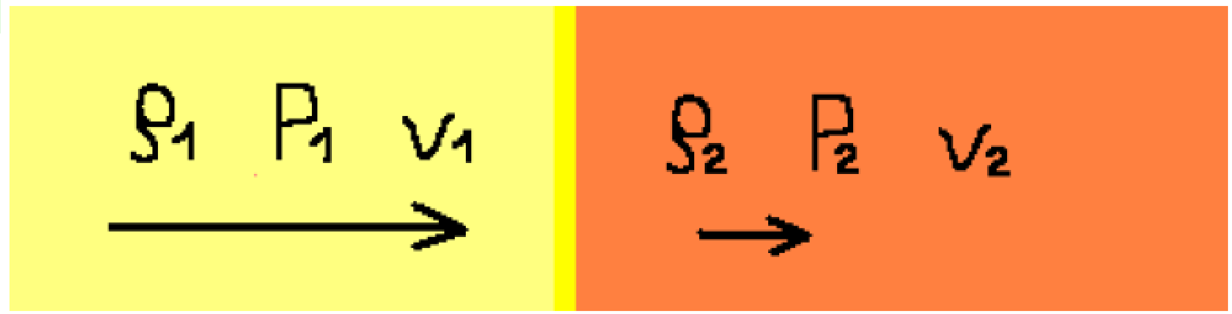
Next we integrate it across the narrow shock zone o the width Δ, remembering that ρv = const, and we go with Δ to zero. The first two terms integrate directly, while the third one will go to zero if $\Delta \longrightarrow 0$.

6. Spherical accretion onto a star – zero velocity condition

We then get the condition: $$P + \rho v^2 = const$$

Similarly, we have to work with the energy equation (for a perfect gas) $$\frac{1}{2} v_2^2 + \frac{5}{2} \frac{P_2}{\rho_2} = const$$

In this way we determine the Rankine-Hugoniot conditions for the parameters change accross the shock.



If the flow was highly supersonic before the shock, then pressure before the shock can be neglected, and the conditions read:

$$\rho_1 v_1 = \rho_2 v_2$$

$$\rho_1 v_1^2 = \rho_2 v_2^2 + P_2$$

$$\frac{1}{2} v_1^2 = \frac{1}{2} v_2^2 + \frac{5}{2} \frac{P_2}{\rho_2}$$

Maximum compression factor is 4.

If we combine these equations and introduce a quantity $x = v_1/v_2$ then we get a quandratic equation:

$$\frac{1}{2} x^2 = \frac{5}{2} x - 2$$

which has two solutions:

x = 4

strong shock

x = 1

no shock

These conditions do not specify the distance from the stellar surface where the shock forms.

7. Applications to real objects

7a) column accretion onto white dwarf or a neutron star

As an example, we consider AM Her system consisting of a magnetized white dwarf and a donor star. If the magnetic field of the accreting star is strong (B > 10^7 G for a white dwarf, B > 10^{12} G for a neutron star), accretion disk never forms or is disrupted in the innermost part, and the inflowing matter follows the magnetic field lines. The magnetic field is approximately a dipol, and it forces the matter to accrete onto the poles. This flow is highly non-spherical, but nevertheless the theory applies.

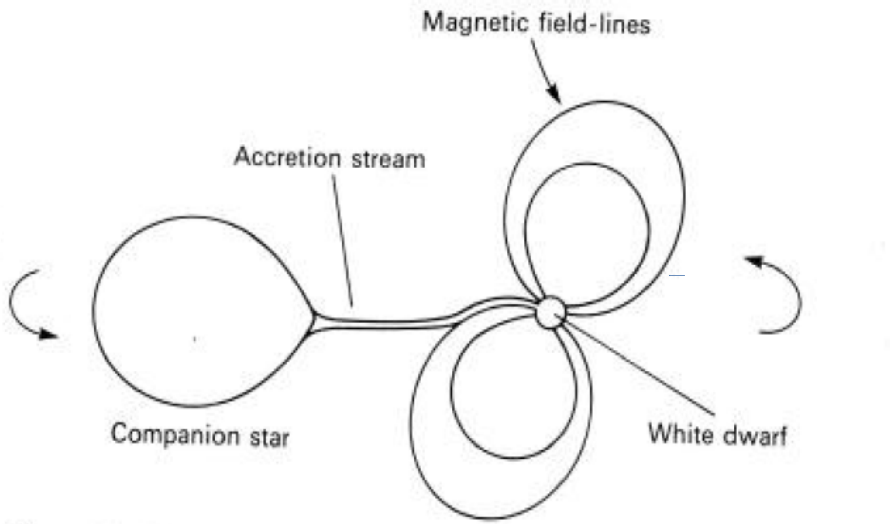


Figure 43. Schematic view of an AM Herculis system. The rotation of the strongly magnetic ($\gtrsim 10^7$ G) white dwarf is locked to that of the binary ($P \lesssim$ 4 h). No accretion disc forms, matter impinging directly on the magnetosphere and following field-lines down to the white dwarf surface.

Figure from Frank, King & Raine book

Wikipedia (zorza polarna)

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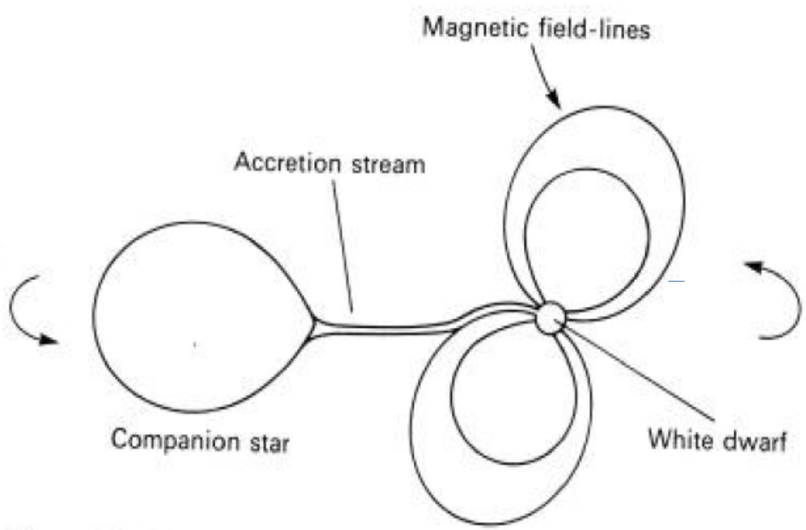

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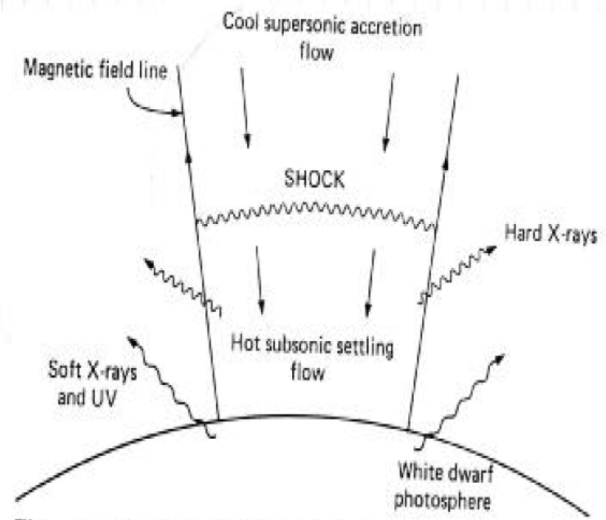

Figure 46. Accretion column geometry for a magnetized white dwarf.

Figures from Frank, King & Raine book

Of course the detailed models of such a flow are much more complex than just the dynamics, colling is important (it affects the position of the shock), and emitted radiation spectrum has to be compared with the data. Still, there were discussions about ‘pencil model’ vs. ‘fan beam’ model.

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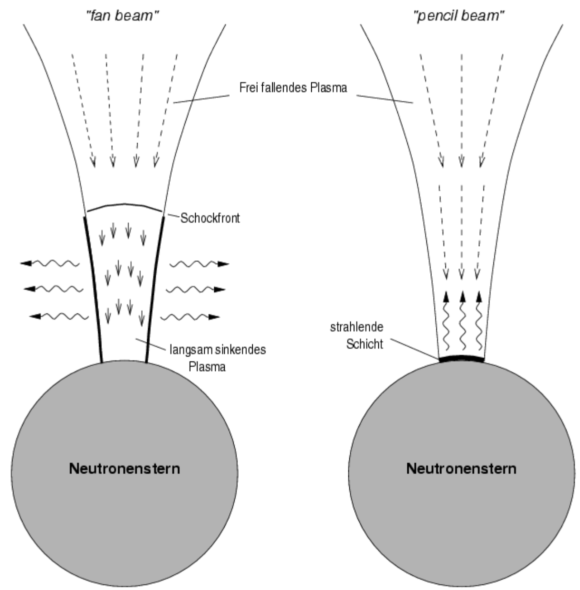
3–60

Fan Beam vs. Pencil Beam



(Kretschmar 1996, Dissertation AIT, Abb. 2.9 [after Harding 1994])

Typical mass accretion rates XXXXXX

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7. Applications to real objects

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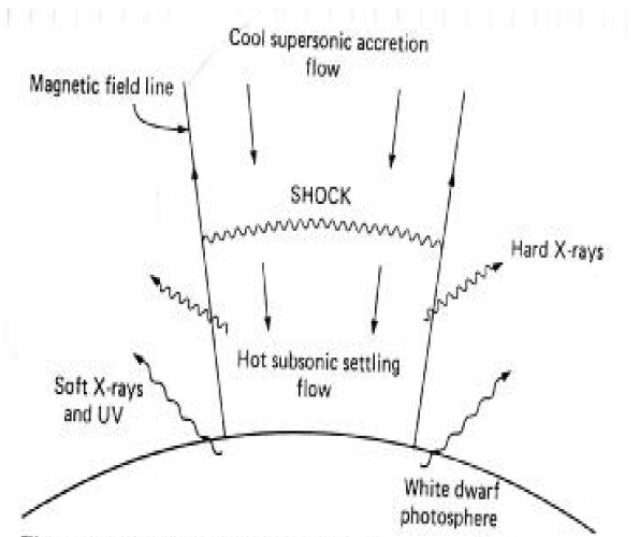


Figure 46. Accretion column geometry for a magnetized white dwarf.

Figure from Frank, King & Raine book

The geometry is not easily determined from observations since what we see is the spectrum (bremstrahlung plus some non-thermal emission) and the variations of the total flux with the rotation period.

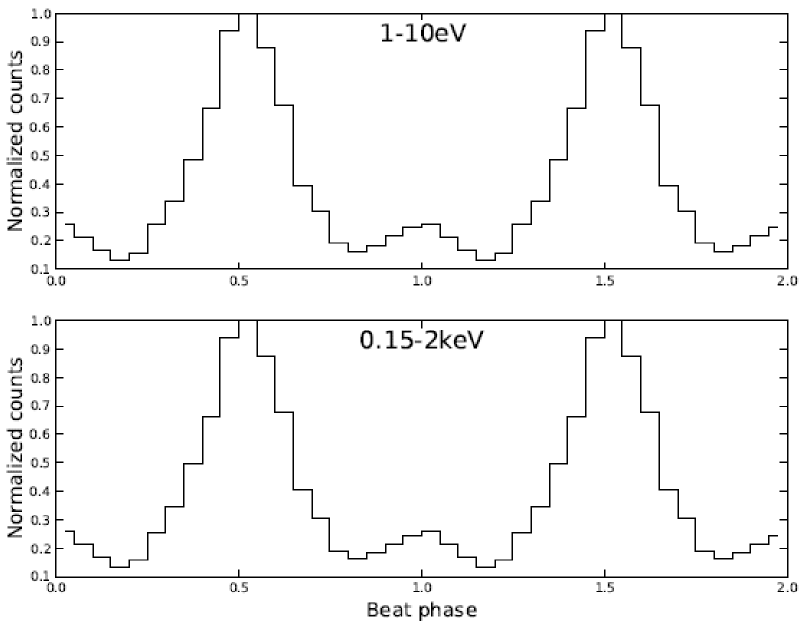


FIG. 11.— Model pulse profiles folded in the beat frequency. Top: 0.1-1 eV energy bands. Bottom: 0.15-2keV energy bands.

Pulse profile in the Intermediate polar AR Scorpii (Takata et al. 2017)

Details of geometry, particularly in the case of neutron stars are still unders discussion: hollow cone, multi-sub-column accretion etc.

7. Applications to real objects

s, and the density of the hot plasma in the Sgr A* region (from Baganoff et al. 2001 X-ray maps we have the density of 26 particles/cm^3, the temperature of 1.3 keV at a distance of 1.3 pc) we can get the flow parameters. Assuming no coefficients in the Bondi flow we get the following:

$$r_s = 3 x 10^{17}\, cm = 0.1\, pc$$

for Bondi radius (or order of 1 arc sec for Solar System observer)

$$\dot{M} = 2 x 10^{21}\, g/s$$

for Bondi accretion rate

And the expected luminosity (for efficiency η = 0.1) would be 10^{42} erg/s. The observed persistent luminosity is 10^{33} erg/s, up to 10^{35} erg/s during flares.

Of course the efficiency can much much lower, but there are also estimated of the flow density close to Sgr A* from Faraday rotation, and they imply less than 10^{18} g/s.

The mass does not reach the center.

Quataert (2004) addressed this issue. He considered more carefully the sources of matter.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \rho v = q(r),$$

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial r} - \rho \frac{GM}{r^2} - q(r) v,$$

$$\rho T \frac{ds}{dt} = q(r)\left[\frac{v^2}{2} + \frac{v_w^2}{2} - \frac{\gamma}{\gamma - 1} c_s^2\right],$$

q(r) is the input of matter from stars.

7. Applications to real objects

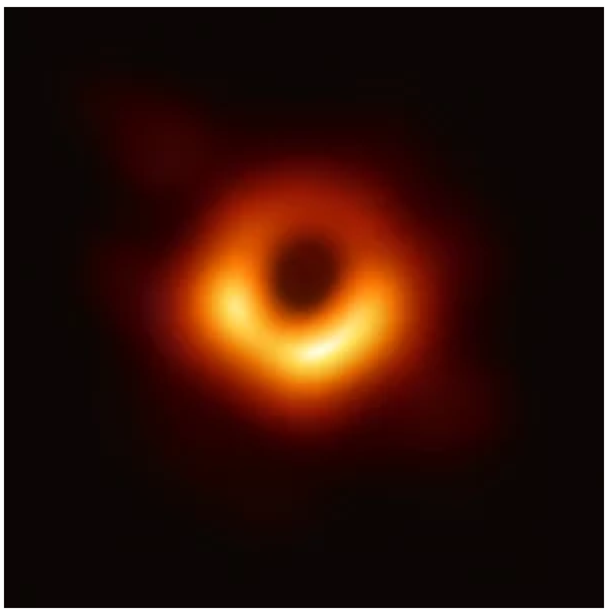
7b) accretion in the M87

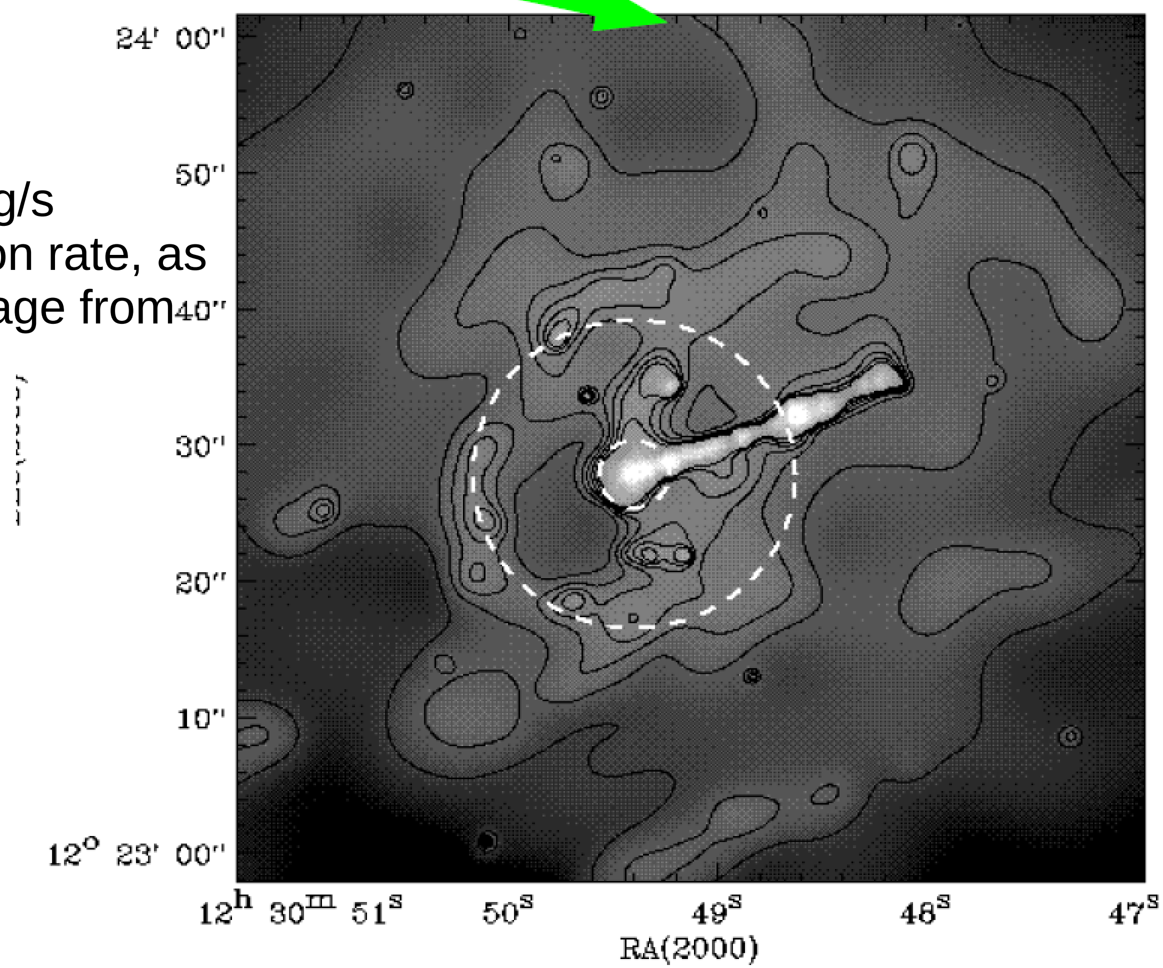
Using the expression for the Bondi radius and the information from X-ray map (Di Matteo et al. 2001) we have density 0.17 particles/cm^3, temperature 0.7 keV. We get the Bondi rate

$$\dot{M} = 2\,x\,10^{25}\,g/s$$

But the observed luminosity 7 x 10^{40} erg/s is far too small for the predicted accretion rate, as before. In this case we also have an image from Event Horizon Telescope..

M 87 is the central elliptical galaxy of the Virgo cluster.





But it is not clear what we see, apart from the shadow, or silhuette, of a black hole. We will return to this issue later on, when we are better prepared.

7. Applications to real objects

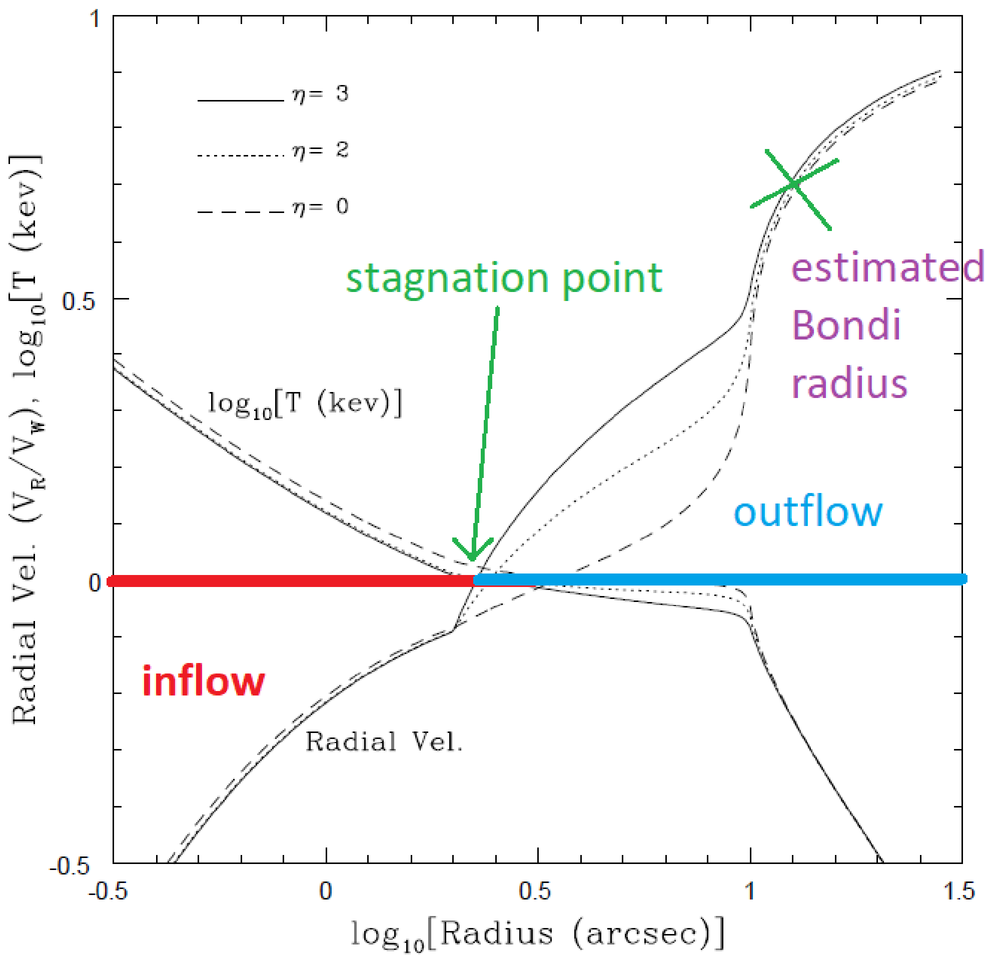
7b) accretion in the Sgr A* region

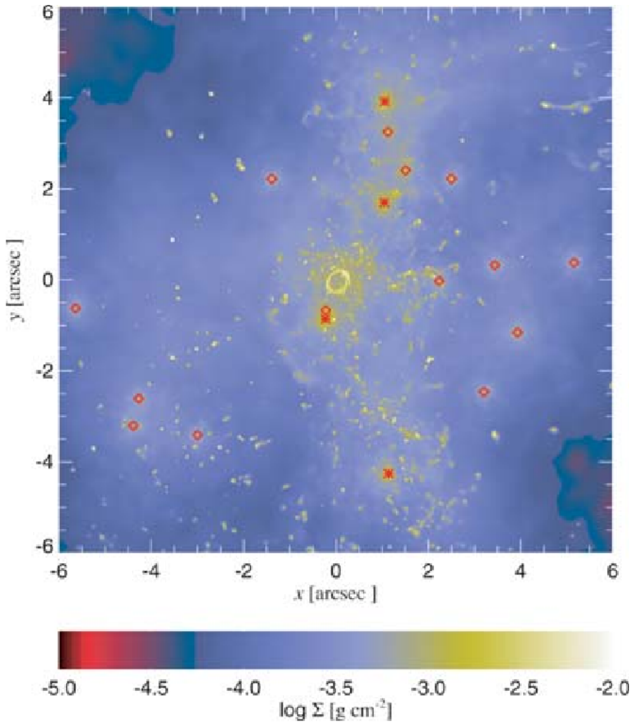
$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}r^2\rho v = q(r),$$

$$\rho\frac{dv}{dt} = -\frac{\partial p}{\partial r} - \rho\frac{GM}{r^2} - q(r)v,$$

$$\rho T\frac{ds}{dt} = q(r)\left[\frac{v^2}{2} + \frac{v_w^2}{2} - \frac{\gamma}{\gamma-1}c_s^2\right],$$

Quataert (2004) addressed this issue. He considered more carefully the sources of matter. q(r) is the input of matter from stars. He got an interesting solution.





This comes from Cuadra et al. (2005), and it is more accurate than Quataert (2004) approximation.

In reality we additionally have the angular momentum problem...