IDENTYFIKACJA MODÓW obserwowanych pulsacji (część 2)

(na podstawie przeglądowego referatu A.A.P. i J. Daszyńskiej-Daszkiewicz dla konferencji w Hiszpanii: „Impact of new instrumentation & new insights in stellar pulsations”, Granada, 5-9 September 2011)

(prezentacja oraz tekst dla Proceedings)

Two approaches to mode identification from photometry:

1) Comparing theoretical and observational values of the amplitude ratios and phase differences (*f* from theory)

2) Making use of the amplitudes and phases themselves (*f* is determined from observations together with ℓ)

both methods need input from model atmospheres !

If we ignore rotation,
the amplitude ratios and phase differences
are independent of the inclination angle, *i* ,
and the azimuthal order, *m* .

Complex amplitude of monochromatic flux variations

$$A_\lambda(i) = \varepsilon Y_\ell^m(i,0) b_\ell^\lambda (D_{1,\ell}^\lambda + D_{2,\ell} + D_{3,\ell}^\lambda)$$

$$D_{1,\ell}^\lambda = \frac{1}{4} f \frac{\partial \log(\mathcal{F}_\lambda |b_\ell^\lambda|)}{\partial \log T_{\rm eff}}$$

temperature term

$$D_{2,\ell} = (2+\ell)(1-\ell)$$

geometrical term

$$D_{3,\ell}^\lambda = -\left(2 + \frac{3\omega^2}{4\pi G <\rho>}\right) \frac{\partial \log(\mathcal{F}_\lambda |b_\ell^\lambda|)}{\partial \log g_{\rm eff}^0}$$

pressure term

$$b_\ell^\lambda = \int_0^1 h_\lambda^0(\mu) \mu P_\ell(\mu) d\mu$$

disc averaging factor

due to Dziembowski 1977, Daszyńska-Daszkiewicz et al. 2002

f - the ratio of the bolometric flux variation to the radial displacement at the photosphere level

$$\frac{\delta \mathcal{F}_{\rm bol}}{\mathcal{F}_{\rm bol}} = {\rm Re}\{\varepsilon f Y_\ell^m {\rm e}^{-{\rm i}\omega t}\},$$

A SYSTEM OF EQUATIONS

$$\mathcal{D}_\ell^\lambda(\tilde{\varepsilon} f) + \mathcal{E}_\ell^\lambda \tilde{\varepsilon} = A^\lambda,$$

$$\tilde{\varepsilon} = \varepsilon Y_\ell^m(i, 0),$$

$$\mathcal{D}_\ell^\lambda = \frac{1}{4} b_\ell^\lambda \frac{\partial \log(\mathcal{F}_\lambda |b_\ell^\lambda|)}{\partial \log T_{\rm eff}},$$

$$\mathcal{E}_\ell^\lambda = b_\ell^\lambda \left[(2+\ell)(1-\ell) - \left(\frac{\omega^2 R^3}{GM} + 2 \right) \frac{\partial \log(\mathcal{F}_\lambda |b_\ell^\lambda|)}{\partial \log g} \right].$$

Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2003

METHOD

Two complex unknown quantities

$$(\tilde{\varepsilon} f),\ \tilde{\varepsilon}.$$

For a given degree, ℓ, the system is solved by the least-square method to find the minimum of χ^2.

ADDING THE RADIAL VELOCITY

$$\mathrm{i}\omega R\left(u_\ell^\lambda + \frac{GM}{R^3\omega^2}v_\ell^\lambda\right)\tilde{\varepsilon} = V_{rad}(i)$$

$$u_\ell = \int_0^1 h\mu^2 P_\ell(\mu)d\mu,$$

$$v_\ell = \ell\int_0^1 h(P_{\ell-1} - \mu P_\ell)\mu d\mu,$$

Dziembowski , 1977, Acta Astr. 27, 203

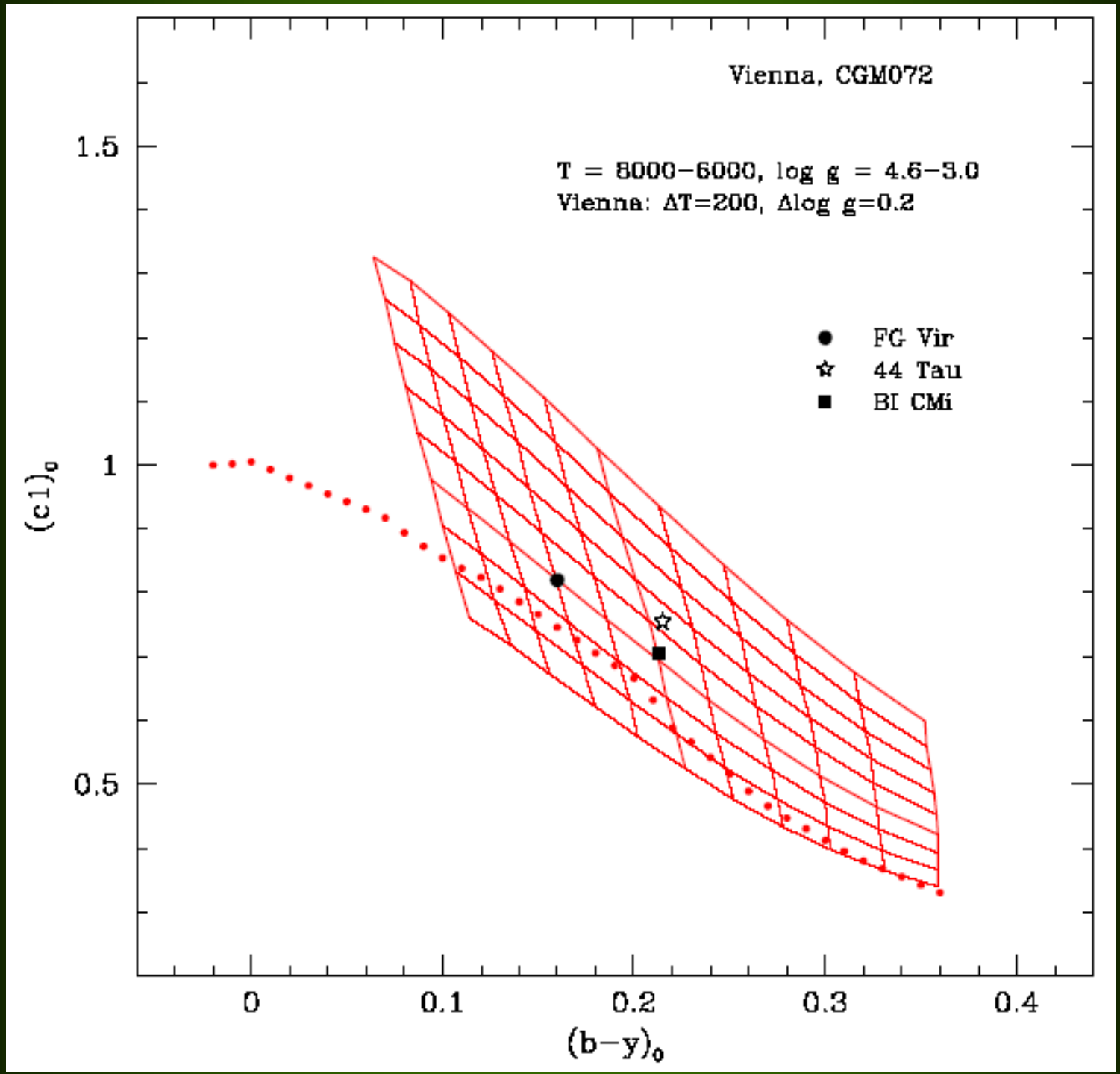
Examples: δ SCUTI STARS

β Cas, FG Vir, 44 Tau, HD144277

Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh, 2003, A&A 407, 999
Daszynska-Daszkiewicz, Dziembowski, Pamyatnykh et al., 2005, A&A 438, 653
Lenz, Pamyatnykh, Breger & Antoci, 2008, A&A 478, 855
Lenz, Pamyatnykh, Zdravkov & Breger, 2010, A&A 509, A90
Zwintz, Lenz, Breger, Pamyatnykh et al., 2011, A&A, 533, id. A133

44 Tauri

15 frequencies (degree *l* is identified for 10 of them),
Sp F2 IV-V,
mV = 5.399,
p = 16.72 +- 0.93 mas,
Vrot = 3 +- 2 km/s,
[m/H] = approx 0.0,
Teff = 6900 +- 100 K,
log g = 3.6 +- 0.1

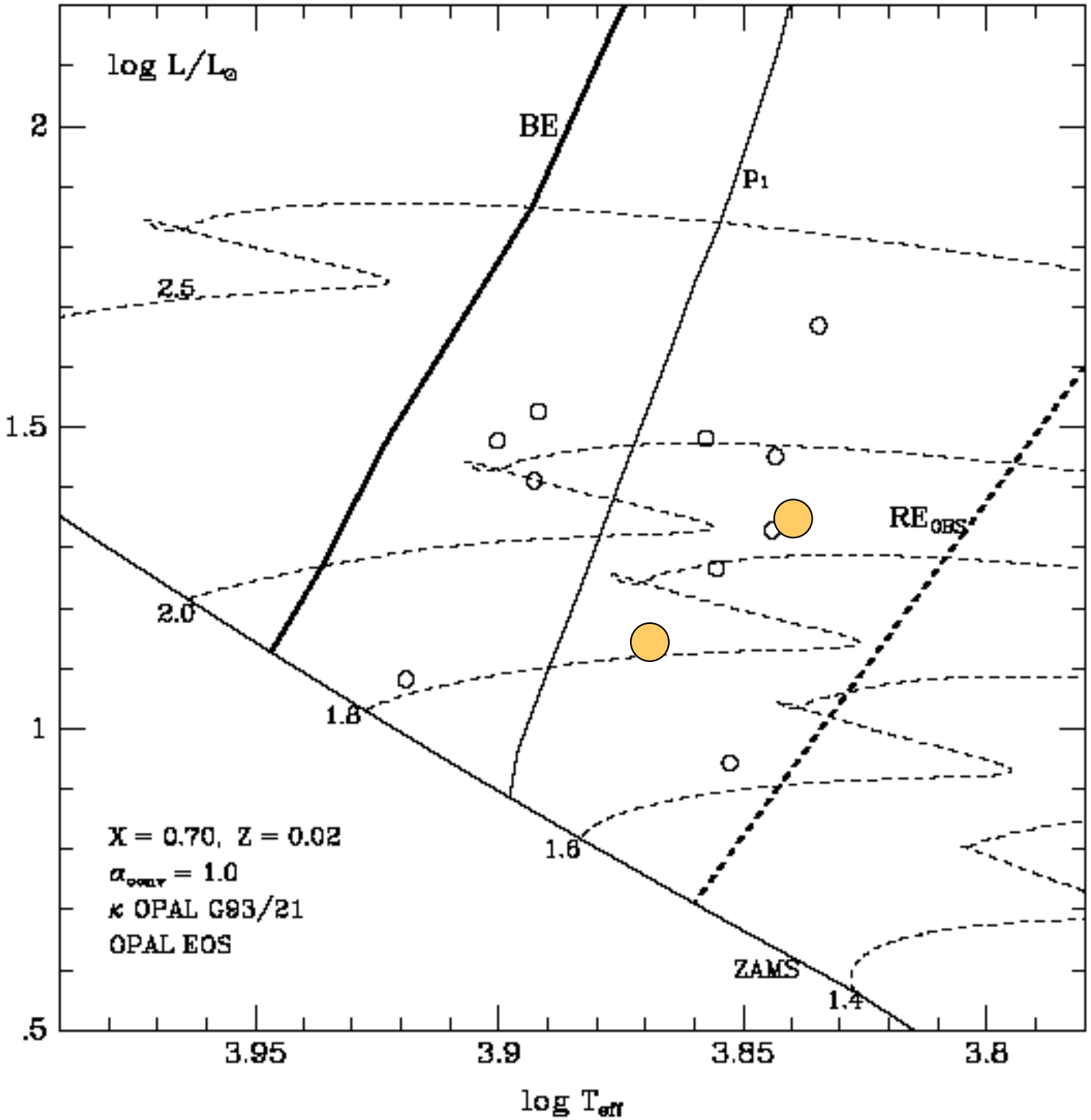

Vienna, CGM072
T = 8000−6000, log g = 4.6−3.0
Vienna: ΔT=200, Δlog g=0.2
FG Vir
44 Tau
BI CMi
1.5
1
0.5
(c1)0
0
0.1
0.2
0.3
0.4
(b−y)0

FG Vir (MS)

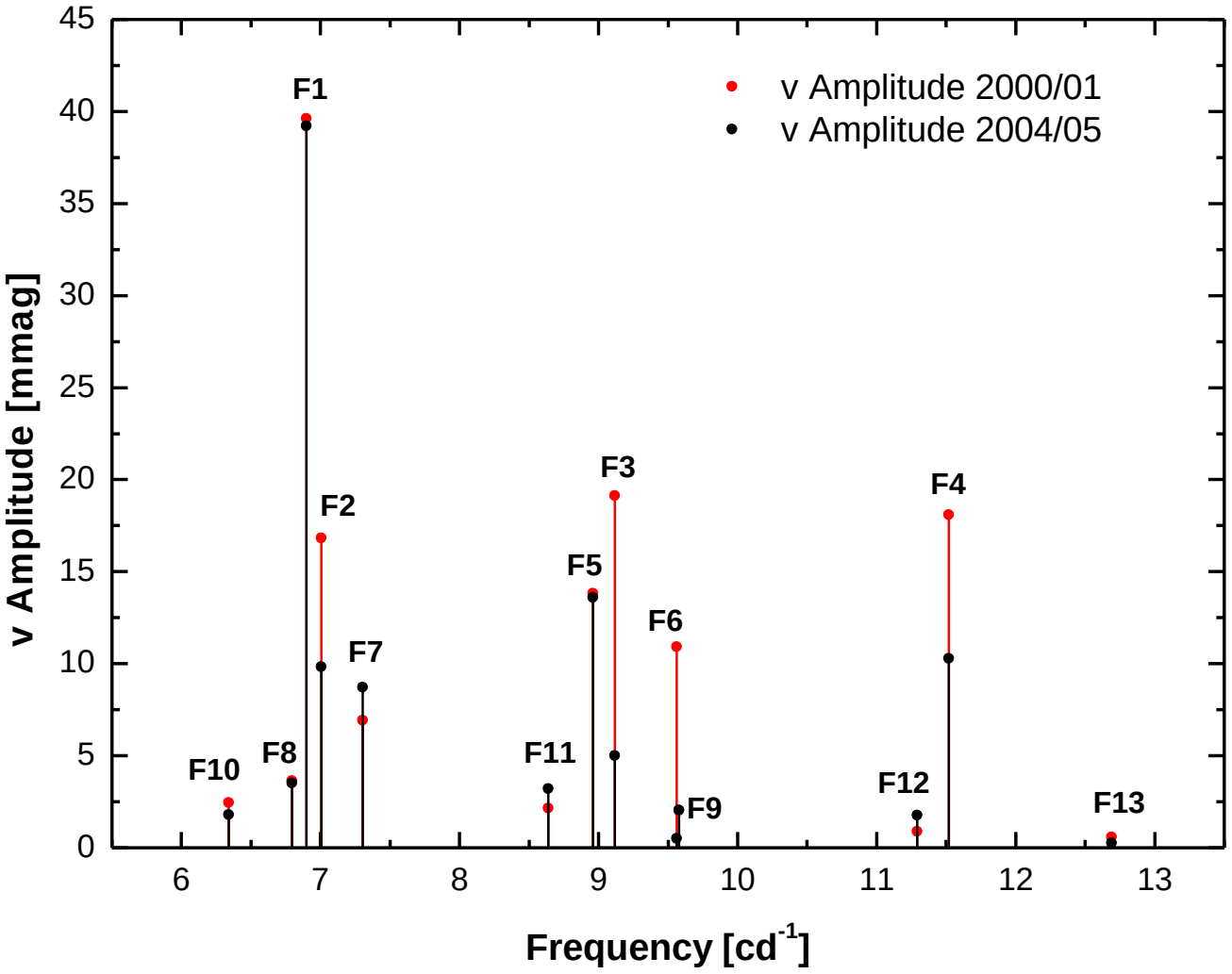
and

44 Tau (post-MS)

in the δ Scuti instability domain

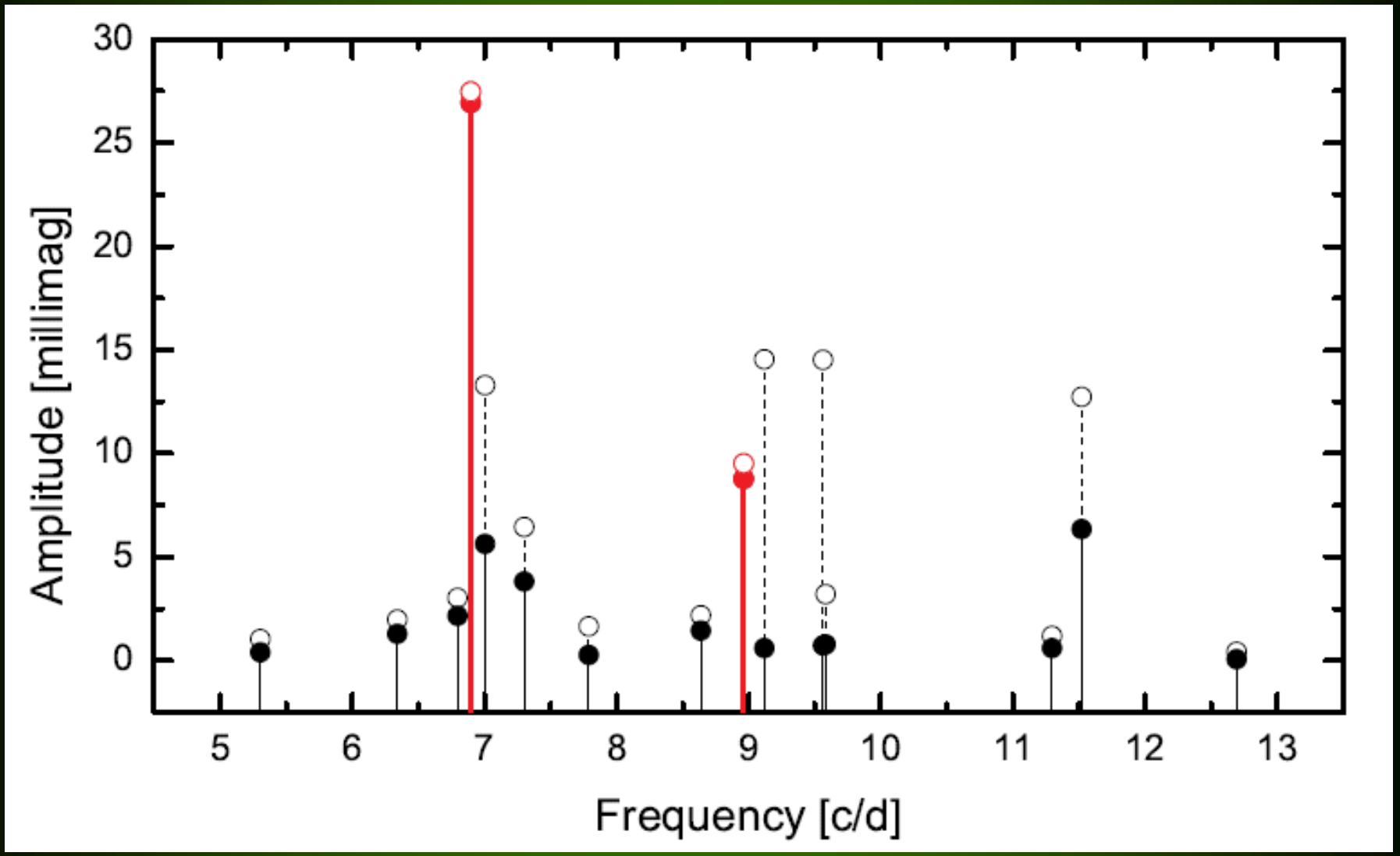


44 Tau

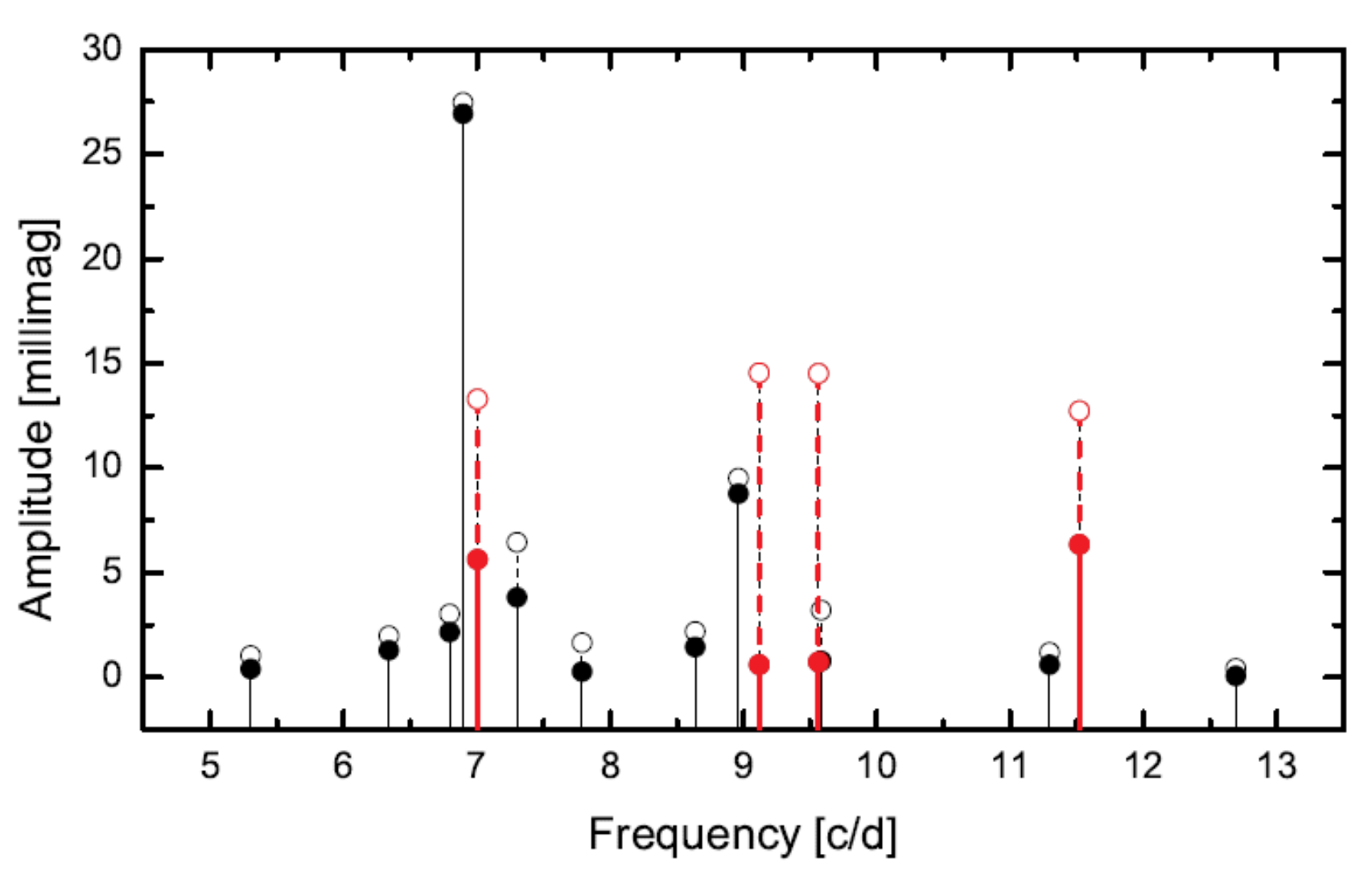


Lenz et al. 2008

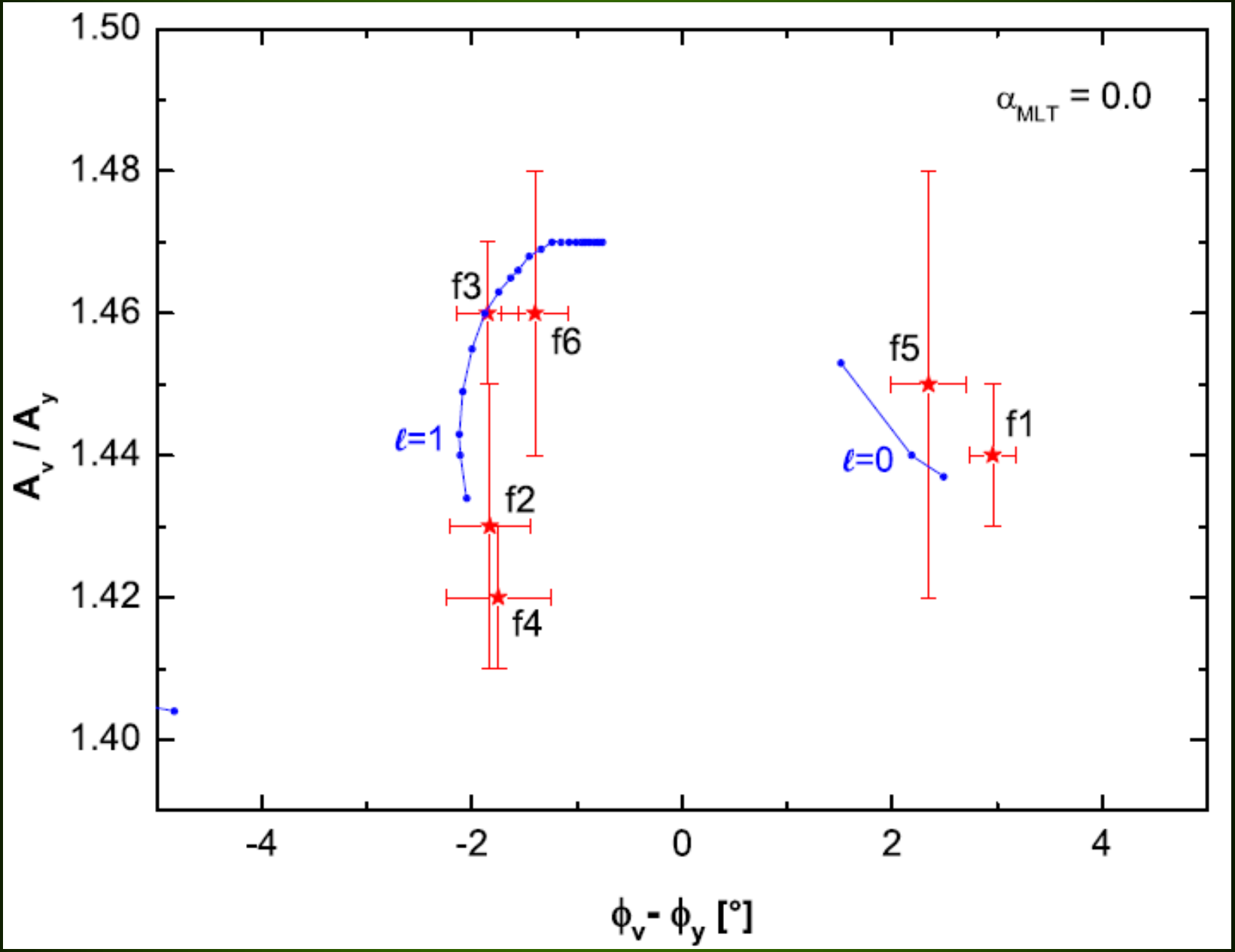
44 Tau, photometry, radial



44 Tau, photometry, dipole

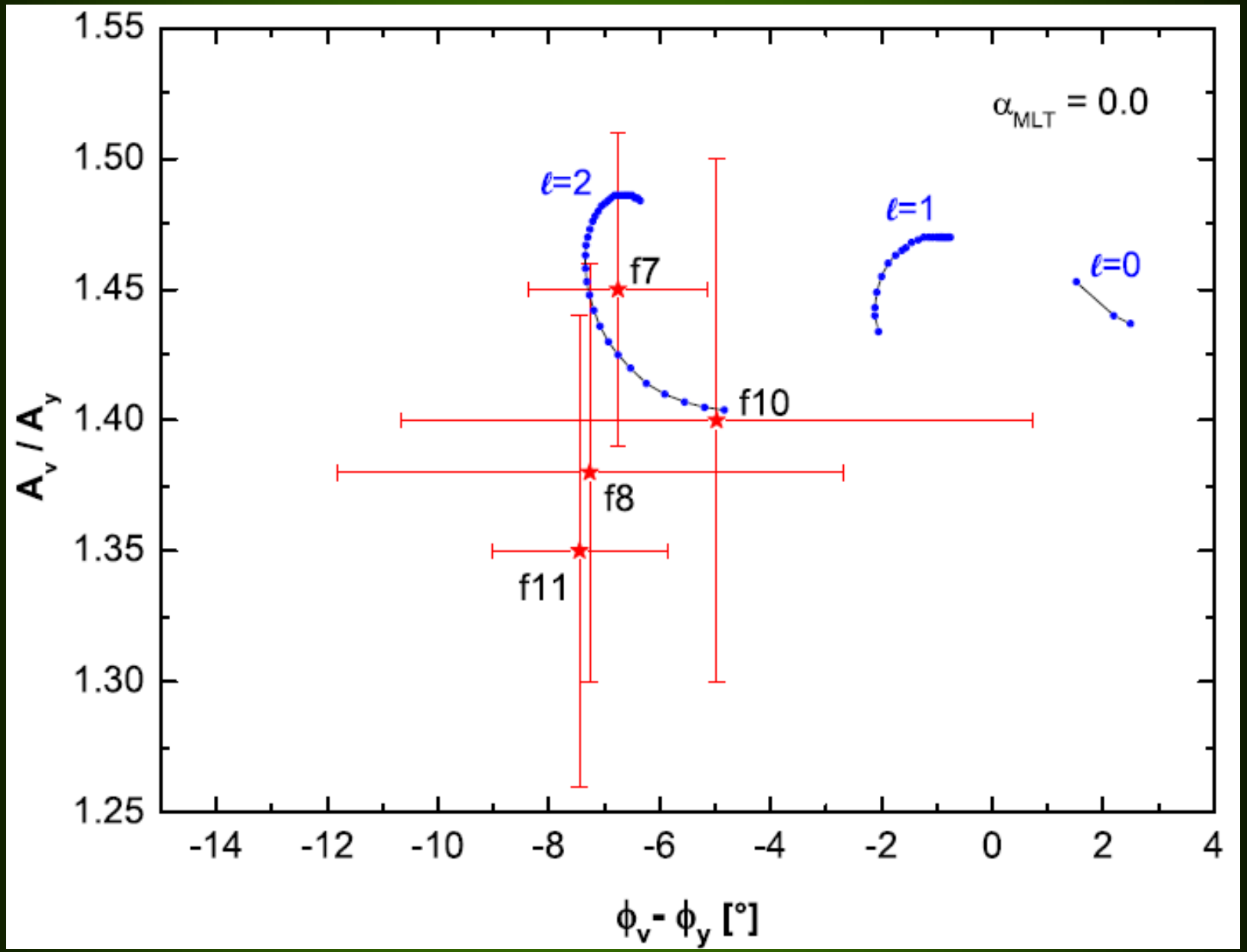


44 Tau, mode identification



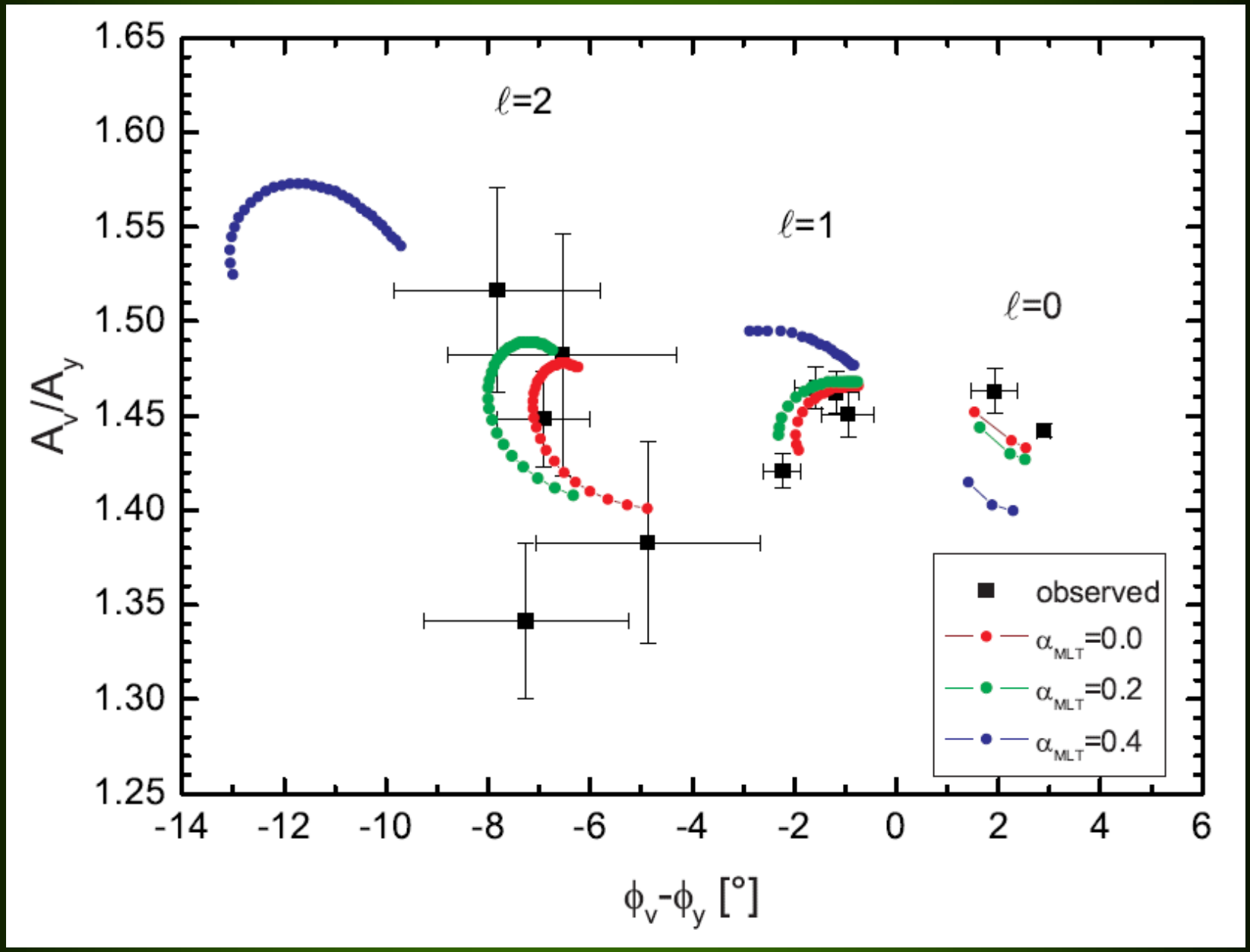
Lenz et al. 2008

44 Tau, mode identification



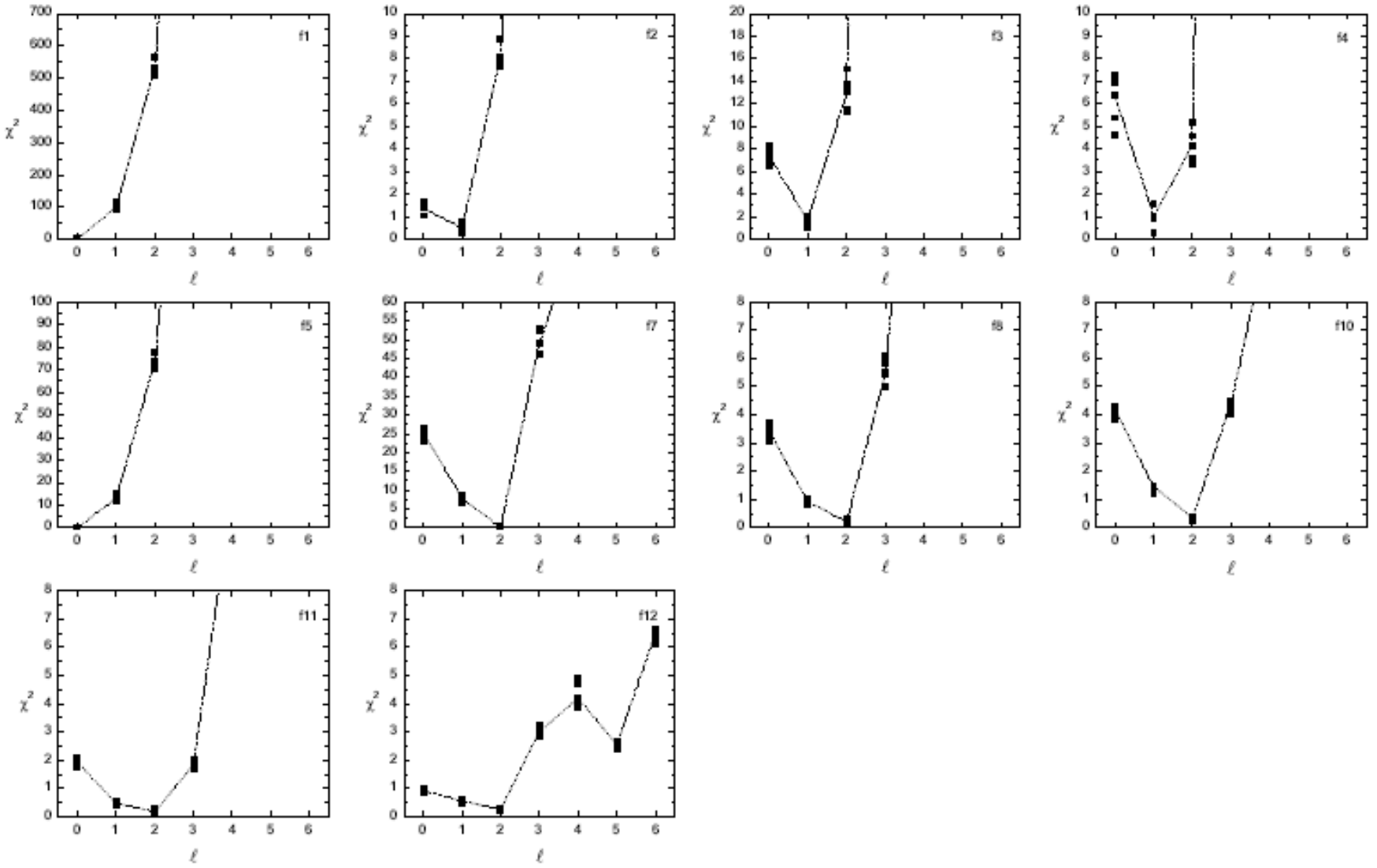
Lenz et al. 2008

44 Tau, mode identification



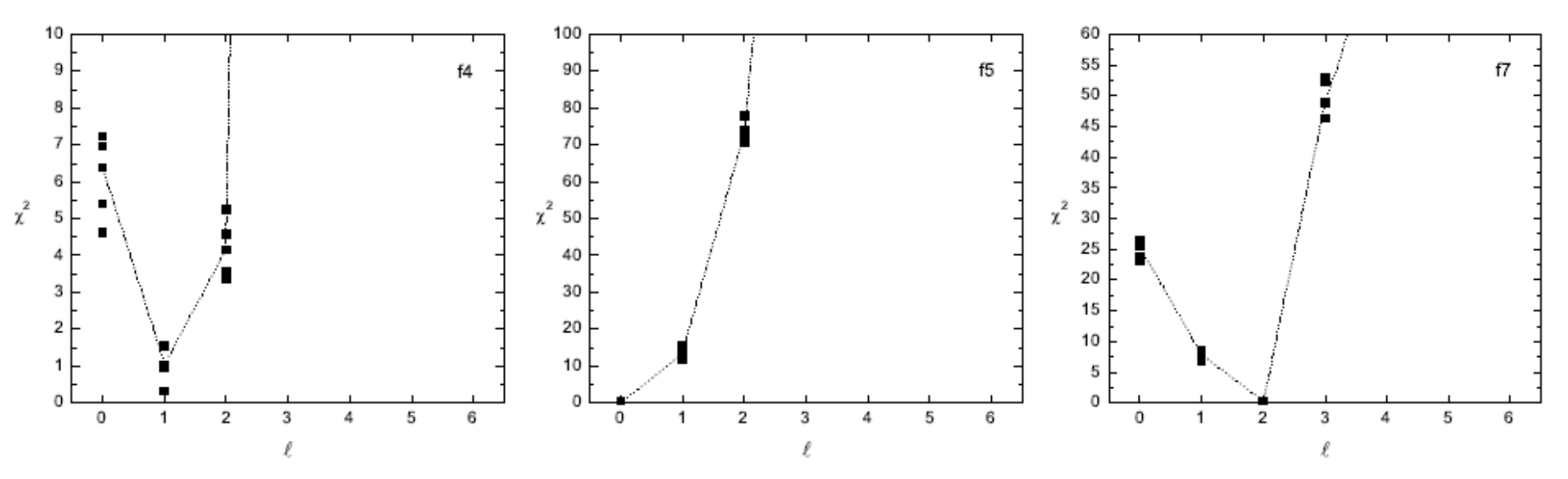
due to Lenz et al. 2008, 2010

44 Tau, mode identification for 10 frequenies.



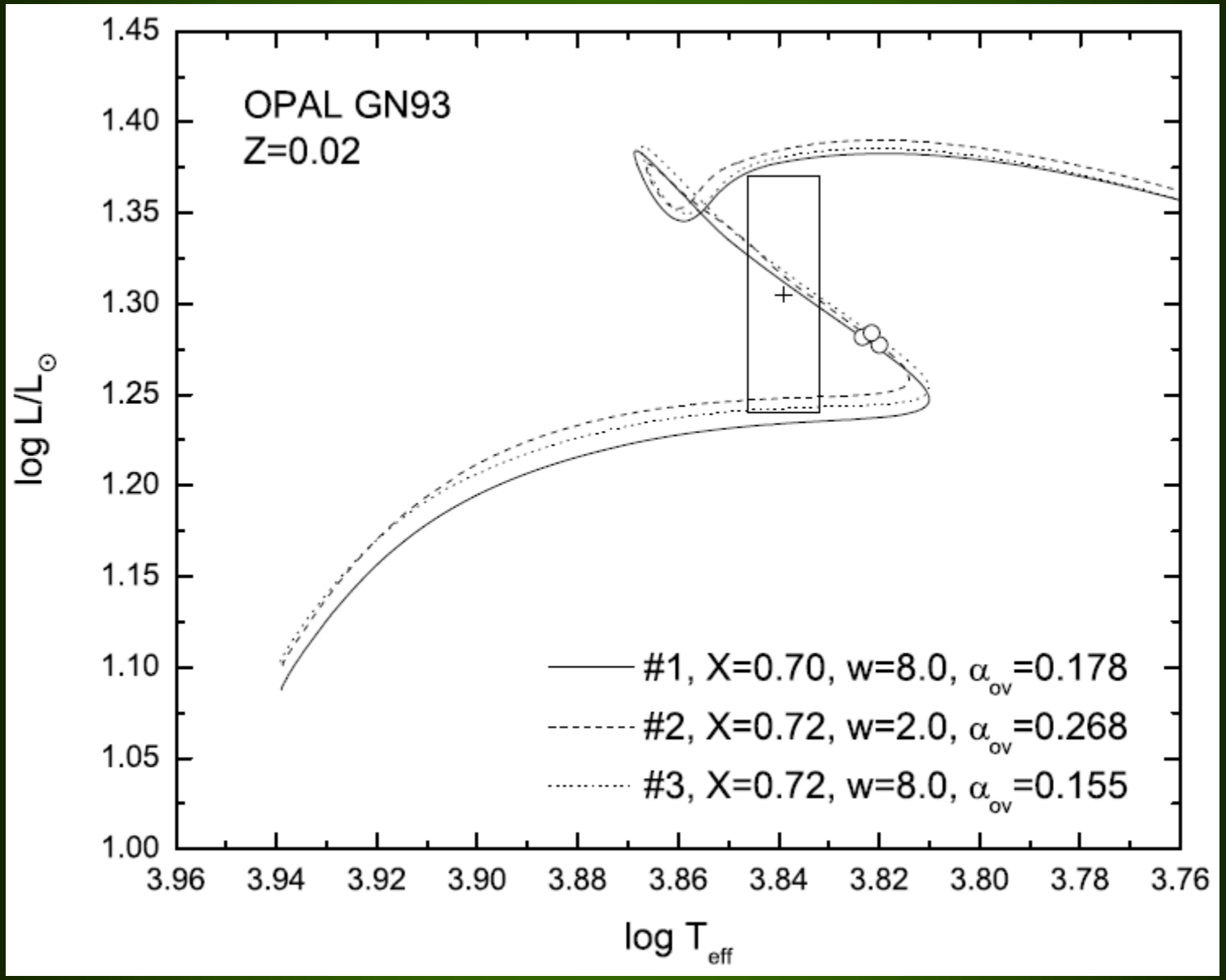
Lenz 2009, PhD Thesis

44 Tau, examples of the mode identification



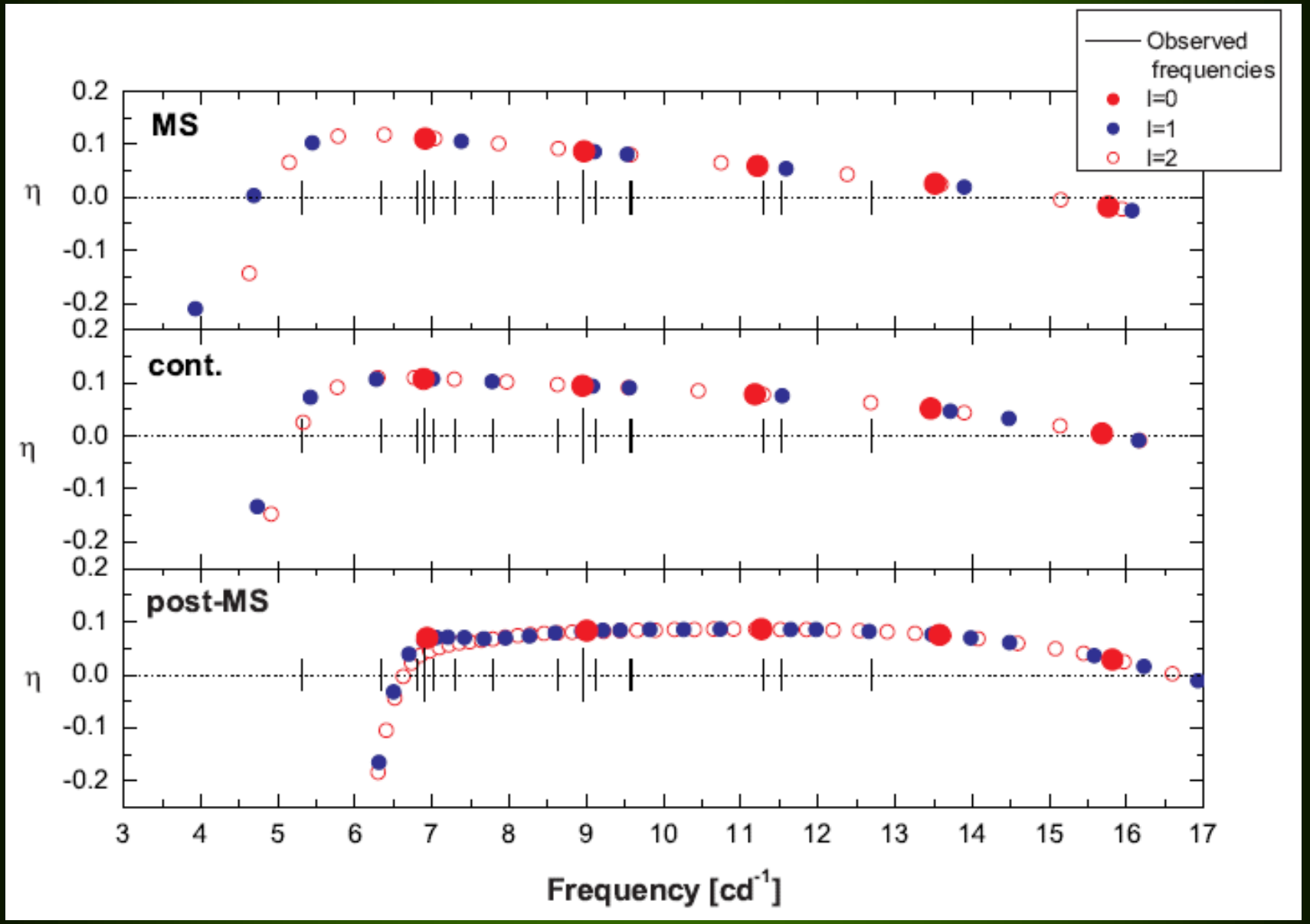
due to Lenz 2009, PhD Thesis

44 Tau. Evolutionary tracks for best models.



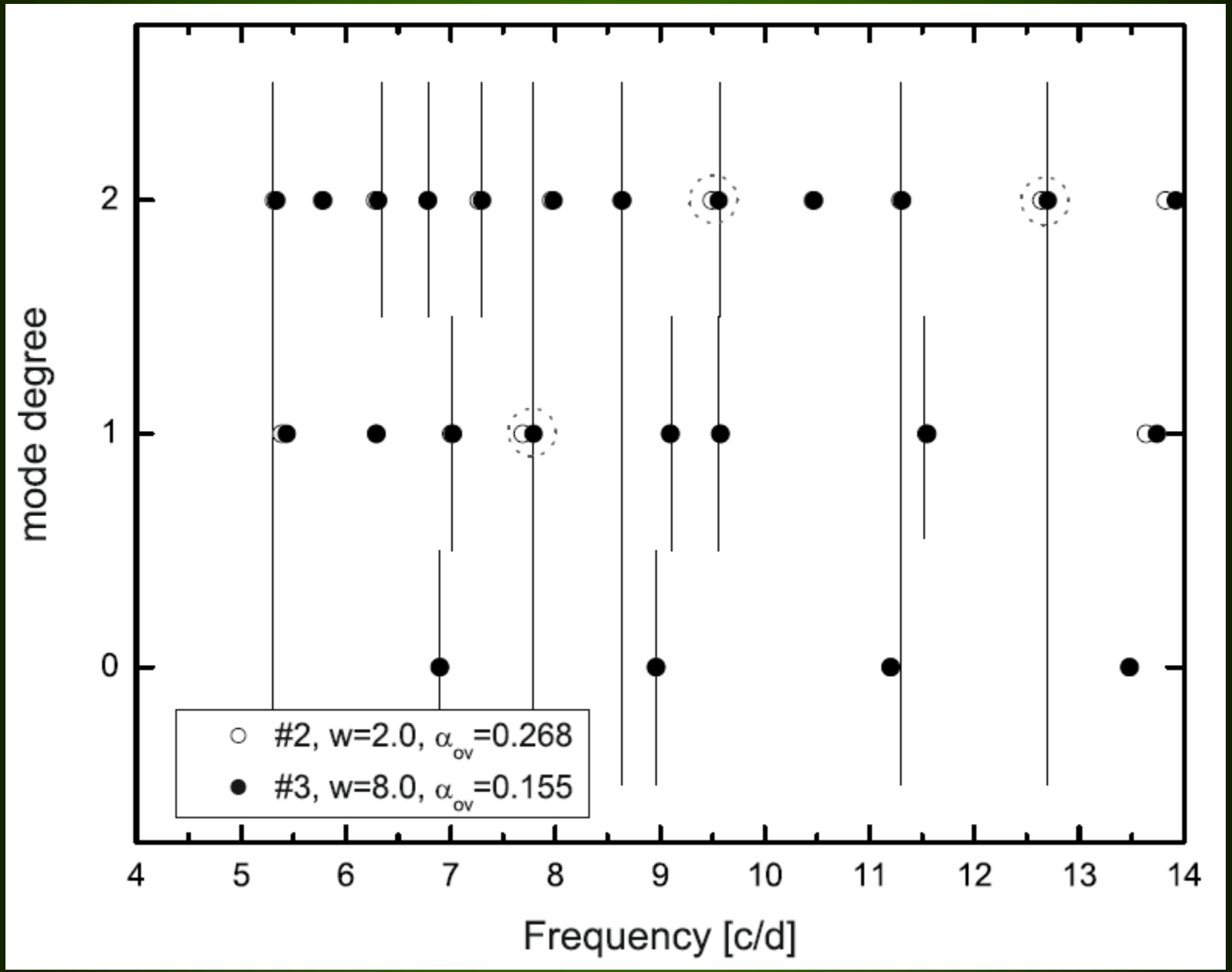
Lenz, Pamyatnykh, Zdravkov & Breger, 2010, A&A 509, A90

44 Tau, pulsational instability range



due to Lenz 2009, PhD Thesis

44 Tau. Frequency fitting for best models.

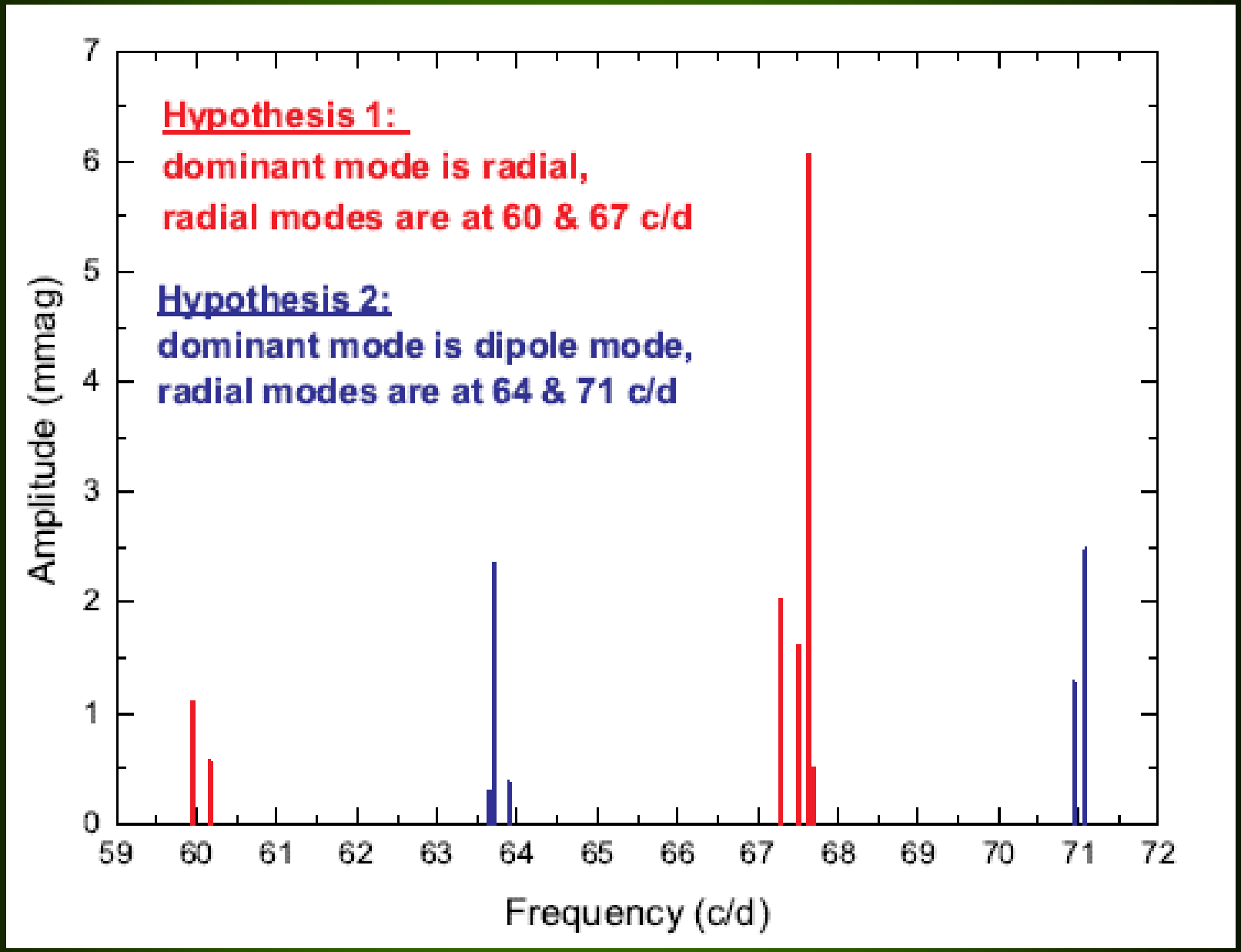


Lenz, Pamyatnykh, Zdravkov & Breger, 2010, A&A 509, A90

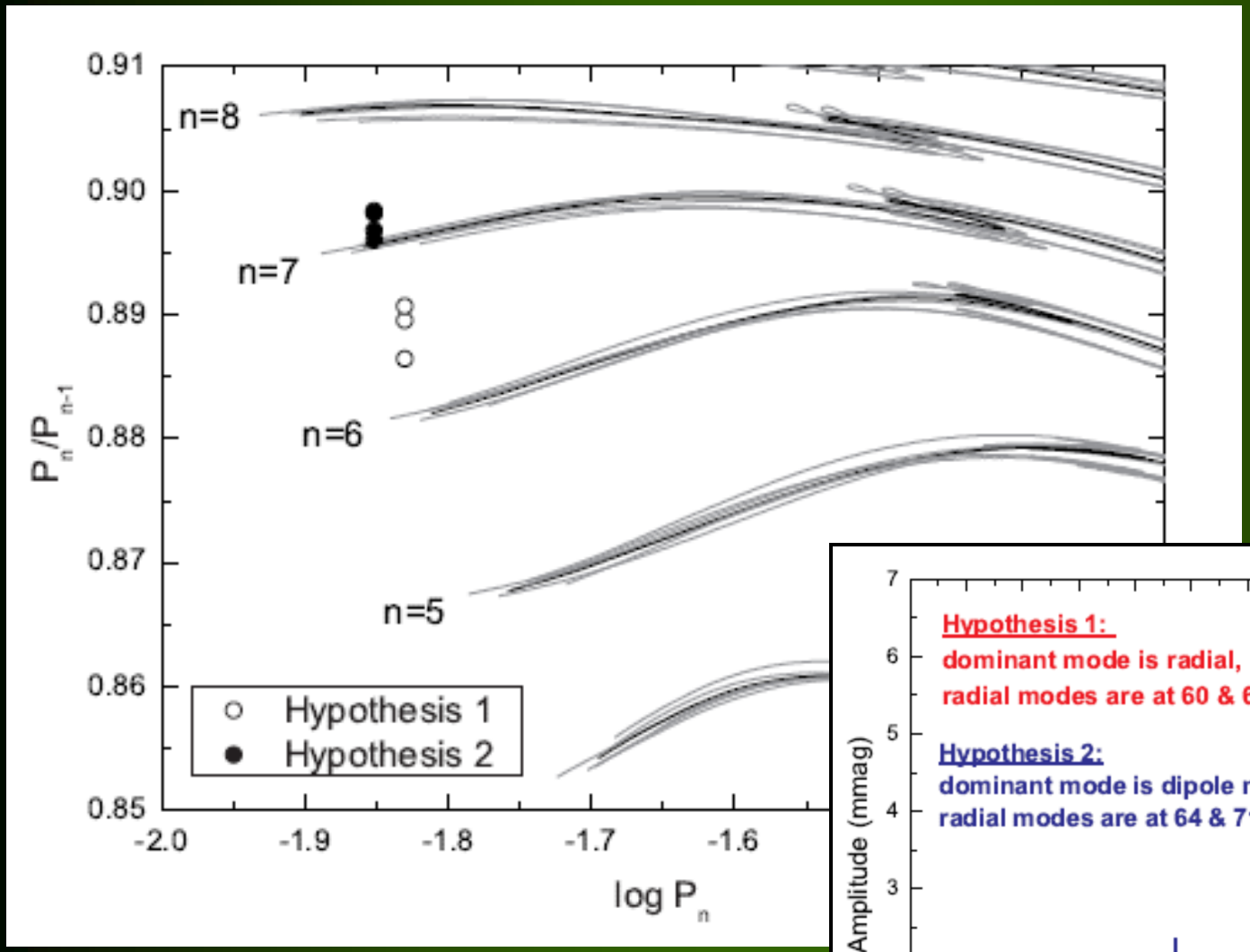
A δ SCUTI STAR HD 144277 (from the MOST observations)

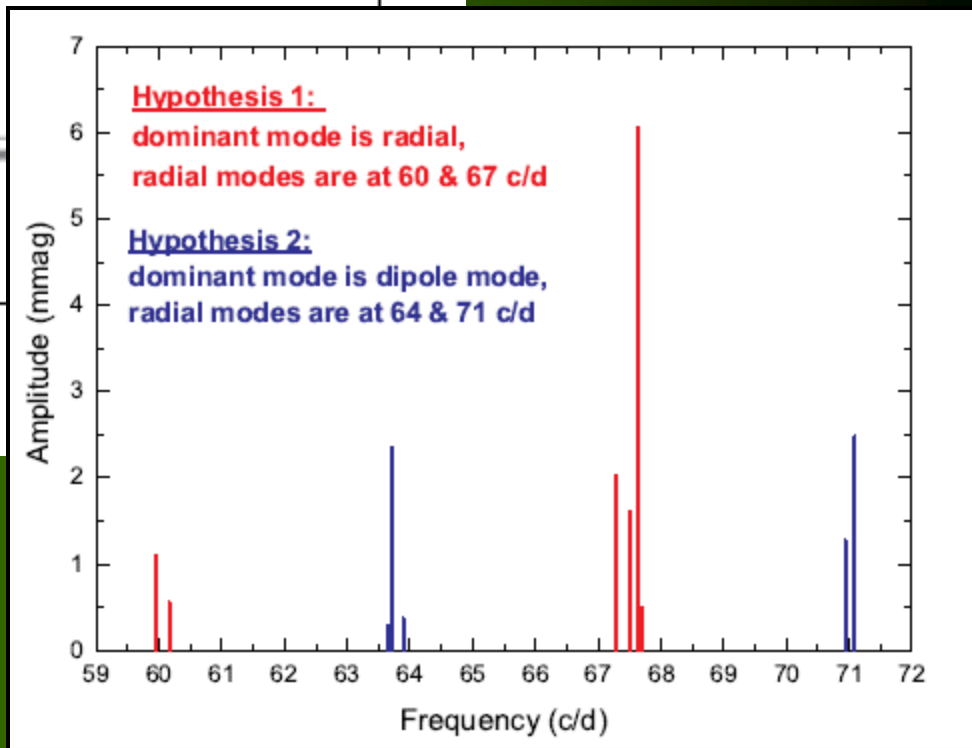
Zwintz, Lenz, Breger, Pamyatnykh et al., 2011, A&A, 533, id. A133.

Pulsation frequency spectrum of HD 144277. Two hypotheses.

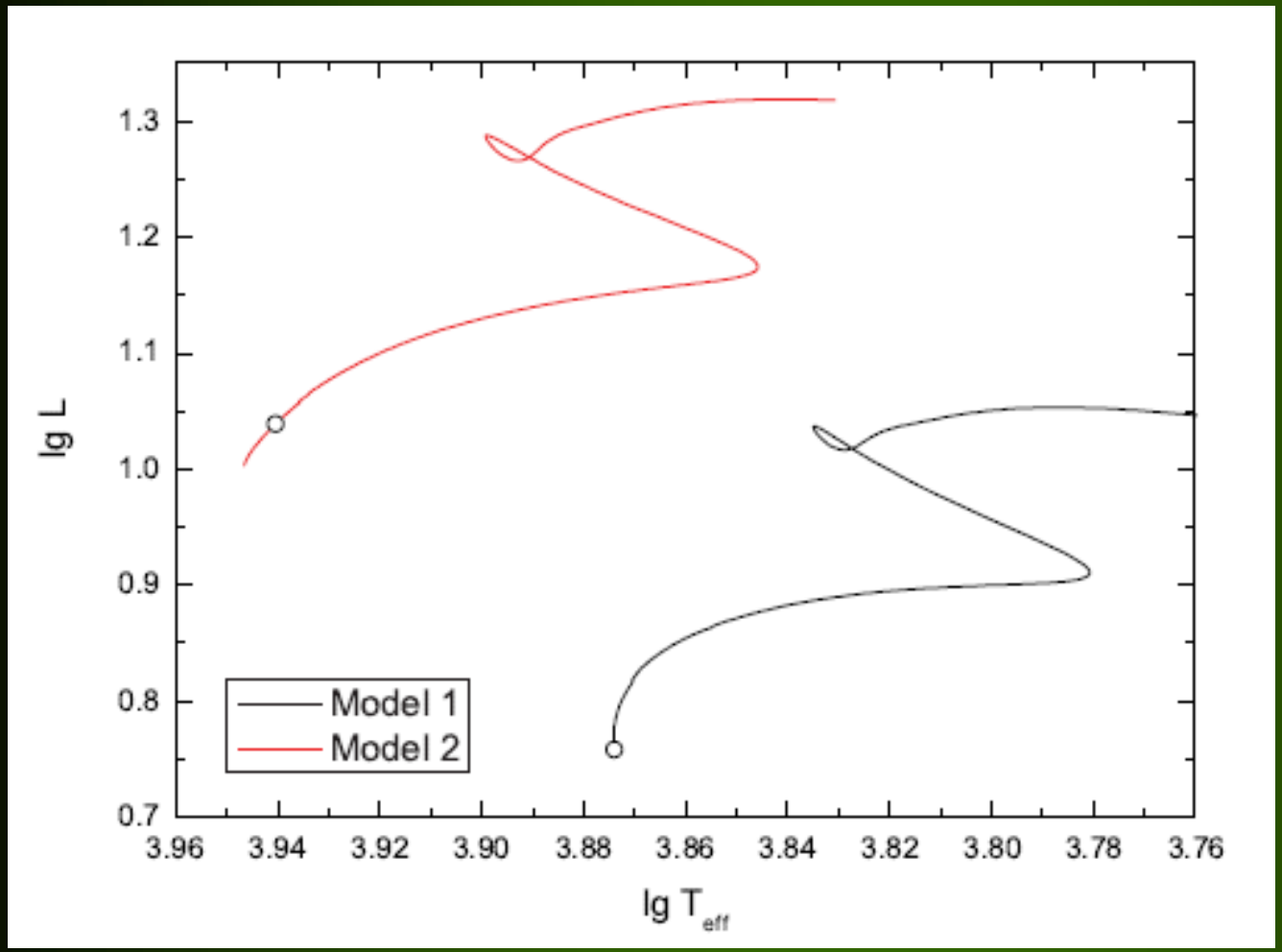


HD 144277. Period ratios of consecutive radial modes versus shorter periods.





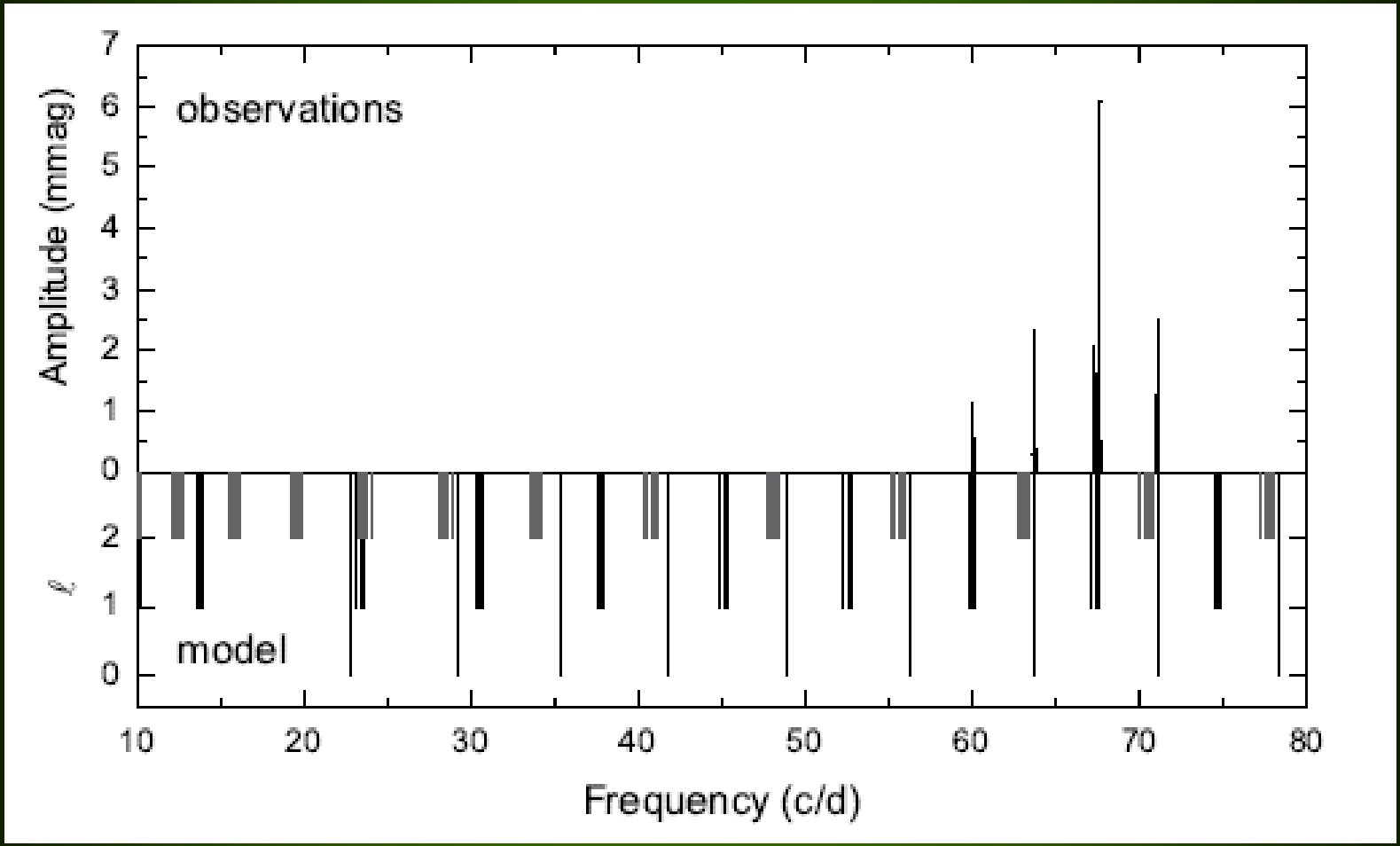
Evolutionary tracks of two representative models which fit two frequencies at 71 and 64 c/d as consecutive **radial** overtones (hypothesis 2). The fitted models are shown by open circles.



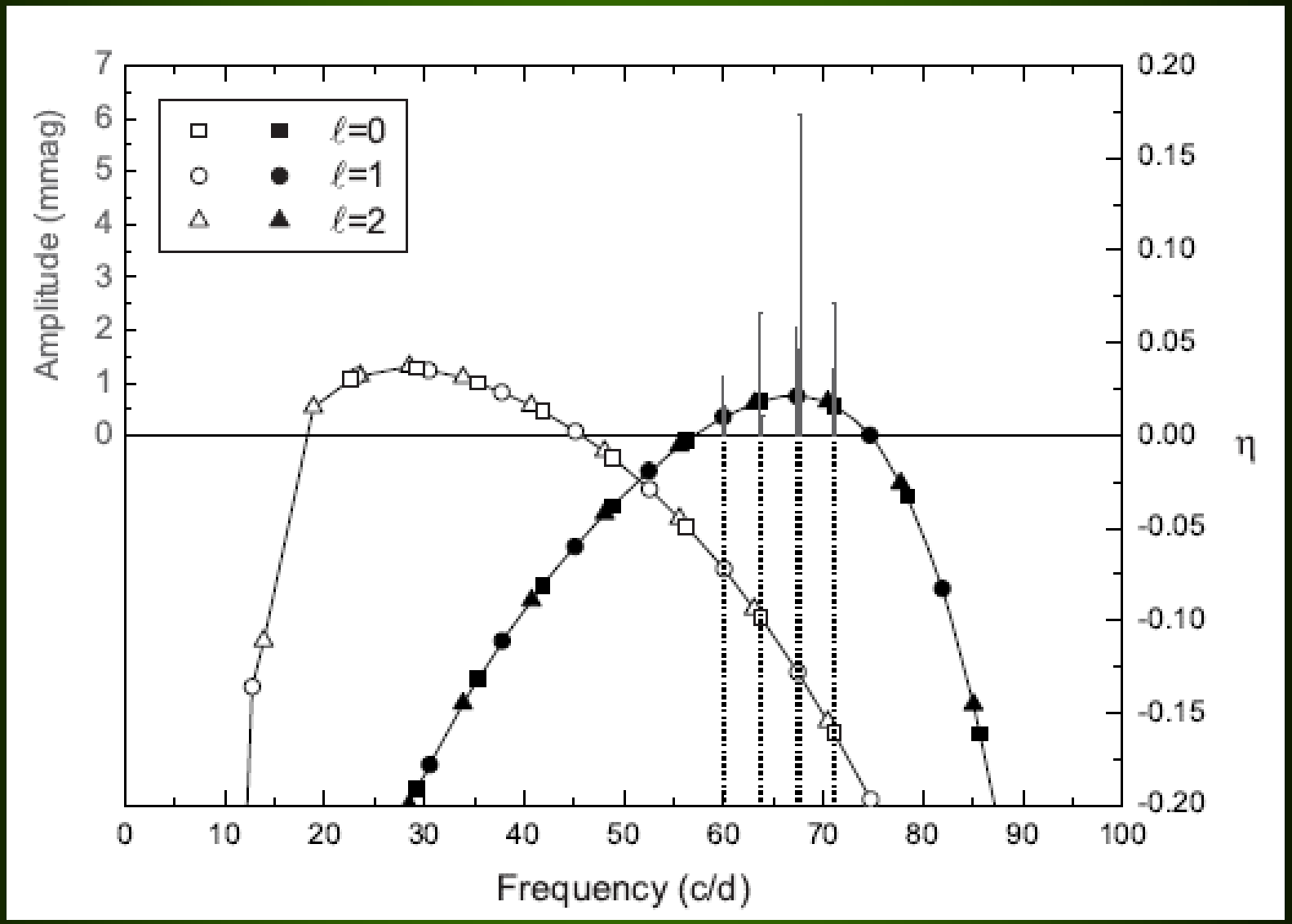
Model 1:
M=1.576
X=0.74
Z=0.0134
log T_{eff} = 3.874
log L = 0.757

Model 2:
M=1.662
X=0.70
Z=0.01
log T_{eff} = 3.941
log L = 1.04

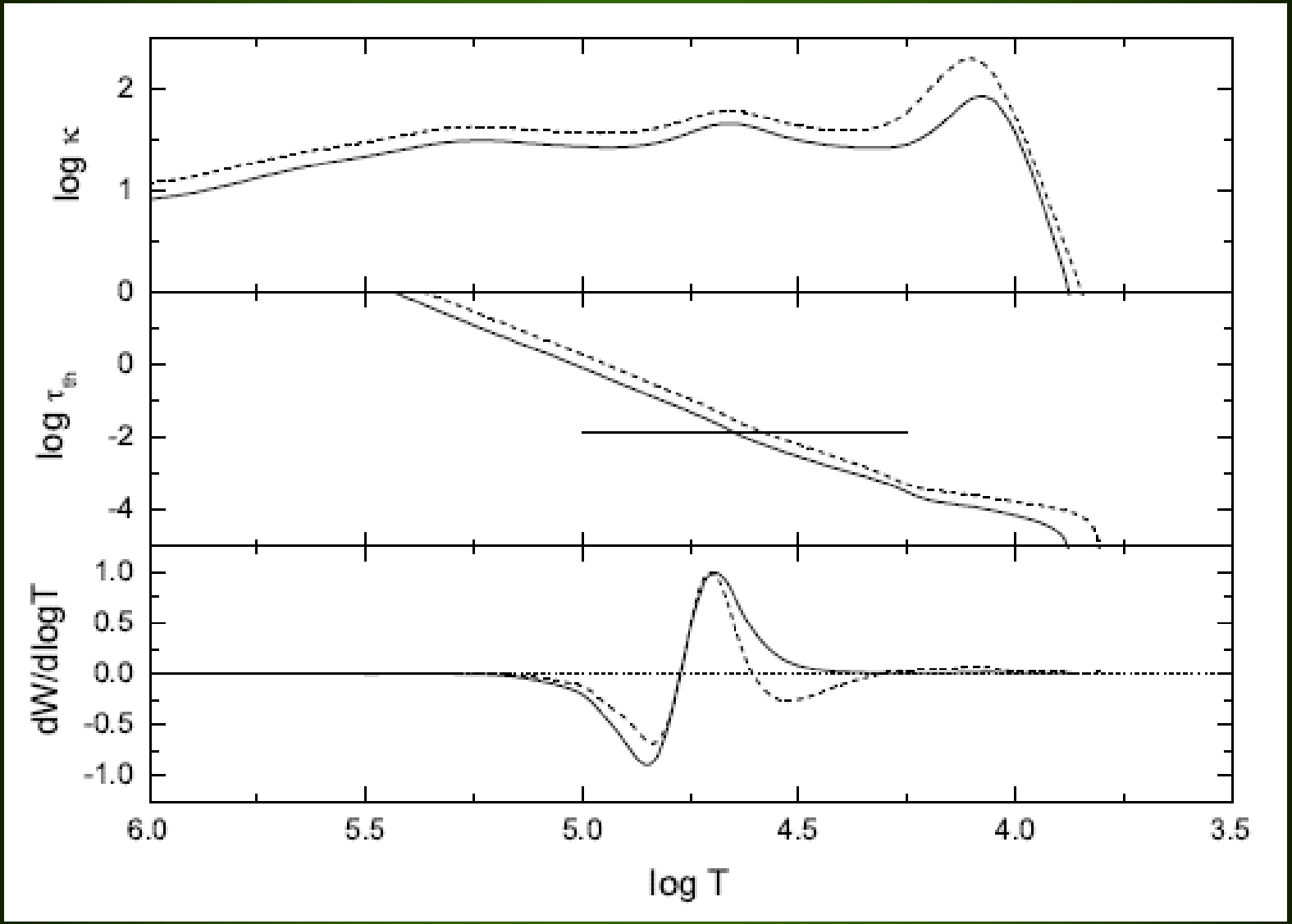
Comparison between observed frequencies and the theoretical Frequency spectrum according to the best model (V_{rot} = 15 km/s)



Instability parameter, η, in a model with solar composition (open symbols) And in a model with modified metallicity and helium abundance (filled symbols) $\eta > 0$ for unstable modes. Both models fit two frequencies at 71 and 64 c/d as consecutive radial modes. Vertical lines mark observed frequencies.



Opacity, local thermal timescale and differential work integral for the dipole mode at 67 c/d (dominant frequency). Dashed line – model of solar composition (damped mode), solid line – modified model (excited mode)



Examples: β CEPHEI STARS

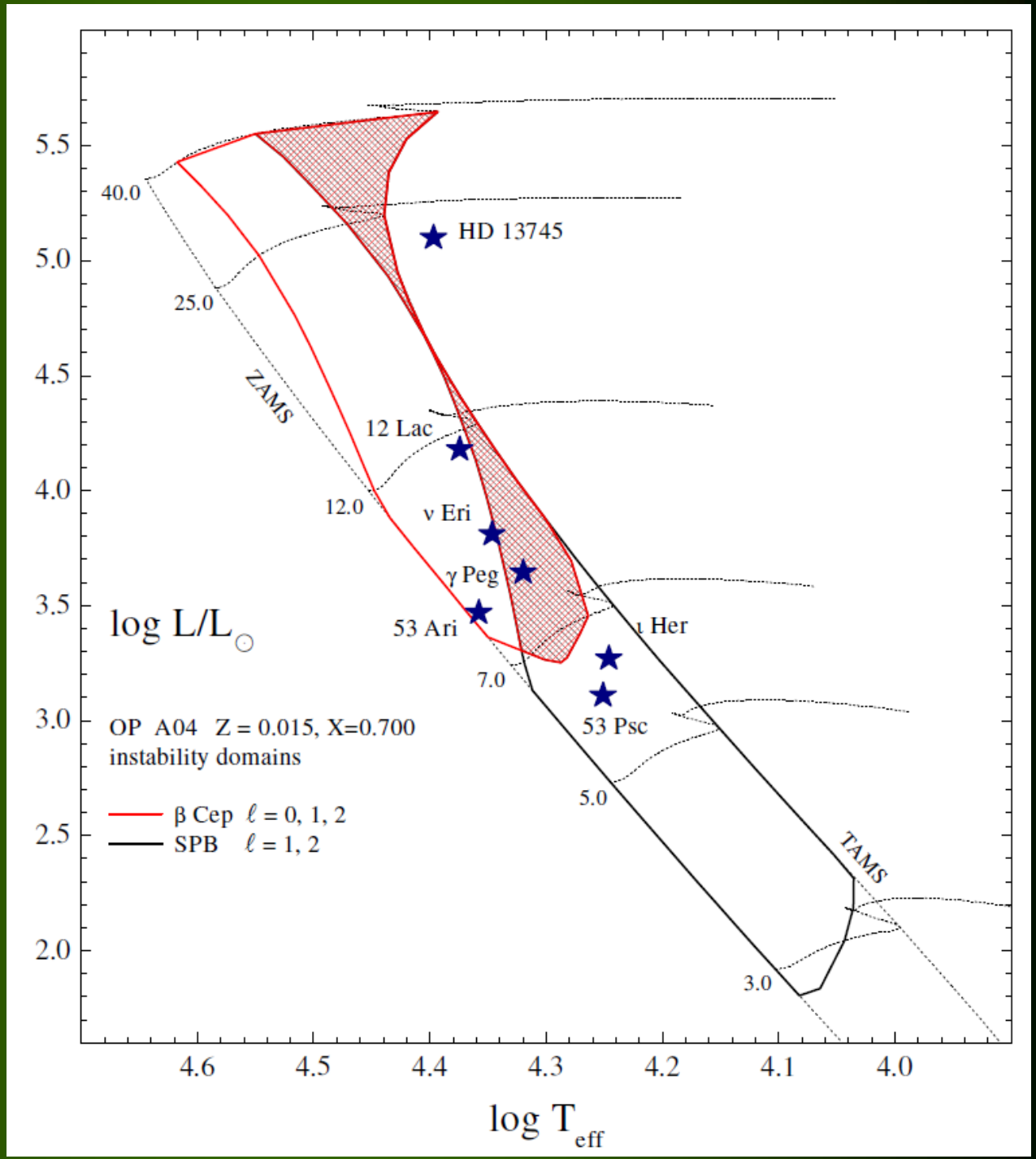
Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh, 2005, A&A 441, 641

See also posters 61 and 62:
W. Szewczuk, P. Walczak & J. Daszyńska-Daszkiewicz

ν Eridani

14 frequencies (degree l is identified for 8 of them),
Sp B2 III,
mV = 3.920,
π = 5.56 +- 0.88 mas,
Vrot = 6 km/s (SIMBAD: Vsini = 7, 20, 25 km/s),
[m/H] = 0.05 +- 0.09,
Teff = 22200 +- 600 K

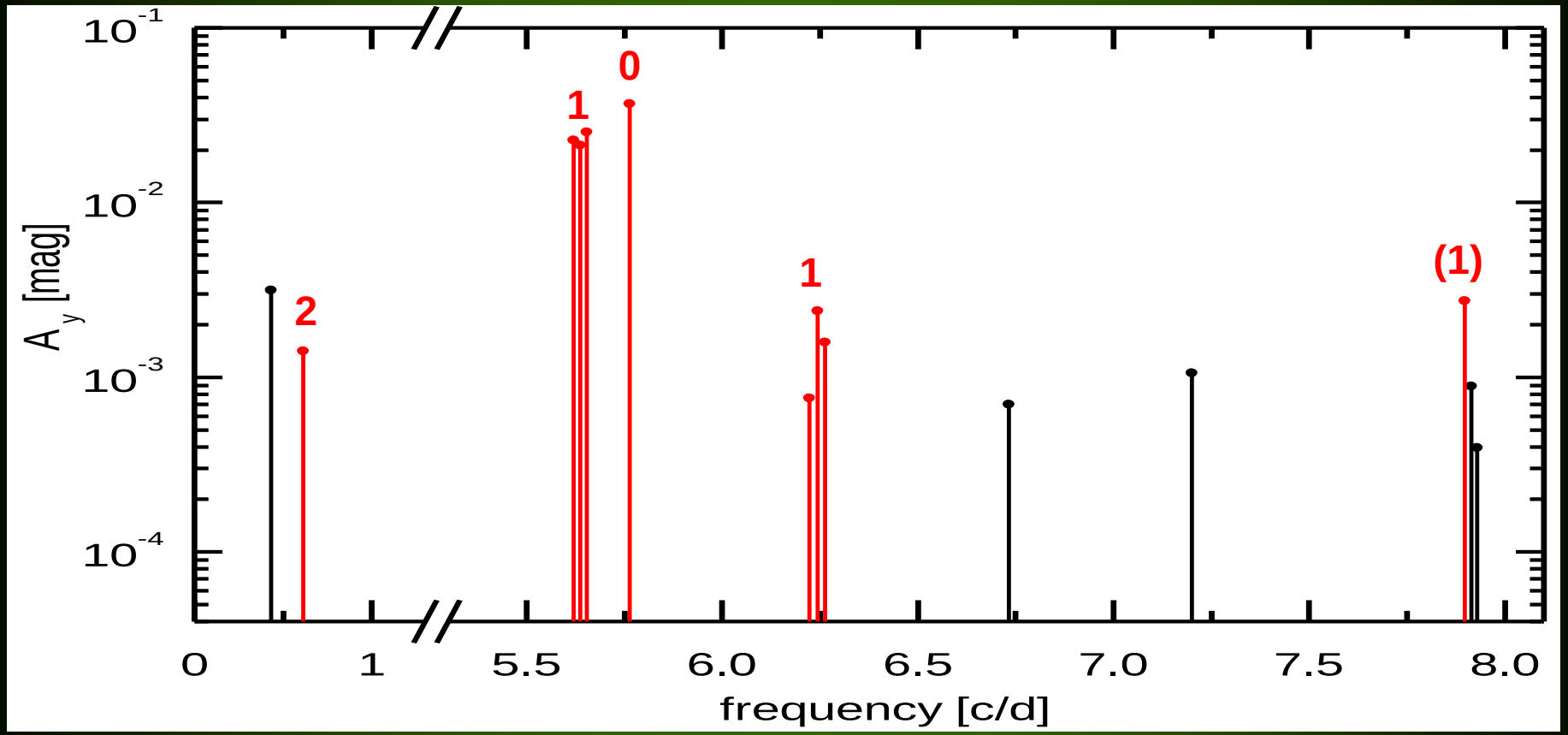
ν Eridani – a hybrid β Cep / SPB star



Zdravkov 2011

ν Eridani – the multiperiodic β Cephei star

14 oscillation frequencies (from ground-based observations)

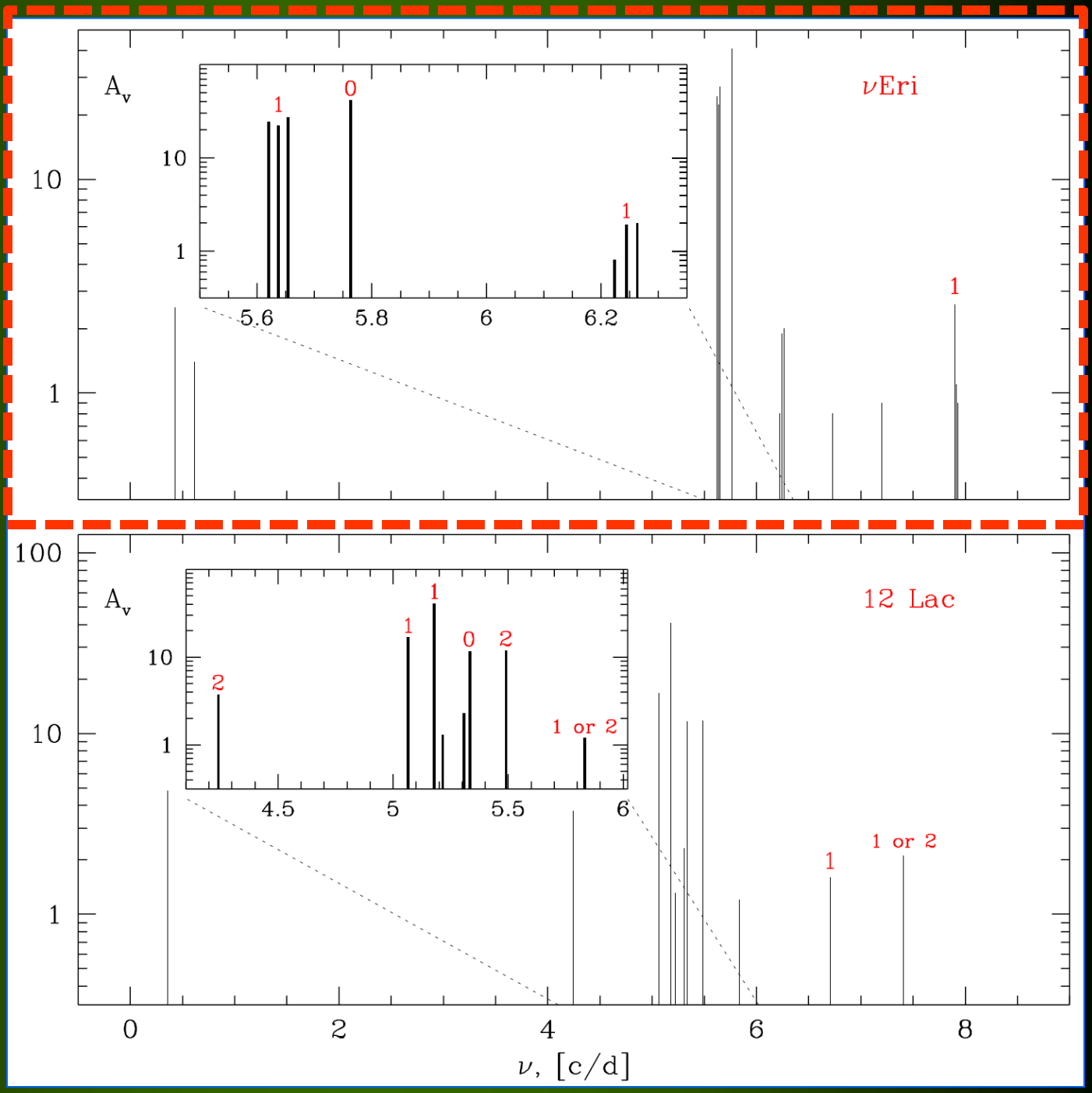


Oscillation spectra of ν Eri and 12 Lac

ν Eri:
Jerzykiewicz et al. 2005

12 Lac:
Handler et al. 2006

Dziembowski &
Pamyatnykh 2008

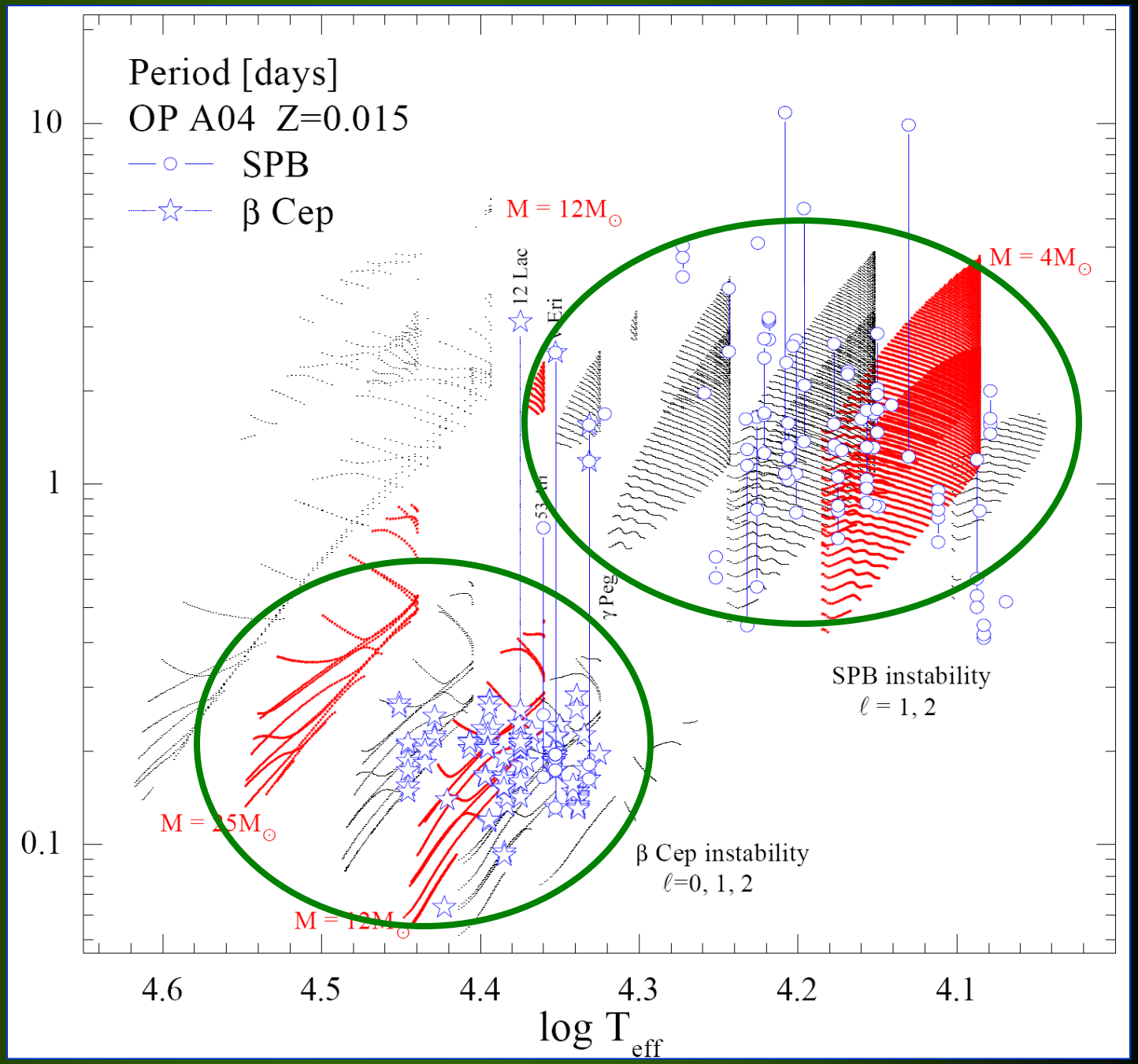


Periods of observed β Cep and SPB stars in domains of theoretical periods of unstable modes

κ OP A04

Z=0.015

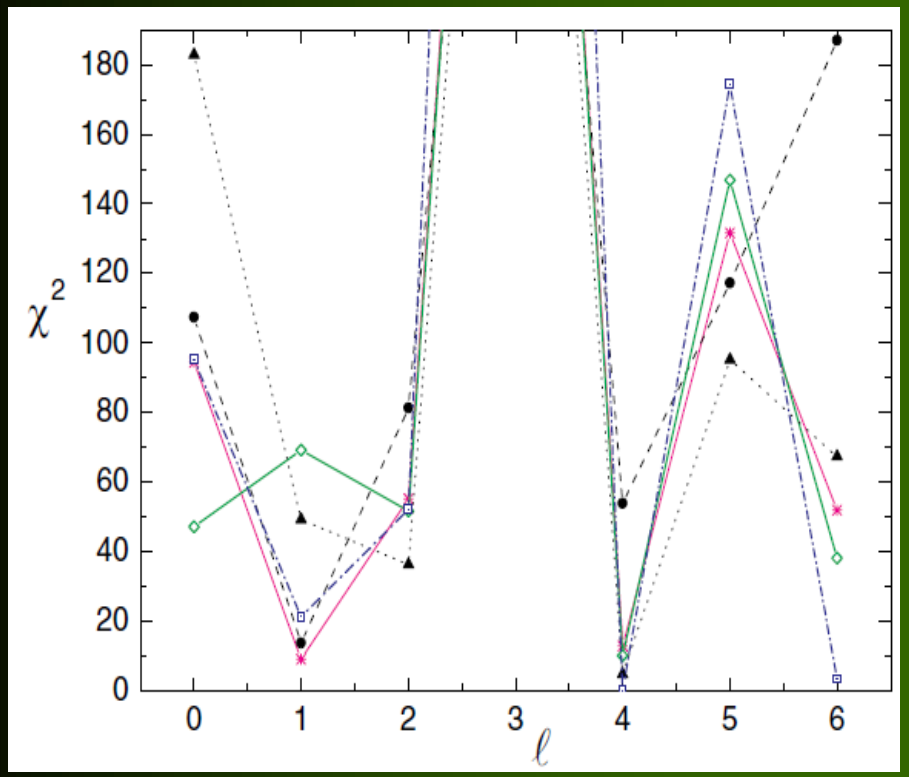
Some hybrid stars (with both β Cep and SPB pulsations) are clearly seen



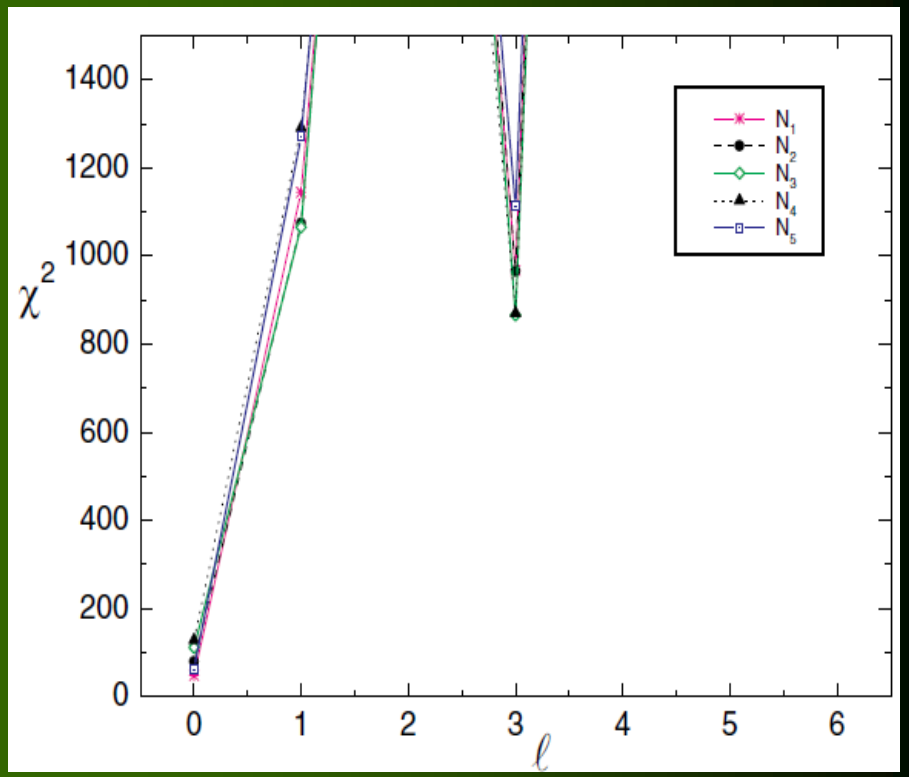
Zdravkov & Pamyatnykh 2008

Identification of ℓ for ν_1 (5.763 c/d)

photometry

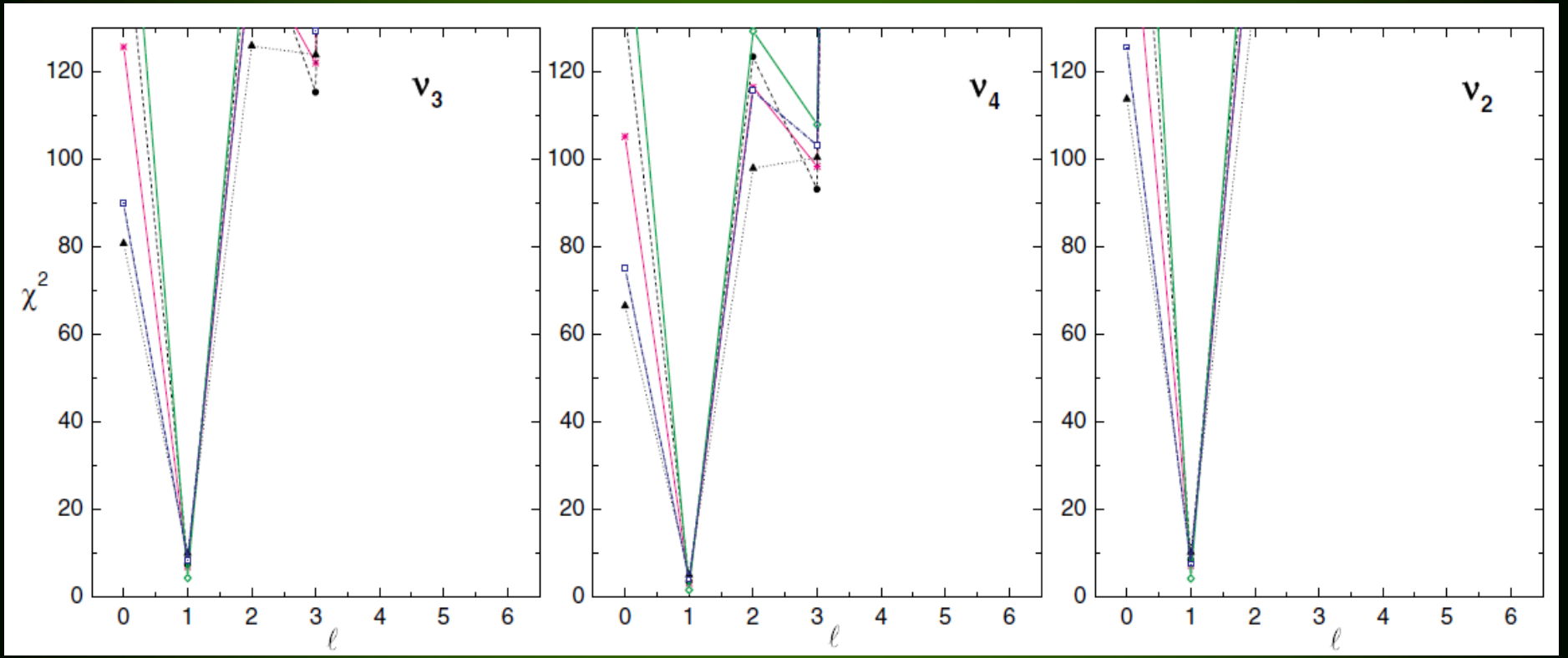


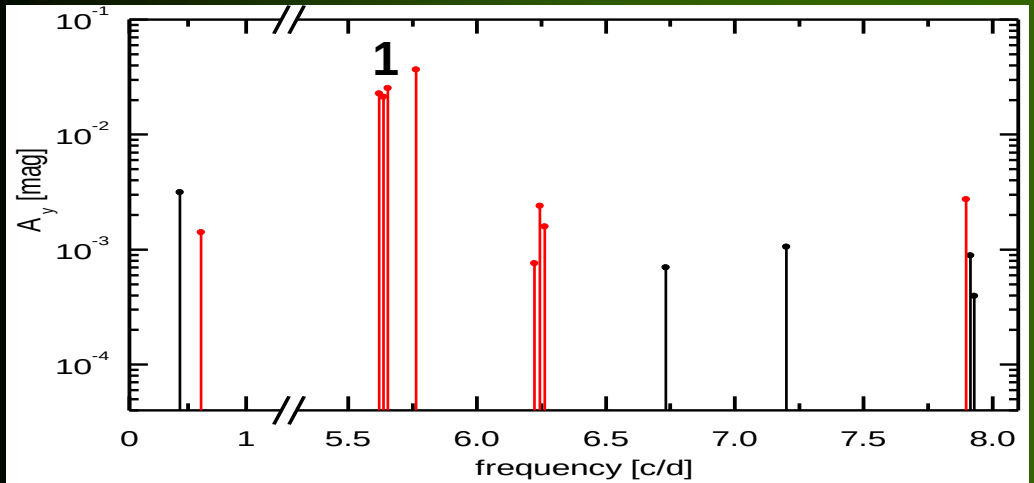
photometry + spectroscopy



Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2005

Identification of ℓ for the close frequency triplet (ν_3 ν_4 ν_2)

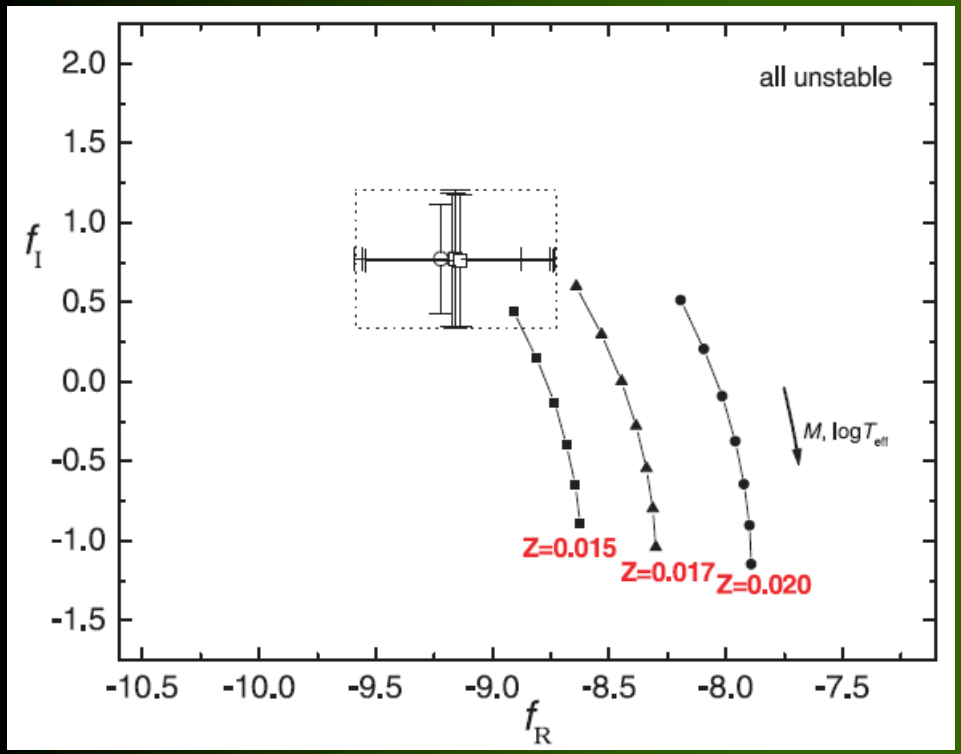


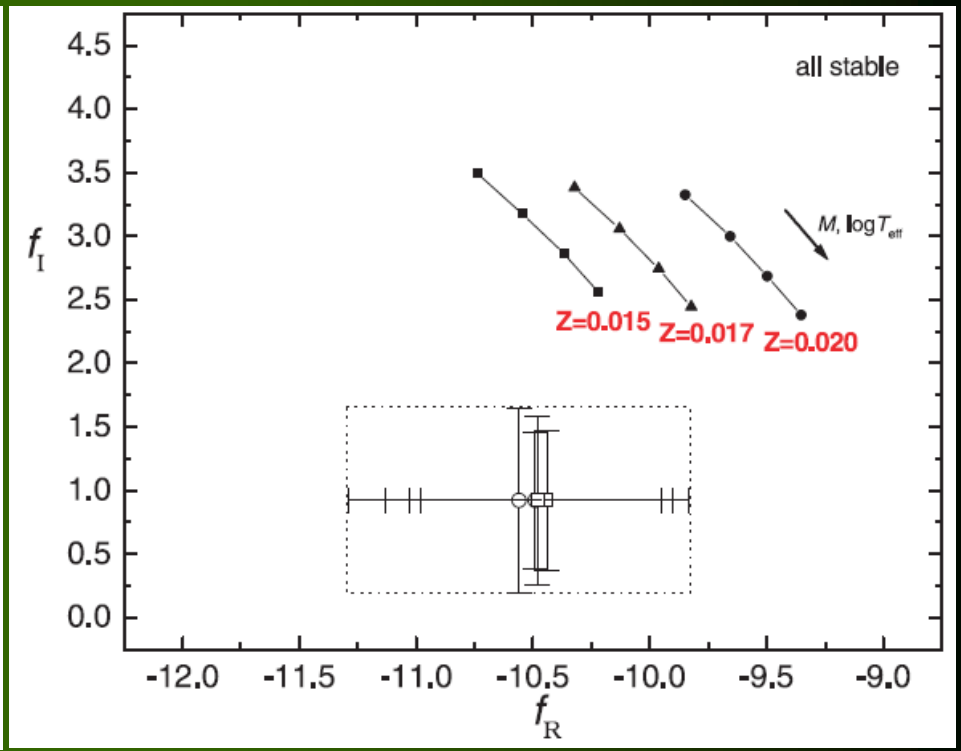


Triplet
5.620, 6.637, 5.654 c/d

Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh, 2005

ν Eri. Theoretical and empirical *f* values for radial mode ν_1 assuming various stellar parameters within the error box.



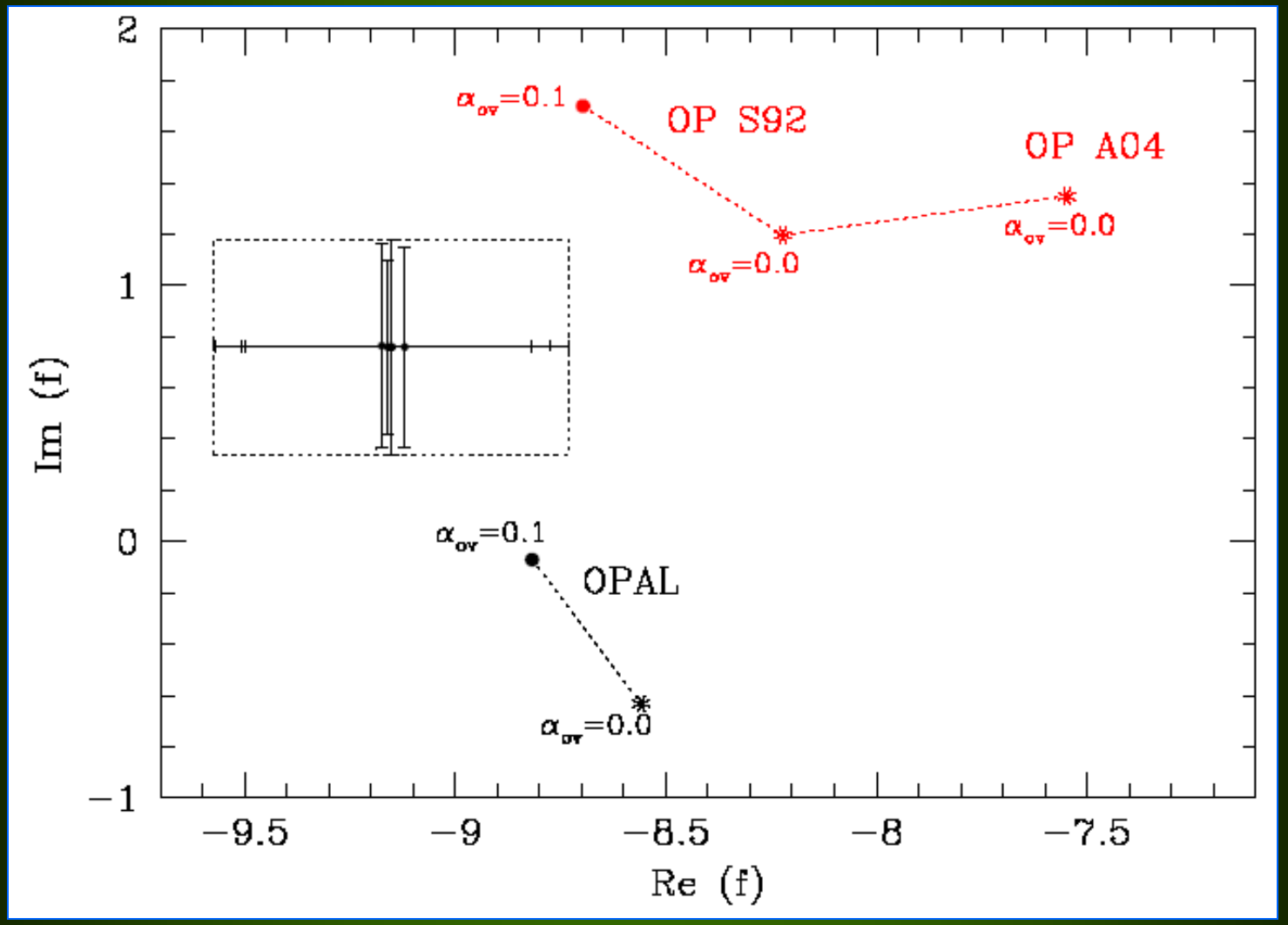


Fundamental mode hypothesis

First overtone hypothesis

Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2005

ν Eri: Relative luminosity variations for different seismic models in comparison with the empirical estimations

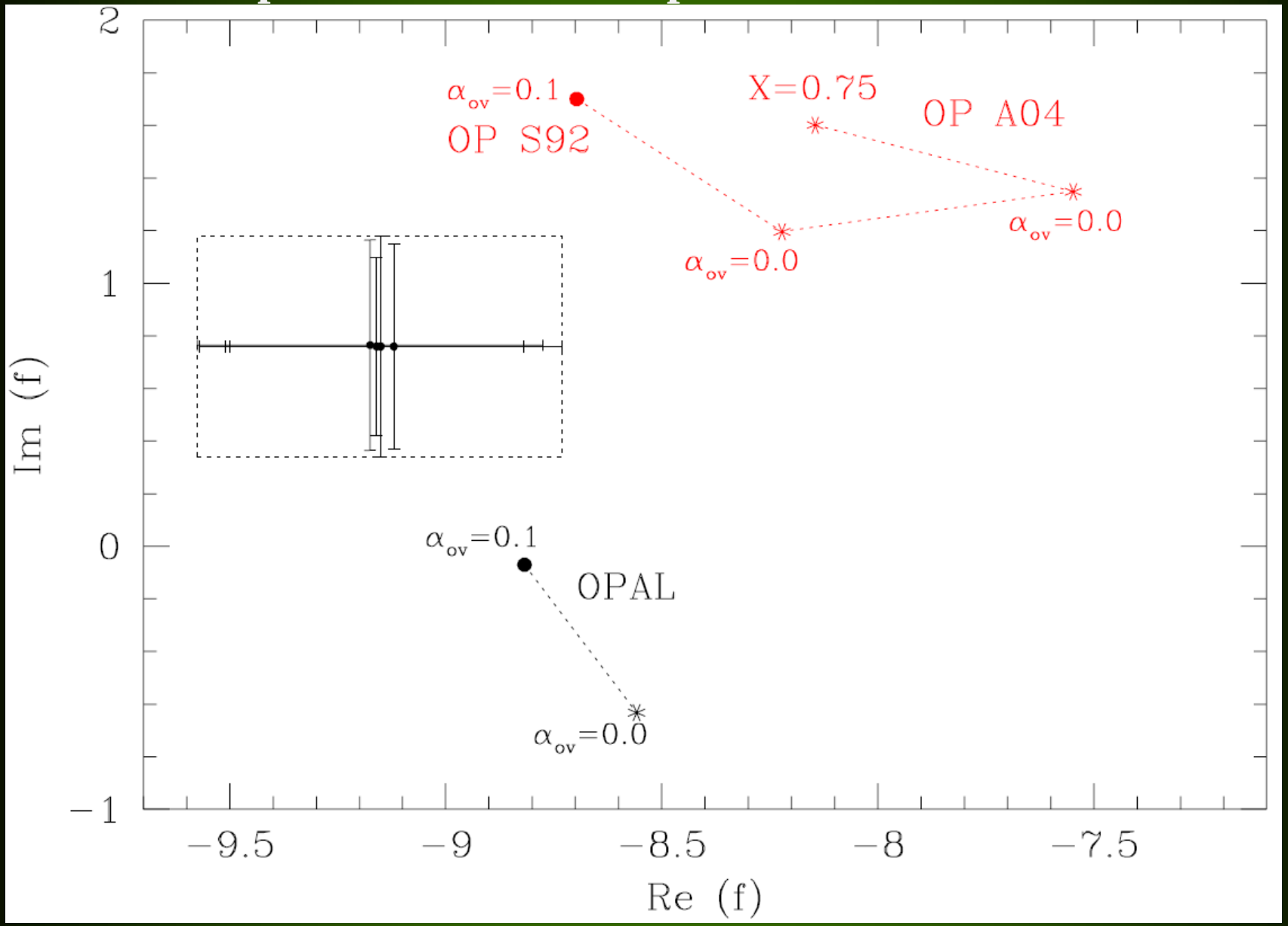


$$\delta r(R,\theta,\varphi) = R\mathrm{Re}\{\varepsilon Y_\ell^m \mathrm{e}^{-\mathrm{i}\omega t}\}$$

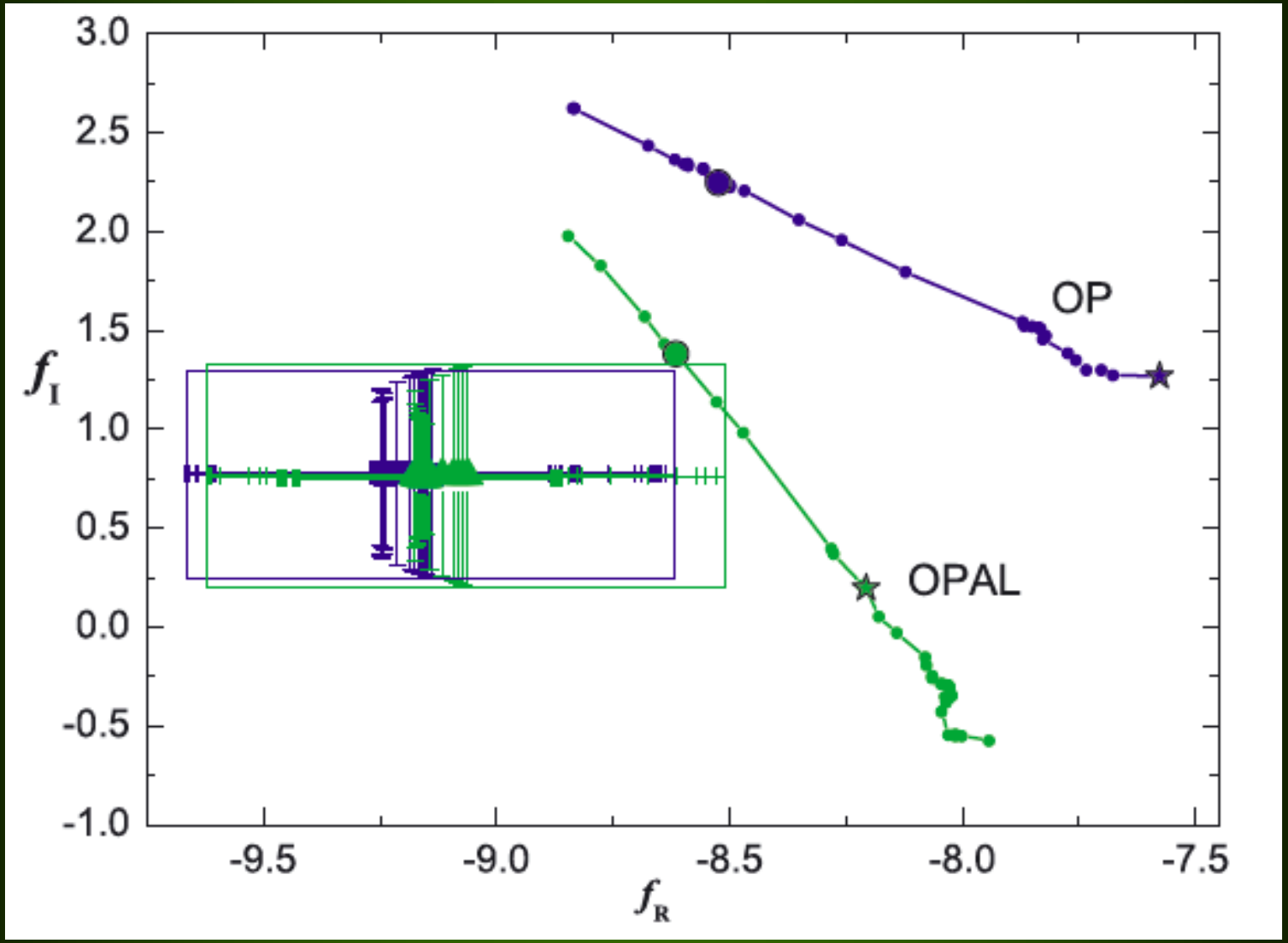
$$\frac{\delta \mathcal{F}_{\mathrm{bol}}}{\mathcal{F}_{\mathrm{bol}}} = \mathrm{Re}\{\varepsilon f Y_\ell^m \mathrm{e}^{-\mathrm{i}\omega t}\}$$

due to Daszyńska-Daszkiewicz et al. 2005

Relative luminosity variations for different seismic models in comparison with the empirical estimations

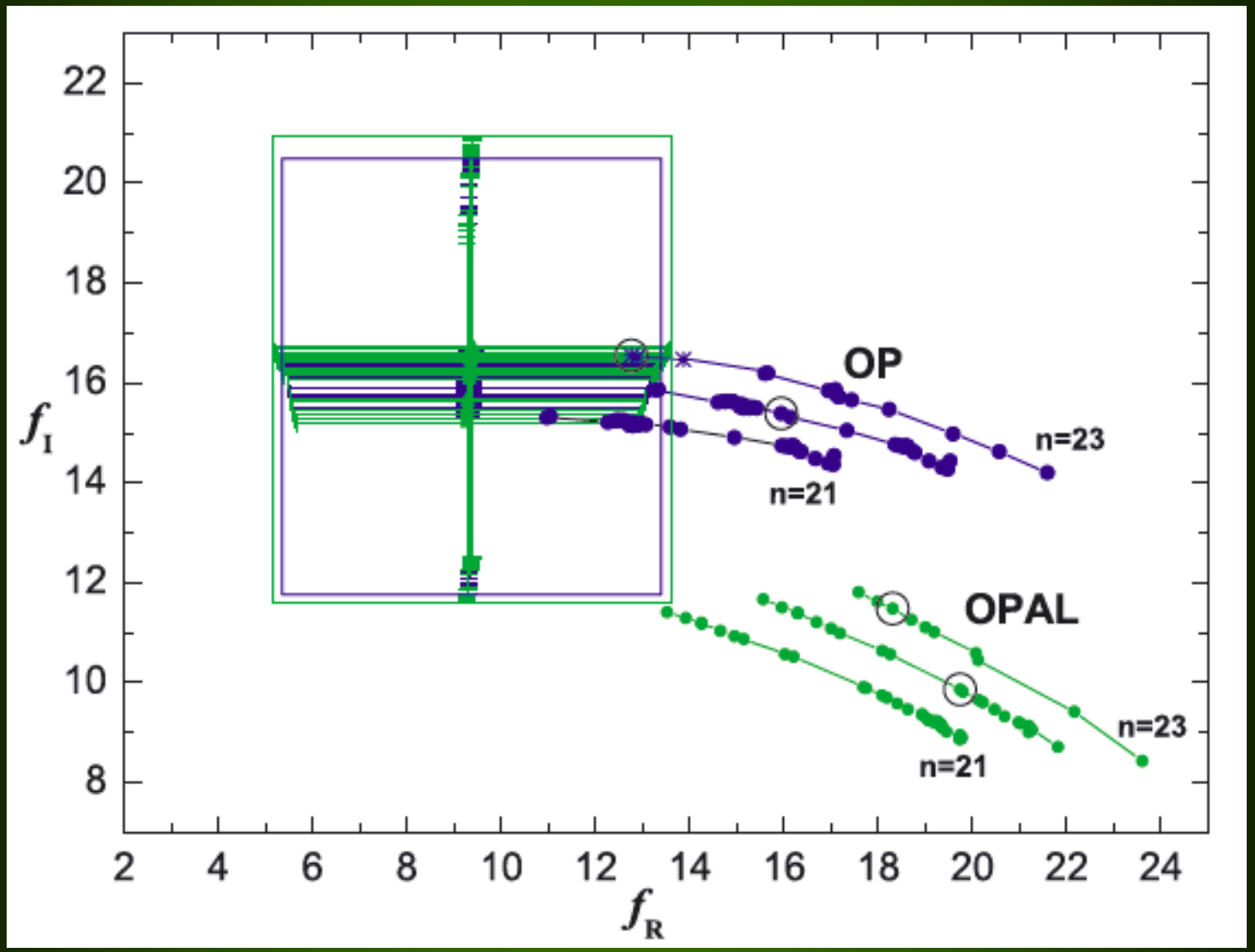


ν Eri. Empirical and theoretical *f* values for radial p1 mode, ν1



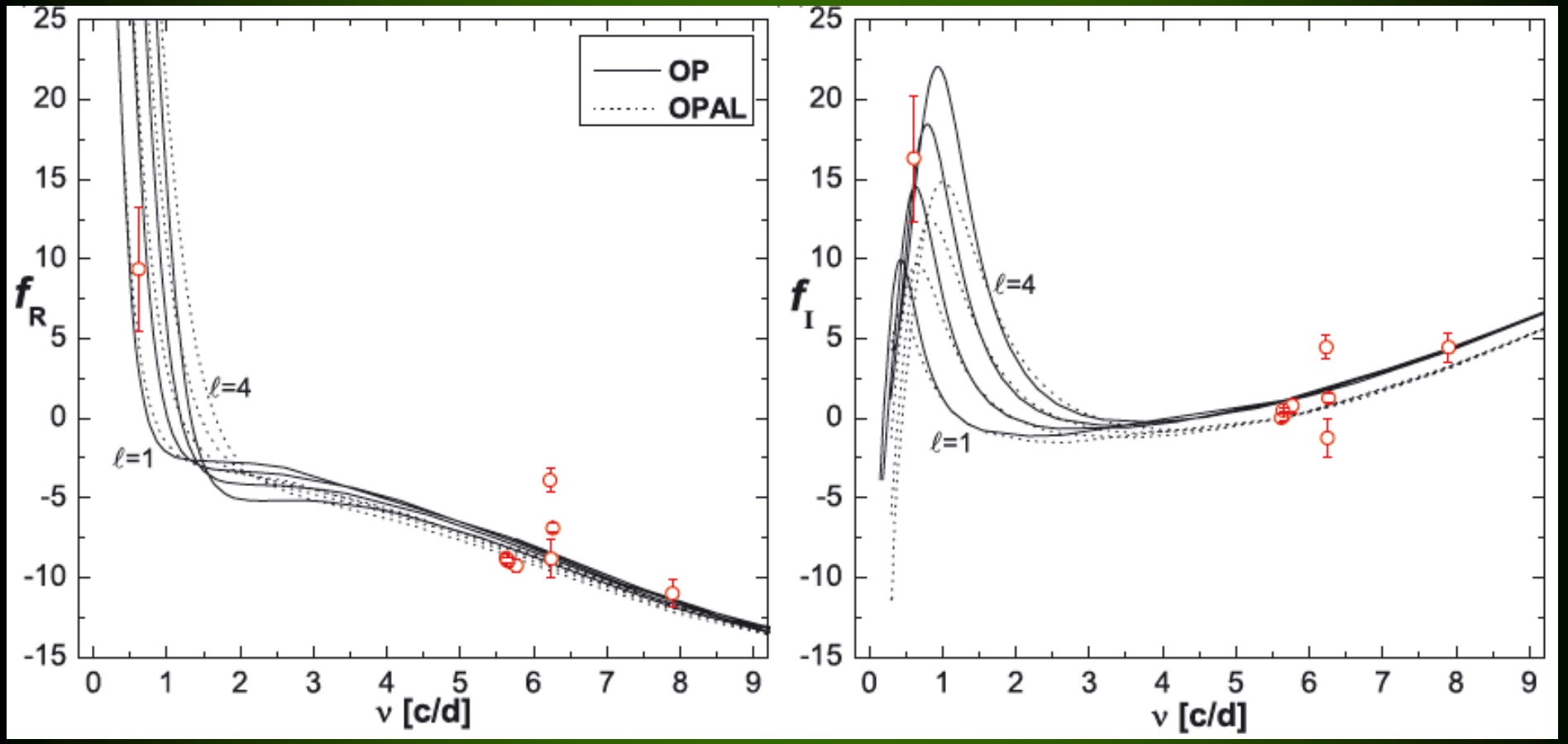
Daszyńska-Daszkiewicz & Walczak 2010, MNRAS, 403, 496.

ν Eri. Empirical and theoretical *f* values for long-period g, *l*=2 mode, νB



Daszyńska-Daszkiewicz & Walczak 2010, MNRAS, 403, 496.

ν Eri. Empirical and theoretical *f* values



Daszyńska-Daszkiewicz & Walczak 2010, MNRAS, 403, 496.

The most recent identification of ℓ

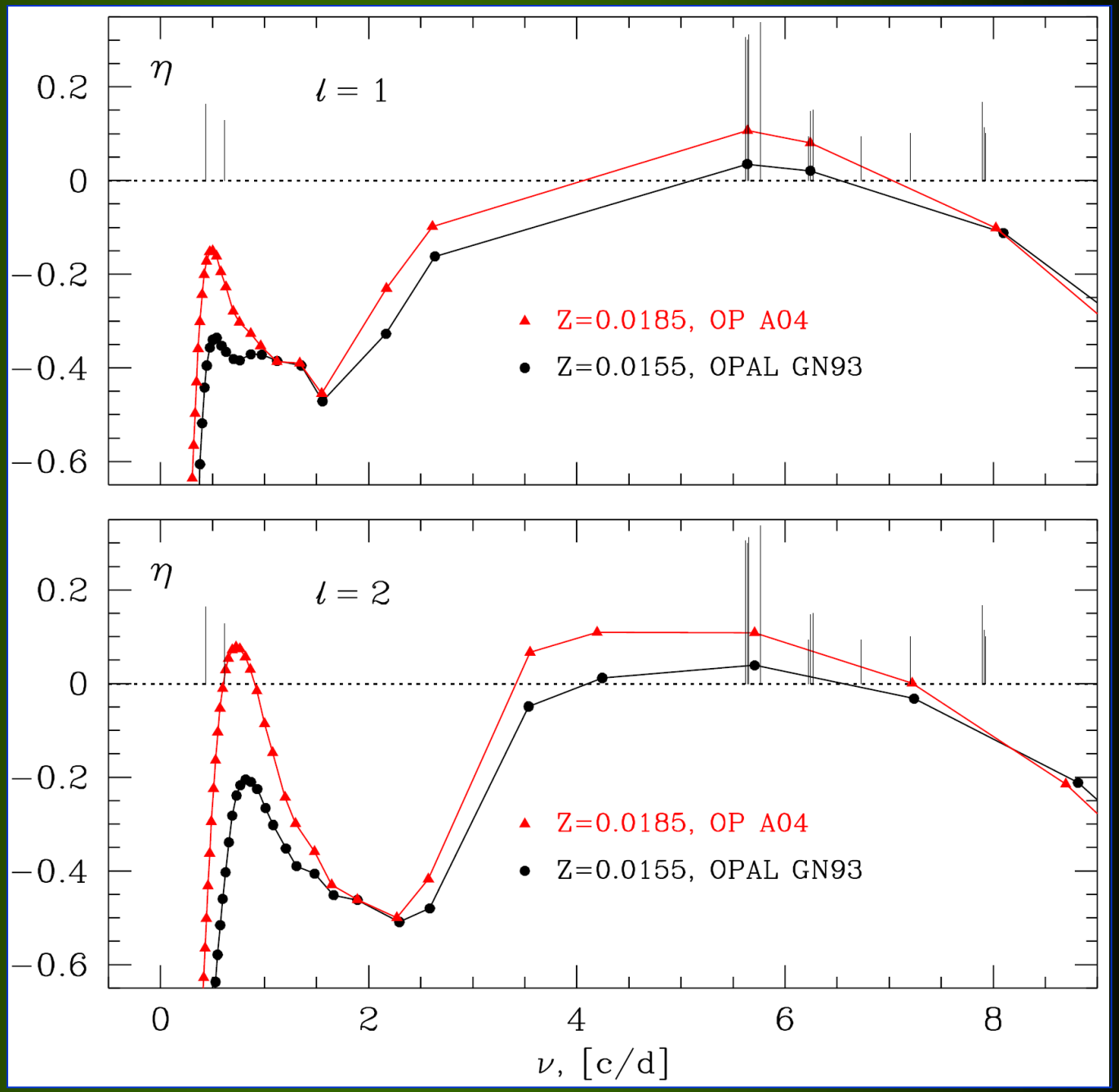
frequency [c/d]	photometry (theoretical f's)	phot.+$V_{\rm rad}$ (empirical f's)
$\nu_1 = 5.7632828$	ℓ=0	ℓ=0
$\nu_2 = 5.6538767$	ℓ=1	ℓ=1
$\nu_3 = 5.6200186$	ℓ=1	ℓ=1
$\nu_4 = 5.6372470$	ℓ=1	ℓ=1
$\nu_5 = 7.898200$	ℓ=1	ℓ=1
$\nu_6 = 6.243847$	ℓ=1	ℓ=1
$\nu_7 = 6.262917$	ℓ=1,3	ℓ=1,3
$\nu_8 = 7.20090$	ℓ=?(2)	-
$\nu_9 = 7.91383$	$\ell \leqslant 3$	-
$\nu_{10} = 7.92992$	ℓ=?	-
$\nu_{11} = 6.73223$	ℓ=?(3)	-
$\nu_{12} = 6.22360$	ℓ=1,2,3	ℓ=1,2,3
$\nu_A = 0.432786$	ℓ=1	-
$\nu_B = 0.61440$	ℓ=2,4	ℓ=2,5

Daszyńska-Daszkiewicz & Walczak, 2010, MNRAS

$$\eta = \frac{W}{\int_0^1 \left| \frac{\mathrm{d}W}{\mathrm{d}r} \right| \mathrm{d}r}$$

Instability coefficient in models computed with OPAL and OP opacities

$\eta > 0$ for unstable modes



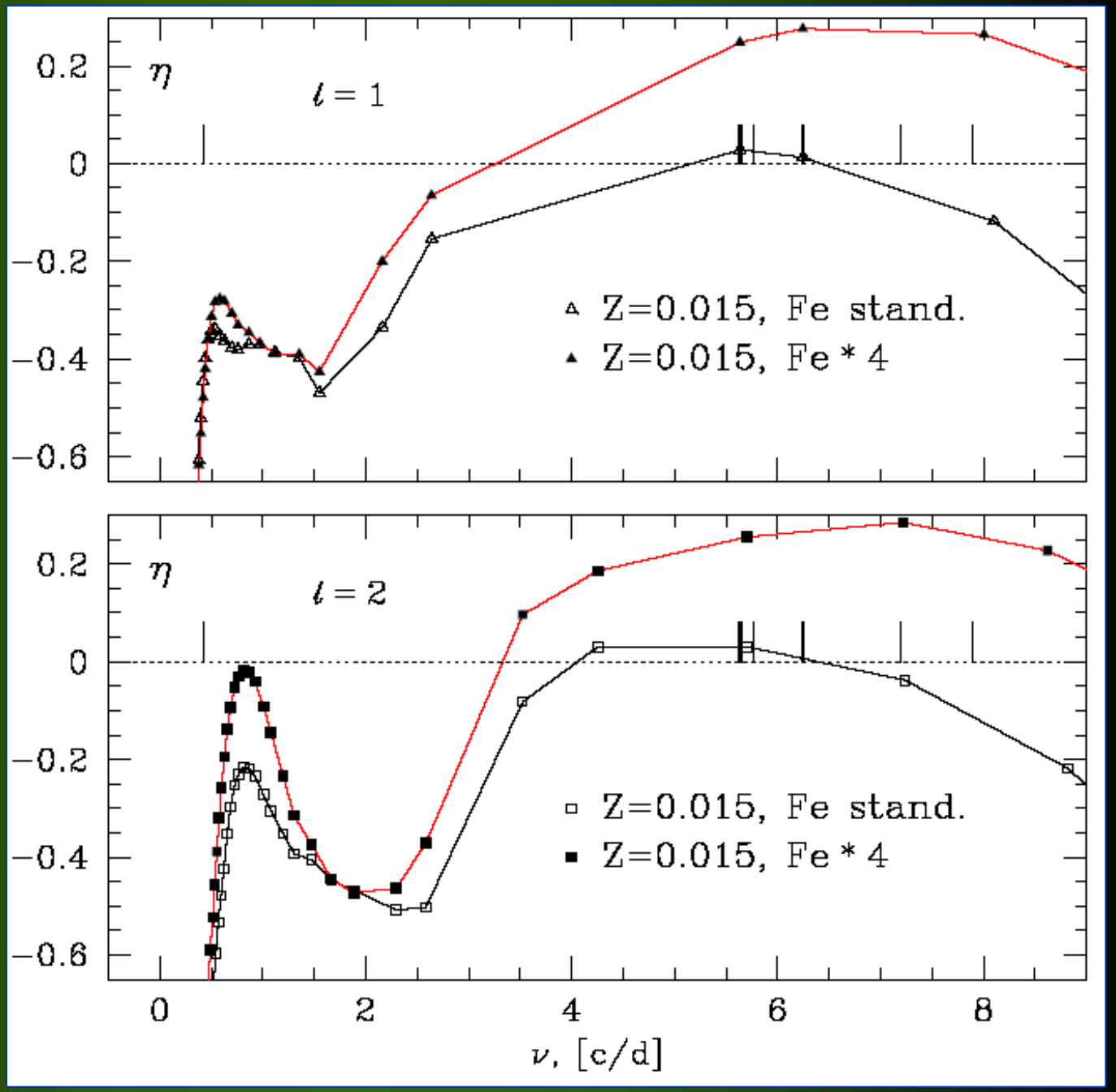
Dziembowski, Pamyatnykh 2008

$$\eta = \frac{W}{\int_0^1 \left| \frac{\mathrm{d}W}{\mathrm{d}r} \right| \mathrm{d}r}$$

Instability coefficient in a standard model and in the model with the Fe accumulation in the driving zone

OPAL opacity in both cases

$\eta > 0$ for unstable modes

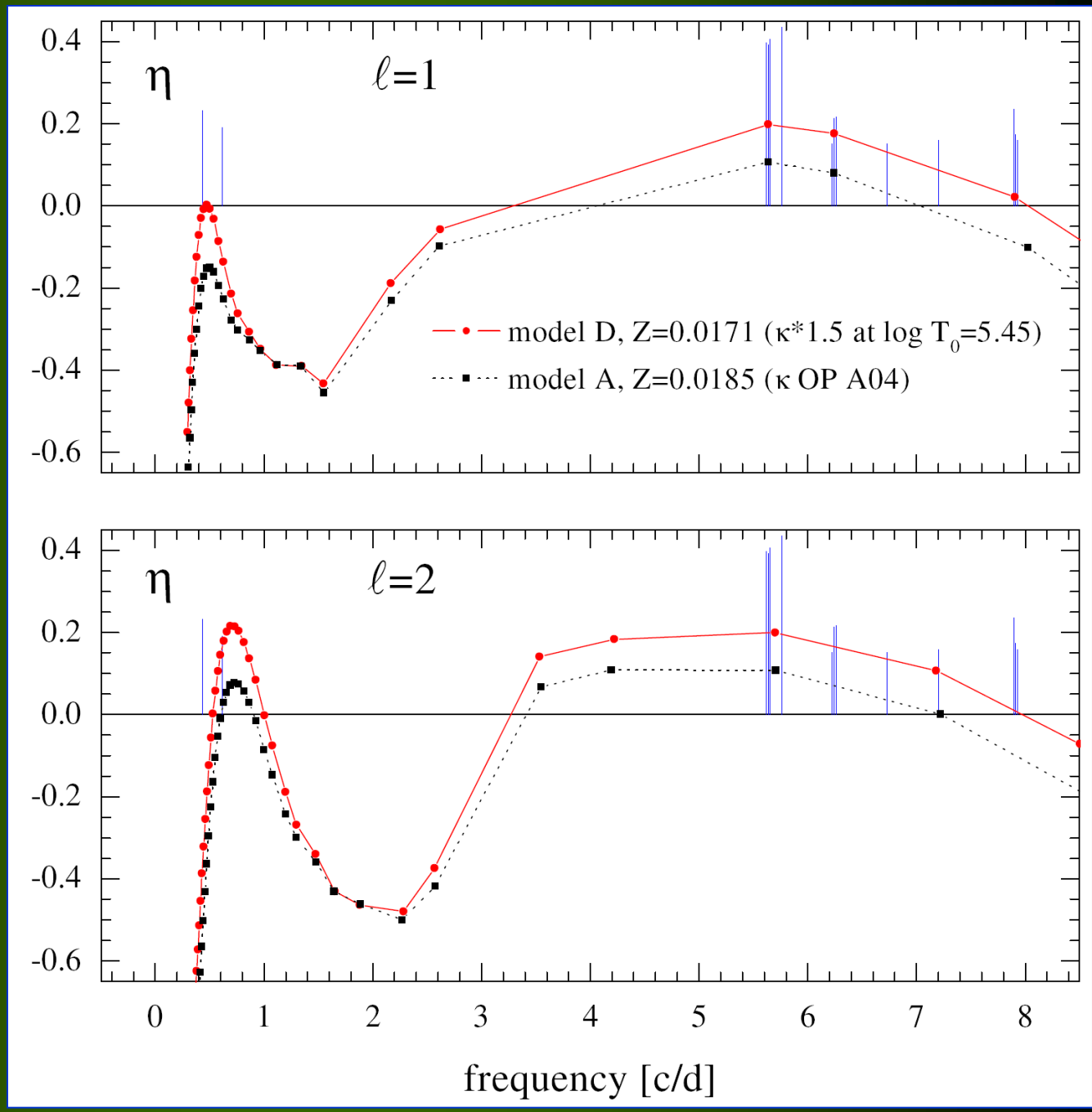


Pamyatnykh, Handler, Dziembowski 2004

$$\eta = \frac{W}{\int_0^1 \left| \frac{\mathrm{d}W}{\mathrm{d}r} \right| \mathrm{d}r}$$

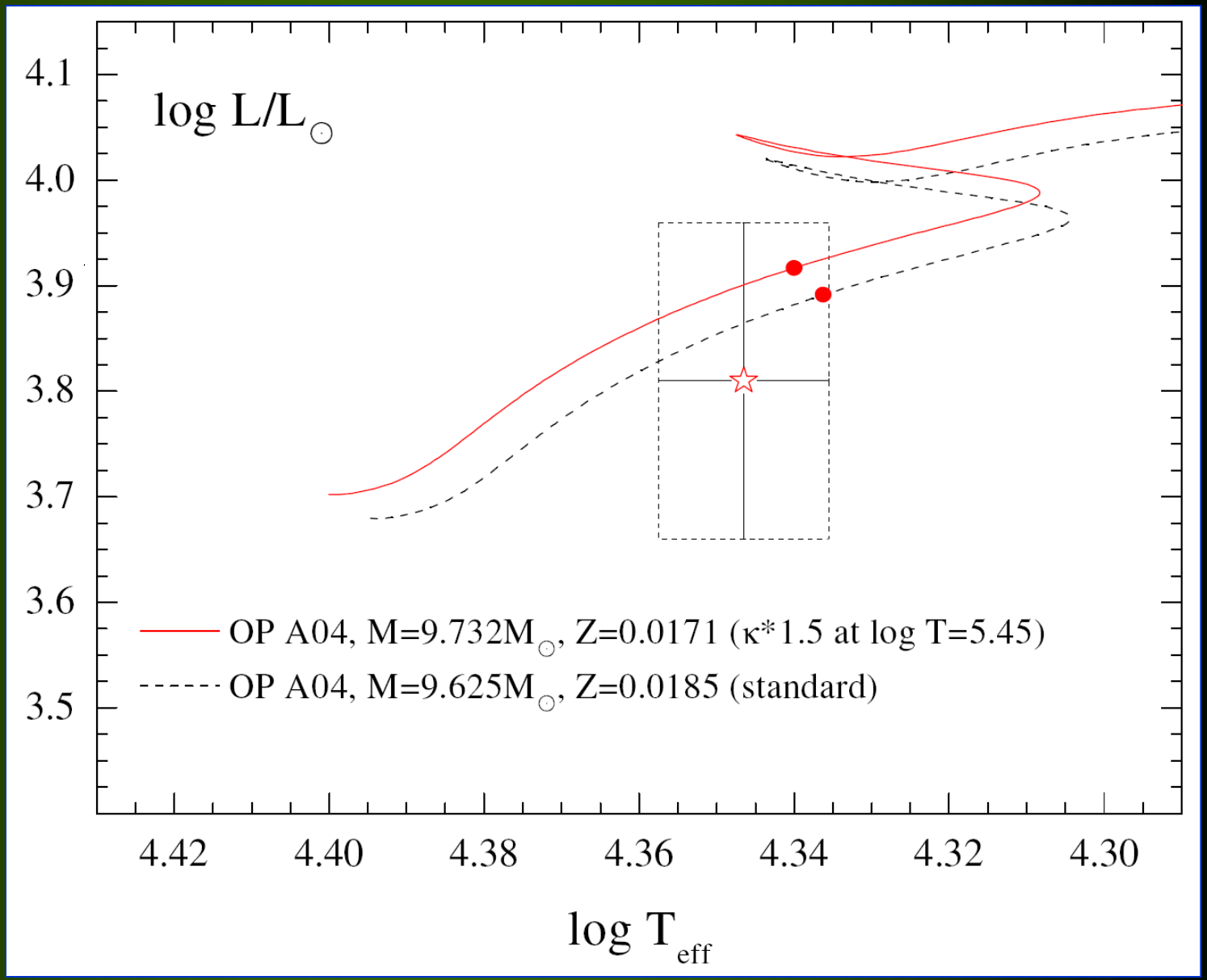
Instability coefficient in a standard model and in the model with enchanced opacity in the Z bump region

$\eta > 0$ for unstable modes



Zdravkov & Pamyatnykh 2008, poster No. 58

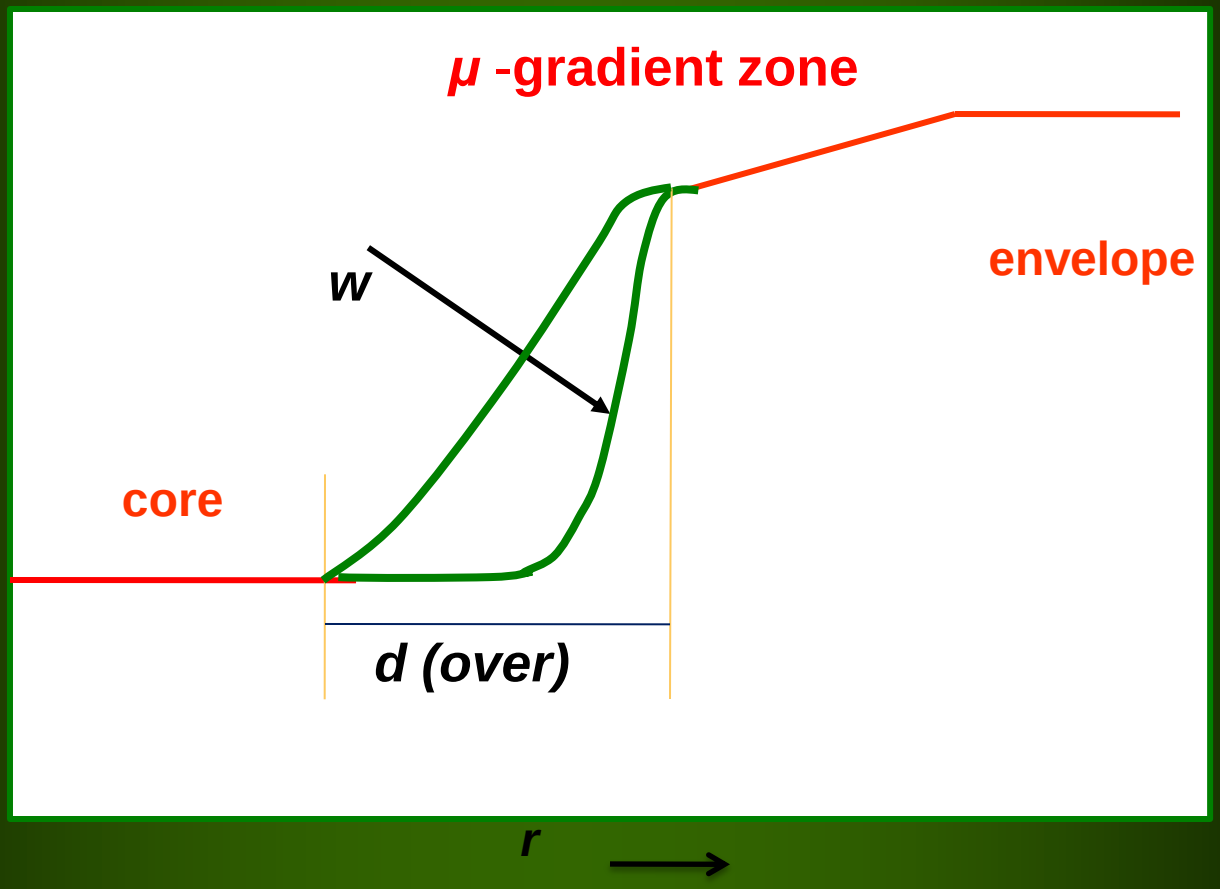
ν Eri : a standard model and the model with enchanced Z bump opacity



Zdravkov & Pamyatnykh 2008

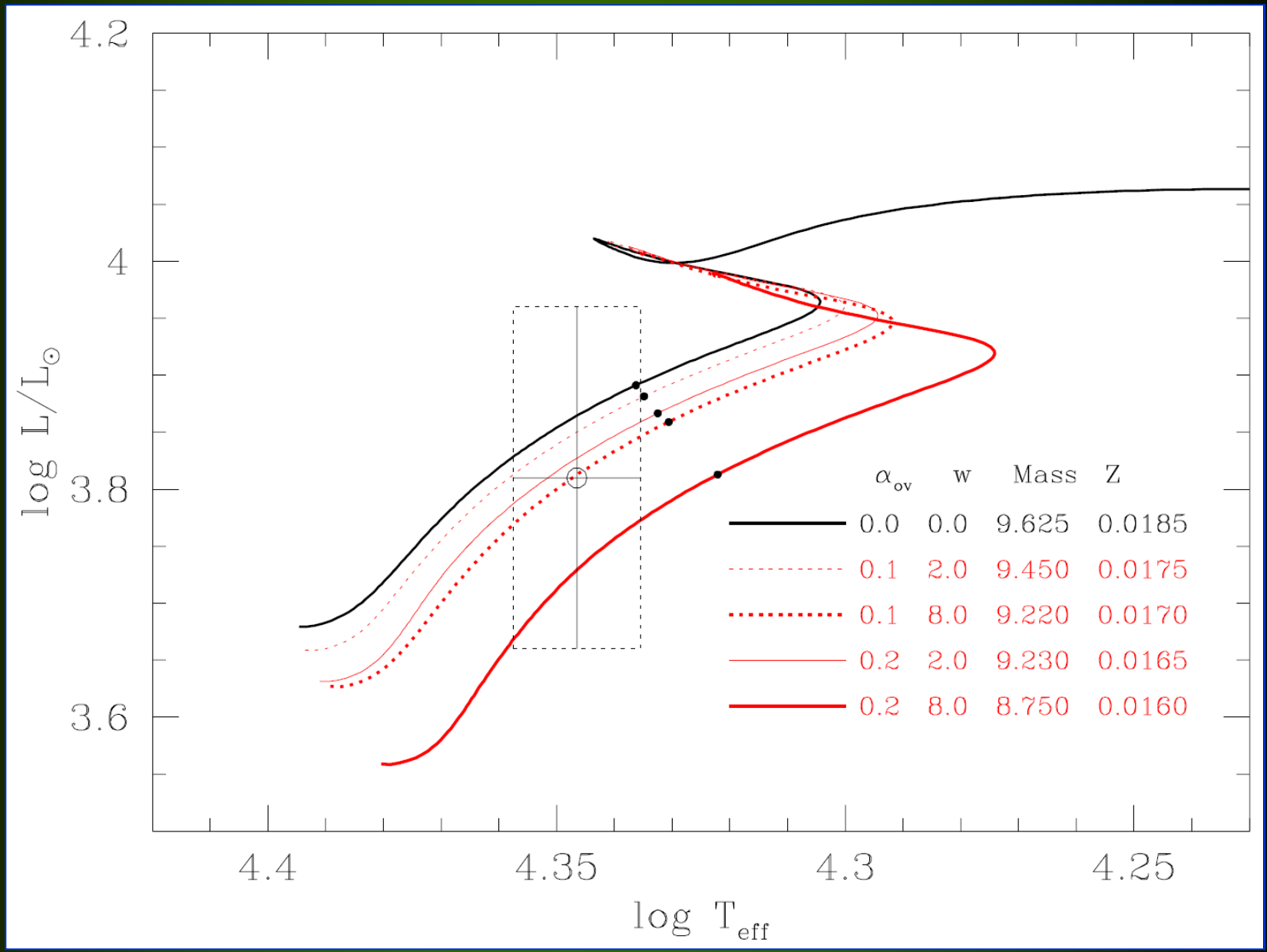
Testing overshooting

Two-parametric overshooting



Dziembowski 2008

Seismic models of ν Eri for different overshooting parameters in the H-R diagram

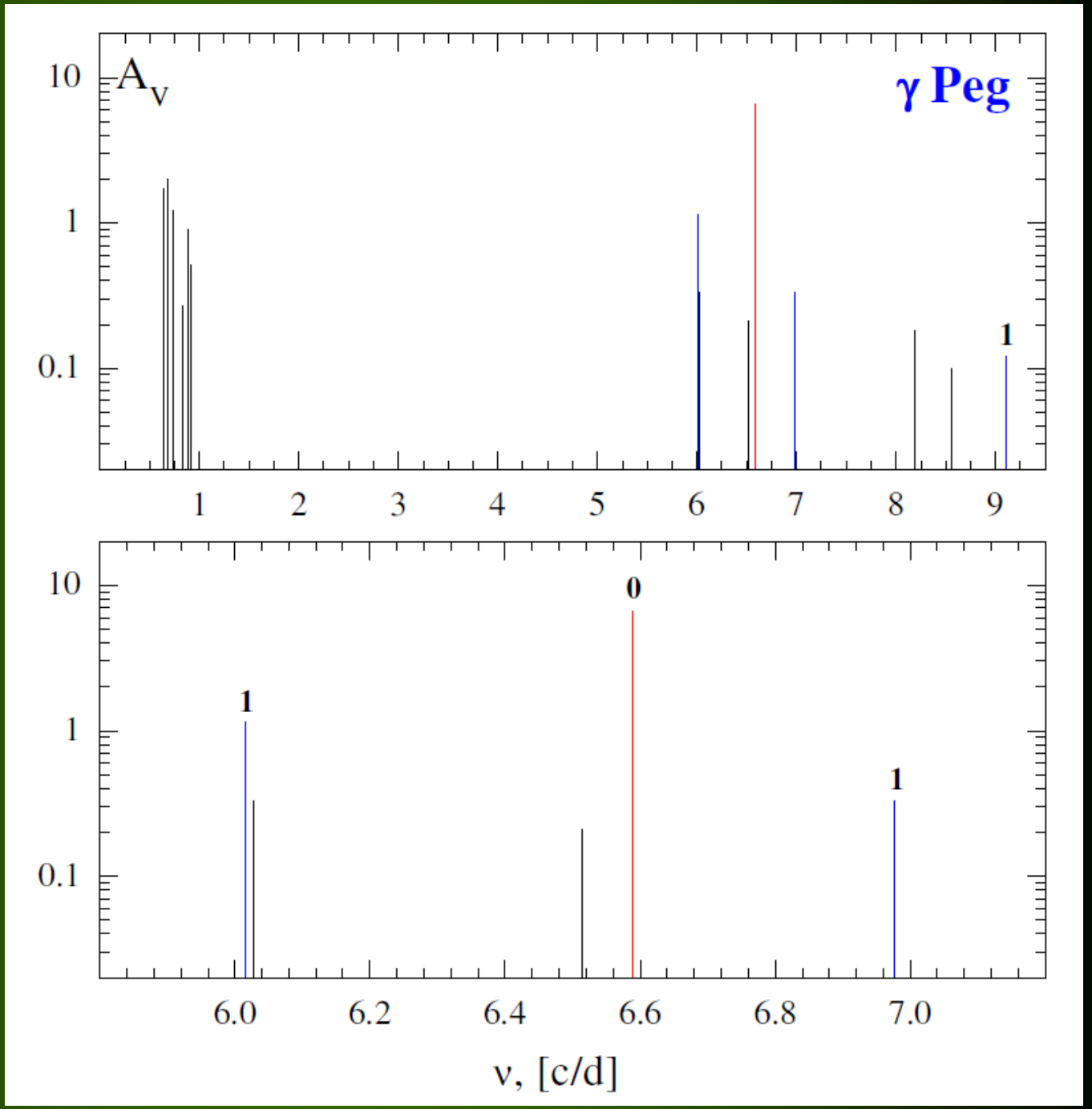


Dziembowski & Pamyatnykh 2008

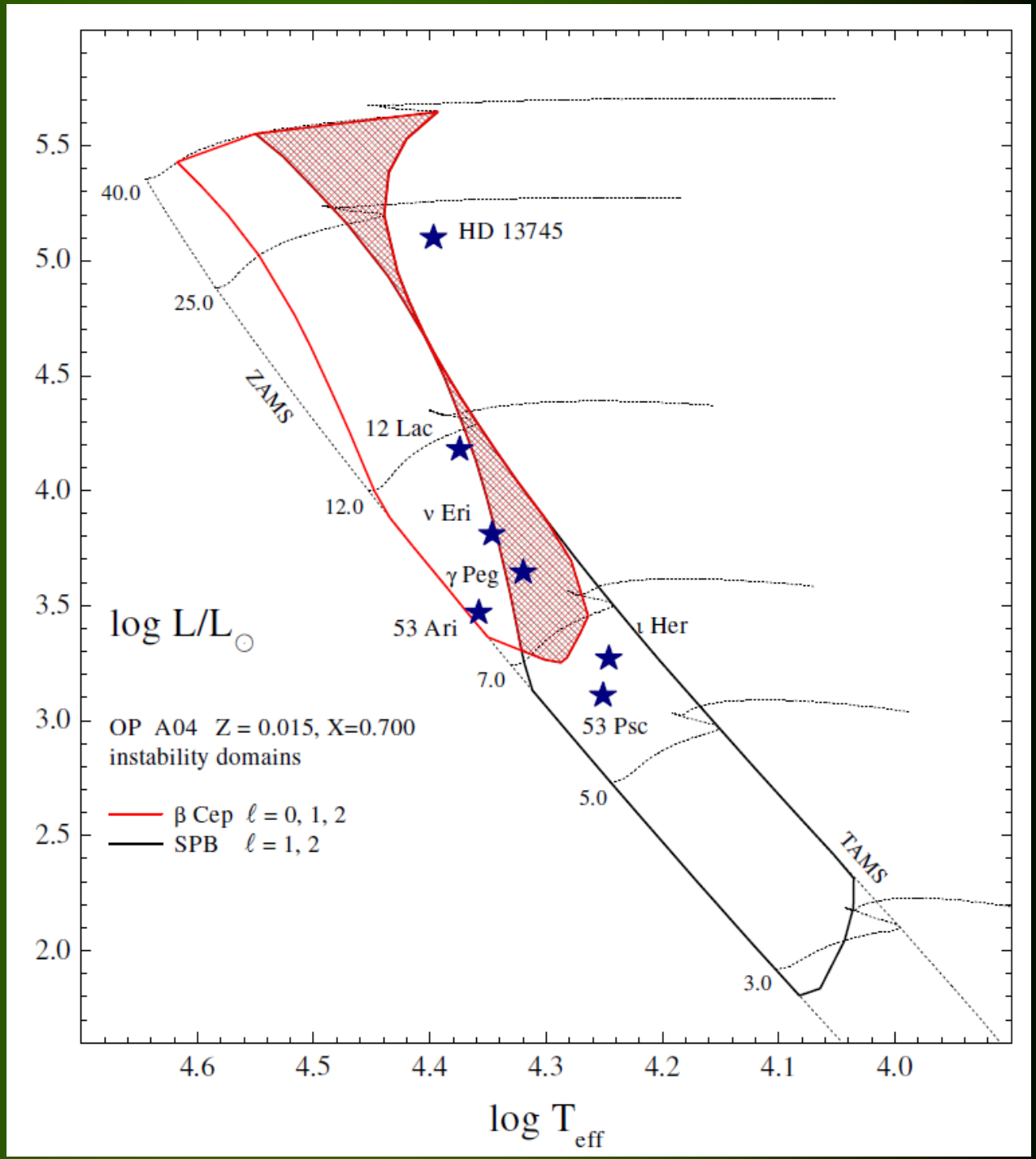
γ Pegasi

14 frequencies (8 - β Cep-type, 6 – SPB-type)
(degree *l* is identified for >2 of them),
Sp B2 IV,
mV = 2.83,
π = 9.79 +- 0.81 mas,
Vrot = 5.0 km/s,
[m/H] = approx. 0,
Teff = 21450 +- 800 K

γ Pegasi – a hybrid β Cep / SPB star

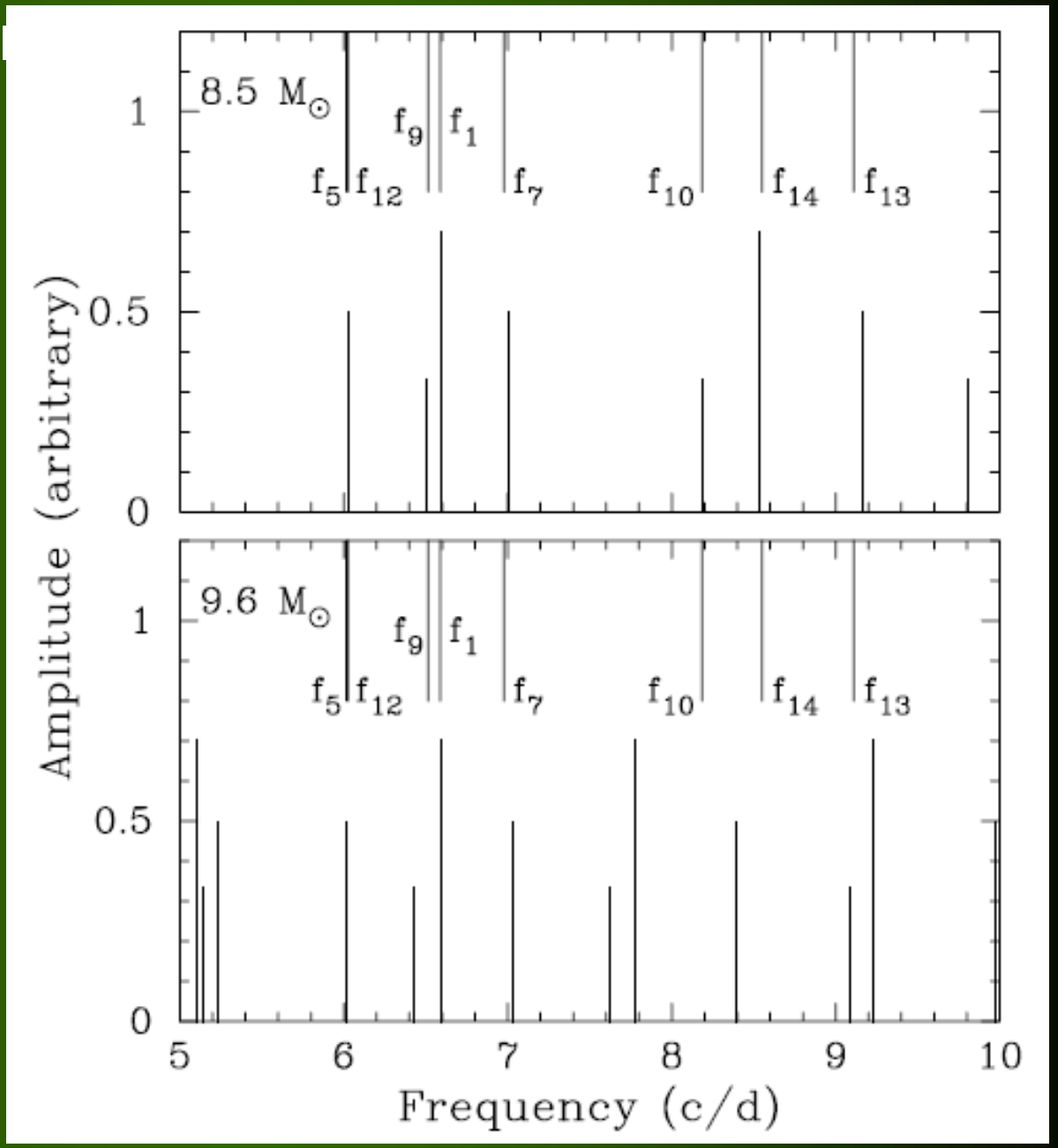


ν Eridani – a hybrid β Cep / SPB star



Zdravkov 2011

Comparison of theoretical and observed frequencies for two models fitting f_1 as radial mode and f_5 as dipole mode.

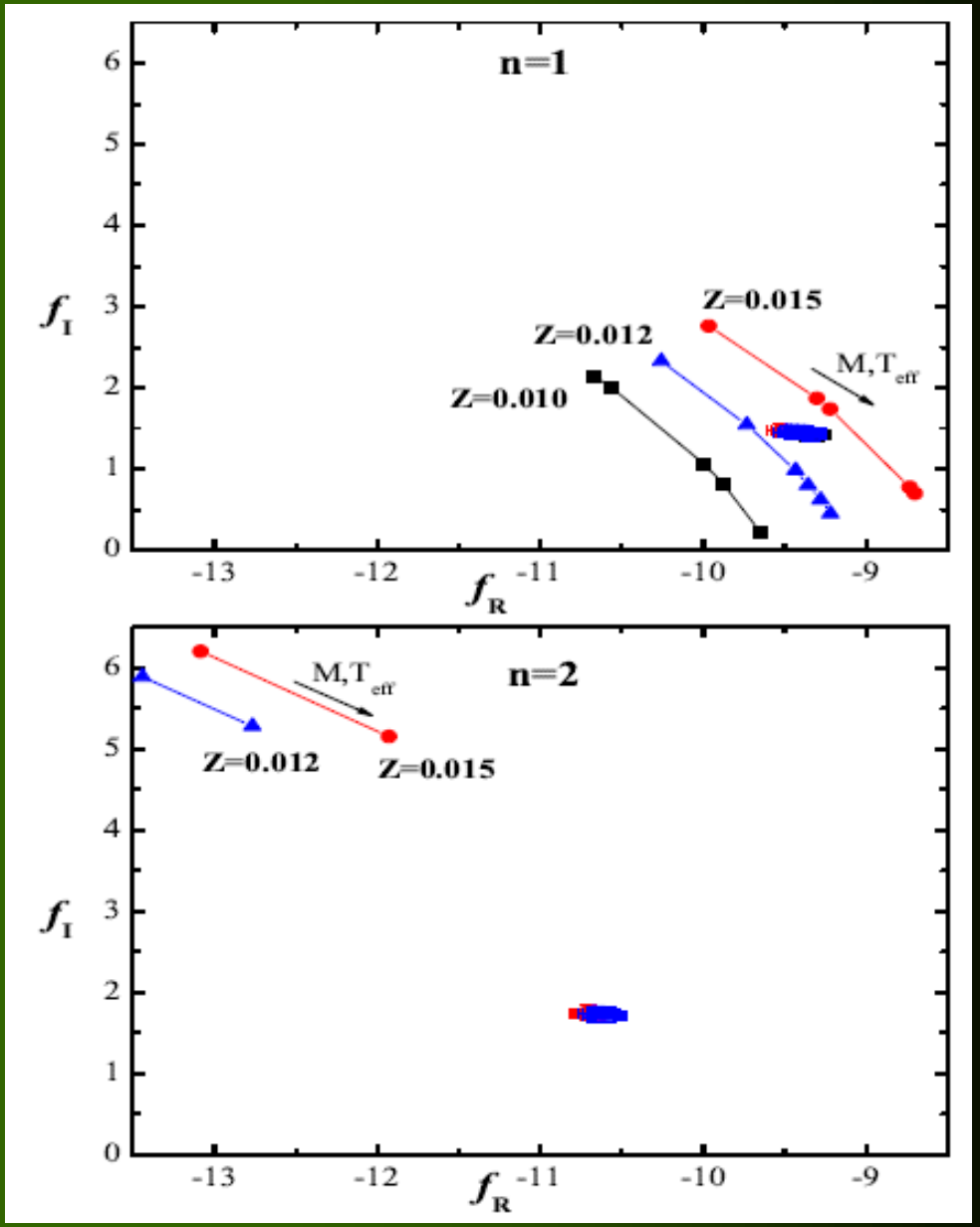


Handler et al. 2009

γ Peg, empirical and theoretical *f* values for dominant frequency

Radial fundamental mode

Radial first overtone



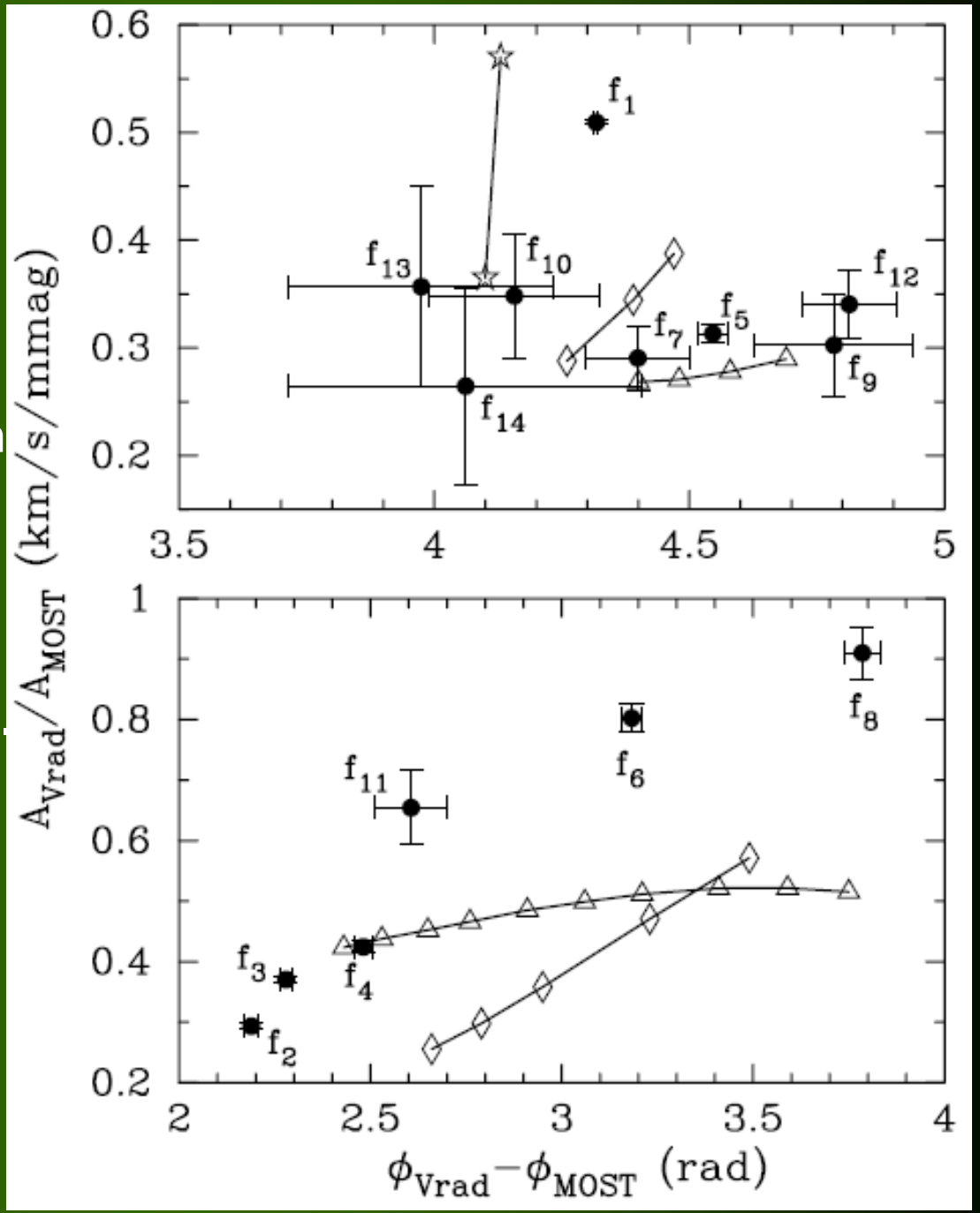
Walczak & Daszyńska-Daszkiewicz, 2011

γ Peg

Observed and theoretical amplitude ratios and phase differences.

Upper panel – β Cep pulsation lower panel – SPB pulsations.

Star symbols – radial modes, diamonds - dipole modes, triangles – quadrupole modes.

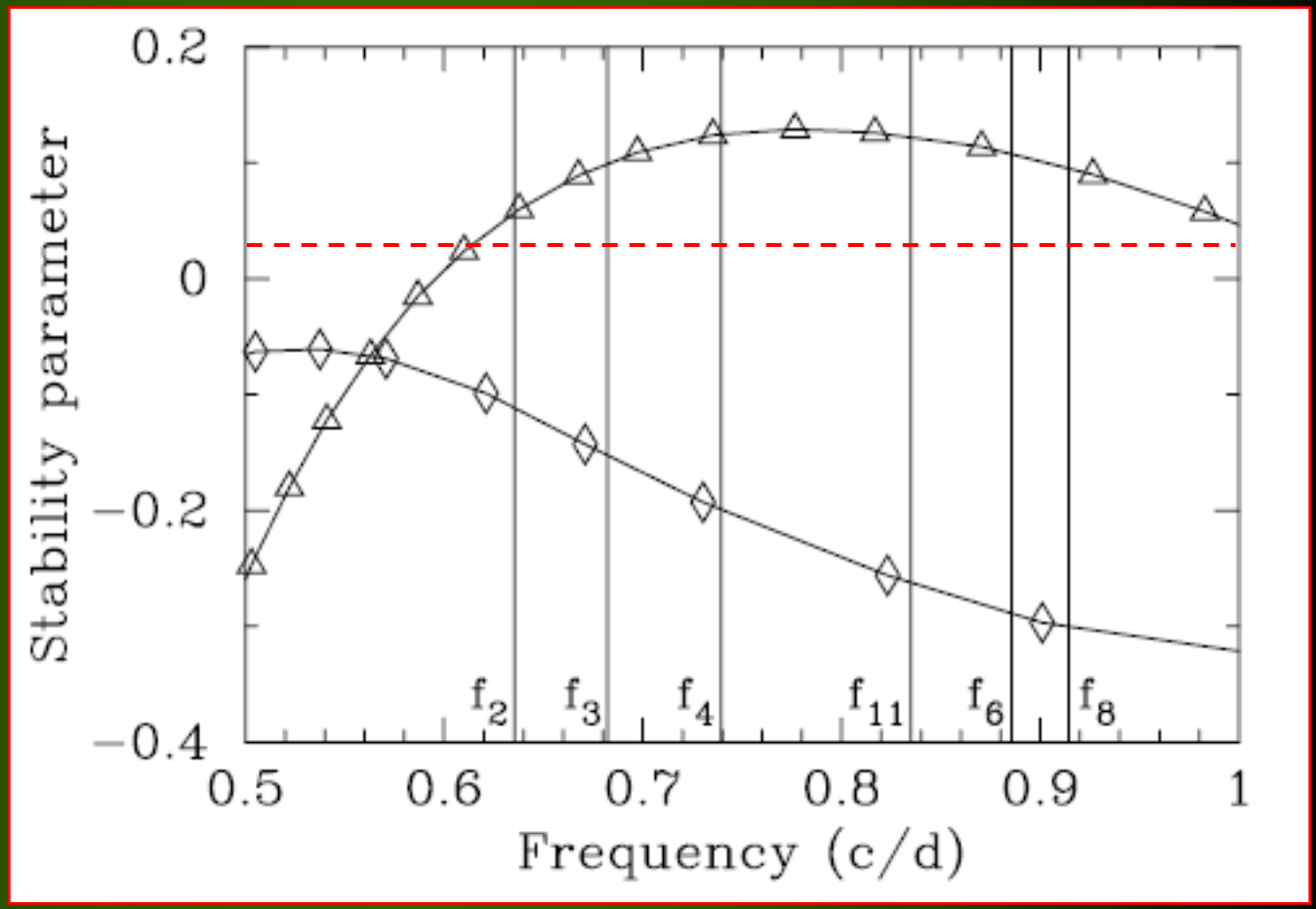


Handler et al. 2009

γ Peg. Comparison of theoretical and observed g-mode pulsation frequencies.

The stability parameter is positive for excited modes

Diamonds - dipole modes,
triangles – quadrupole modes.



Handler et al. 2009

γ Peg, mode identification

Table 1 The most probable identification of ℓ for the pulsational frequencies of γ Peg from three approaches.

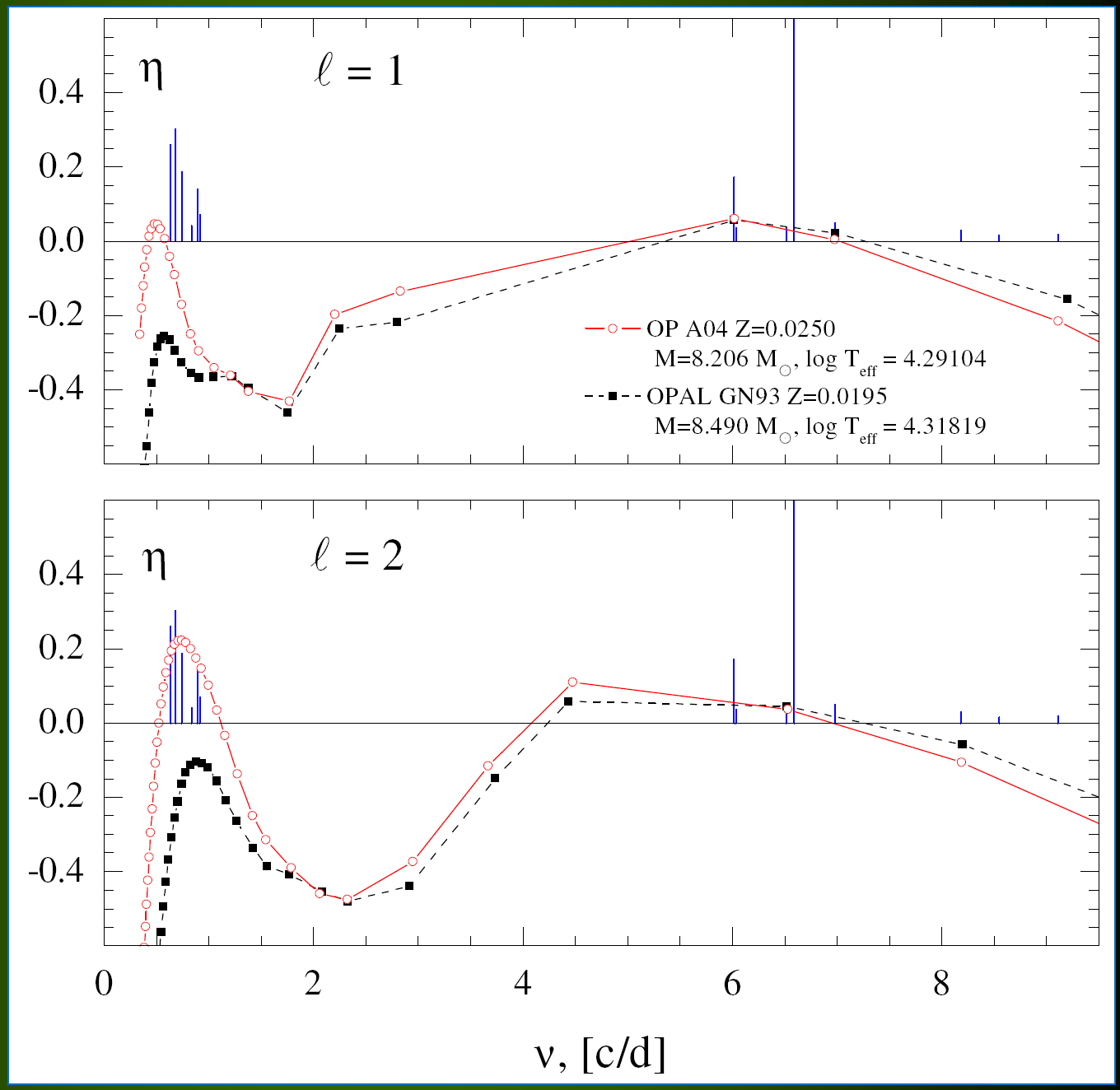
frequency [c/d]	phot. theoretical f	phot.+$V_{\rm rad}$ theoretical f	phot.+$V_{\rm rad}$ empirical f
ν_1=6.58974	ℓ=0	ℓ=0	ℓ=0
ν_2=0.63551	ℓ=2	ℓ=2	ℓ=2
ν_3=0.68241	ℓ=1	ℓ=2	ℓ=1,2
ν_4=0.73940	ℓ=2,1	ℓ=2	ℓ=1,2
ν_5=6.01616	ℓ=1,3	ℓ=1,2	ℓ=1,0,3
ν_6=0.88550	ℓ=2	ℓ=2	ℓ=2,5
ν_7=6.9776	ℓ=?	ℓ=1,2,3,6	ℓ=?
ν_8=0.91442	ℓ=2,3,4	ℓ=1,2	ℓ=2,3,5
ν_9=6.5150	$\ell \geq 1$	ℓ=1,2,3,6	ℓ=?
ν_{10}=8.1861	ℓ=?	ℓ=1,2,3,6	ℓ=?
ν_{11}=0.8352	ℓ=1,2	ℓ=1	ℓ=1,2
ν_{12}=6.0273	ℓ=?	ℓ=1,2	ℓ=2,5
ν_{13}=9.1092	ℓ=?	ℓ=1,2,6	ℓ=?
ν_{14}=8.552	$\ell \geq 4$	ℓ=6	ℓ=?

Walczak & Daszyńska-Daszkiewicz, 2011

γ Pegasi

Instability coefficient in a standard model and in the model with enchanced opacity in the Z bump region

η > 0 for unstable modes



Zdravkov & Pamyatnykh 2008

EFFECTS OF ROTATION (Ω)

Rotational mode coupling

$\omega_j - \omega_k \approx \Omega$; $\ell_j = \ell_k \pm 2$; $m_j = m_k$

Daszyńska-Daszkiewicz et al. 2002, A&A 392,151

Low-frequency modes, $\omega \approx \Omega$

perturbation approach fails

Daszyńska-Daszkiewicz et al. 2007, AcA 57,11

The photometric diagrams become dependent on:

1) **the inclination angle, i,**
2) **the azimuthal order, m,**
3) **the rotational velocity, V_{rot}**

THE PHOTOMETRIC COMPLEX AMPLITUDE OF THE COUPLED MODE

$$\mathcal{A}_\lambda(i) = \sum_k a_k A_{\lambda,k}(i)$$

Daszyńska-Daszkiewicz et al. 2002, A&A 392, 151.

Regularities in theoretical frequency spectra:

- high-order p-modes:

$$\nu_{nl} \approx \Delta\nu \cdot \left(n + \frac{l}{2} + \alpha\right) - \frac{l(l+1)}{4\pi^2 n} \cdot f_c$$

$$\Delta\nu = \left(2\int \frac{dr}{c}\right)^{-1}, \quad f_c = \int \frac{dc}{dr} \cdot \frac{dr}{r}$$

large separation:

$$\nu_{nl} - \nu_{n-1,l} \approx \Delta\nu$$

small separation:

$$\nu_{nl} - \nu_{n-1,l+2} = \delta\nu_{nl} \approx -\frac{4l+6}{4\pi^2 n} \cdot f_c$$

- high-order g-modes:

$$\Pi_{nl} \approx \frac{n}{\sqrt{l(l+1)}} \cdot \Delta\Pi, \quad \Delta\Pi = 2\pi^2 \left(\int \frac{N}{r}\, dr\right)^{-1}$$

$$N = g\left(\frac{1}{\Gamma_1}\frac{d\ln P}{dr} - \frac{d\ln \rho}{dr}\right)$$

- Rotational splitting: (~ Goupil et al. 2000)

$$\nu_m = \nu_0 + m\left(1 - C_{nl}\right)\frac{\Omega}{2\pi} + \frac{\Omega^2}{2\pi\nu_0}\left(D_0 + m^2 D_1\right) + m\frac{\Omega^3}{\nu_0^2} T$$

$$\Rightarrow \frac{\nu_m - \nu_{m=0}}{m} = \frac{\Omega}{2\pi}\left(1 - C_{nl} + m\mu D_1 + \mu^2 T\right)$$

$$\mu = \frac{\Omega}{\nu_0}$$

- rotation rate:

$$\frac{\nu_m - \nu_{-m}}{2m} = \Omega\left(1 - C_{nl} + \mu^2 T\right)$$

- Effective gravity:

$$g_{eff} = g - \frac{2}{3}\Omega^2 r$$

- Ledoux constant:

$$C_{nl} = \frac{\int (2yz + z^2)\rho r^4 dr}{\int (y^2 + \Lambda z^2)\rho r^4 dr} \qquad \Lambda = l(l+1)$$

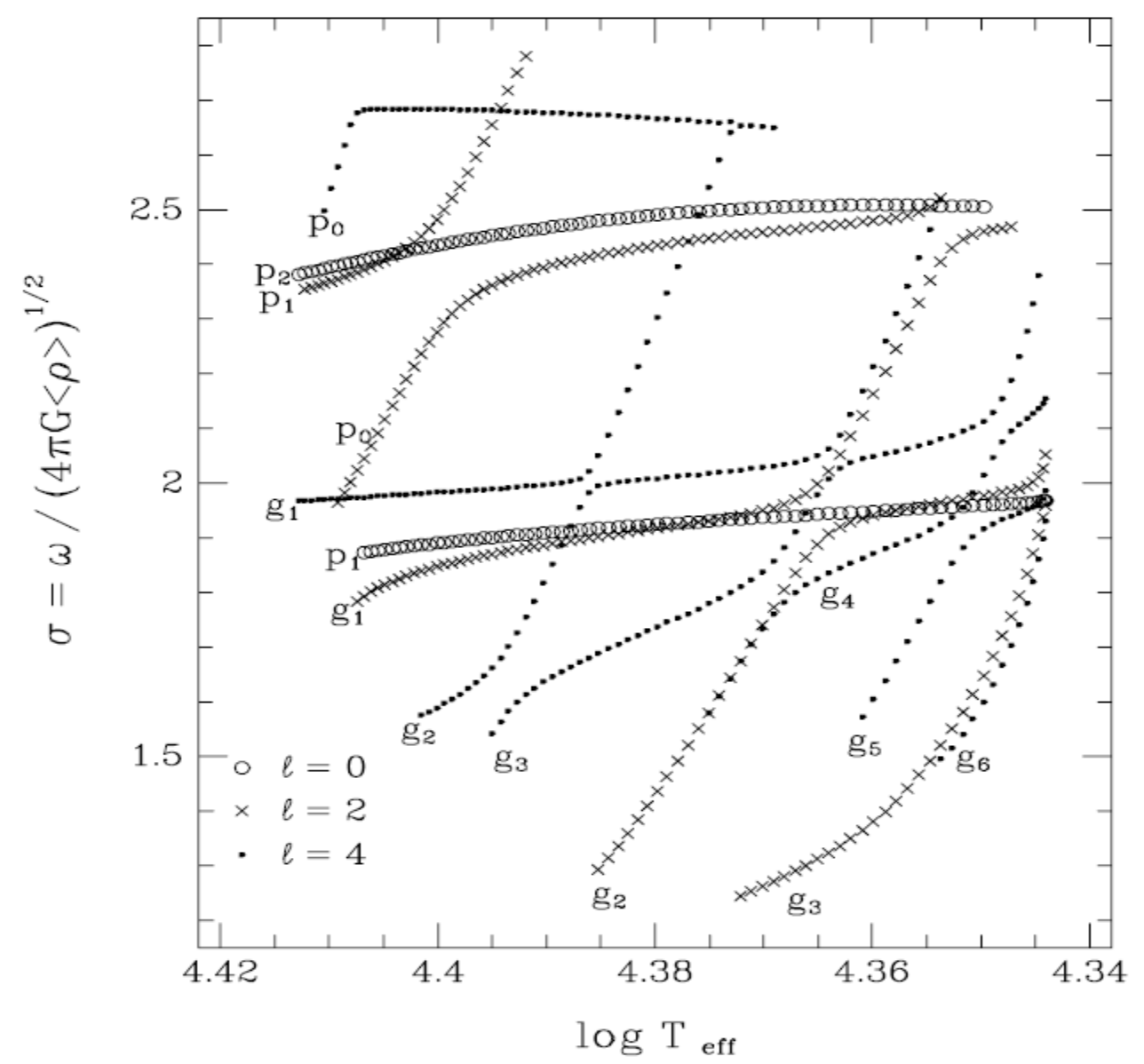
y, z – radial and horizontal components of displacement

$C_{nl} \ll 1$ for high-order p-modes $(y \gg z)$

$C_{nl} \simeq \frac{1}{l(l+1)}$ for g-modes $(z \gg y)$

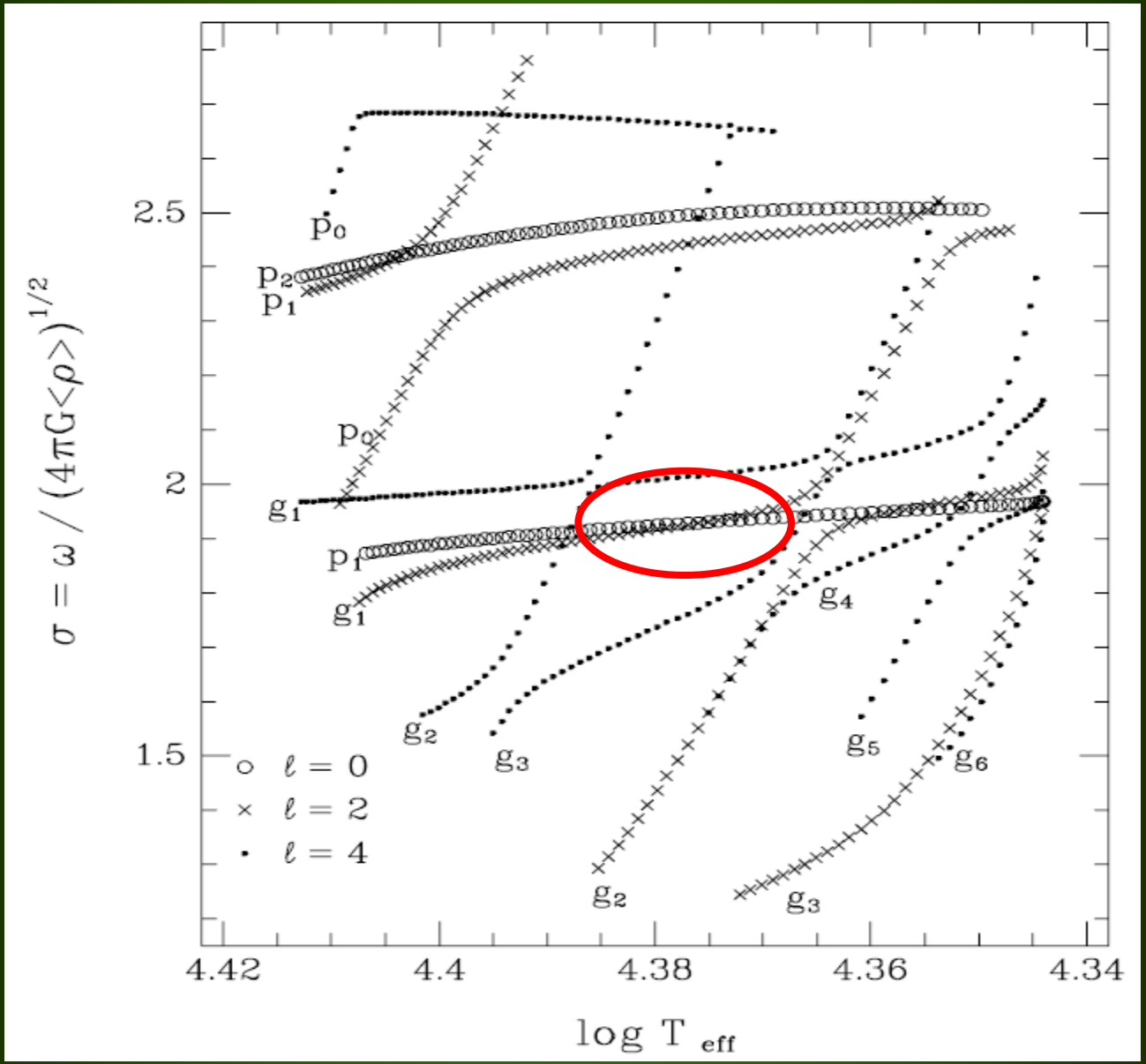
$C \sim \frac{1}{2}$ for gravity modes of $l = 1$

Unstable modes of $l = 0, 2, 4$ in β Cep star models



Daszyńska-Daszkiewicz et al. 2002, A&A 392, 151.

Unstable modes of l = 0, 2, 4 in a β Cep star model



Daszyńska-Daszkiewicz et al. 2002, A&A 392, 151.

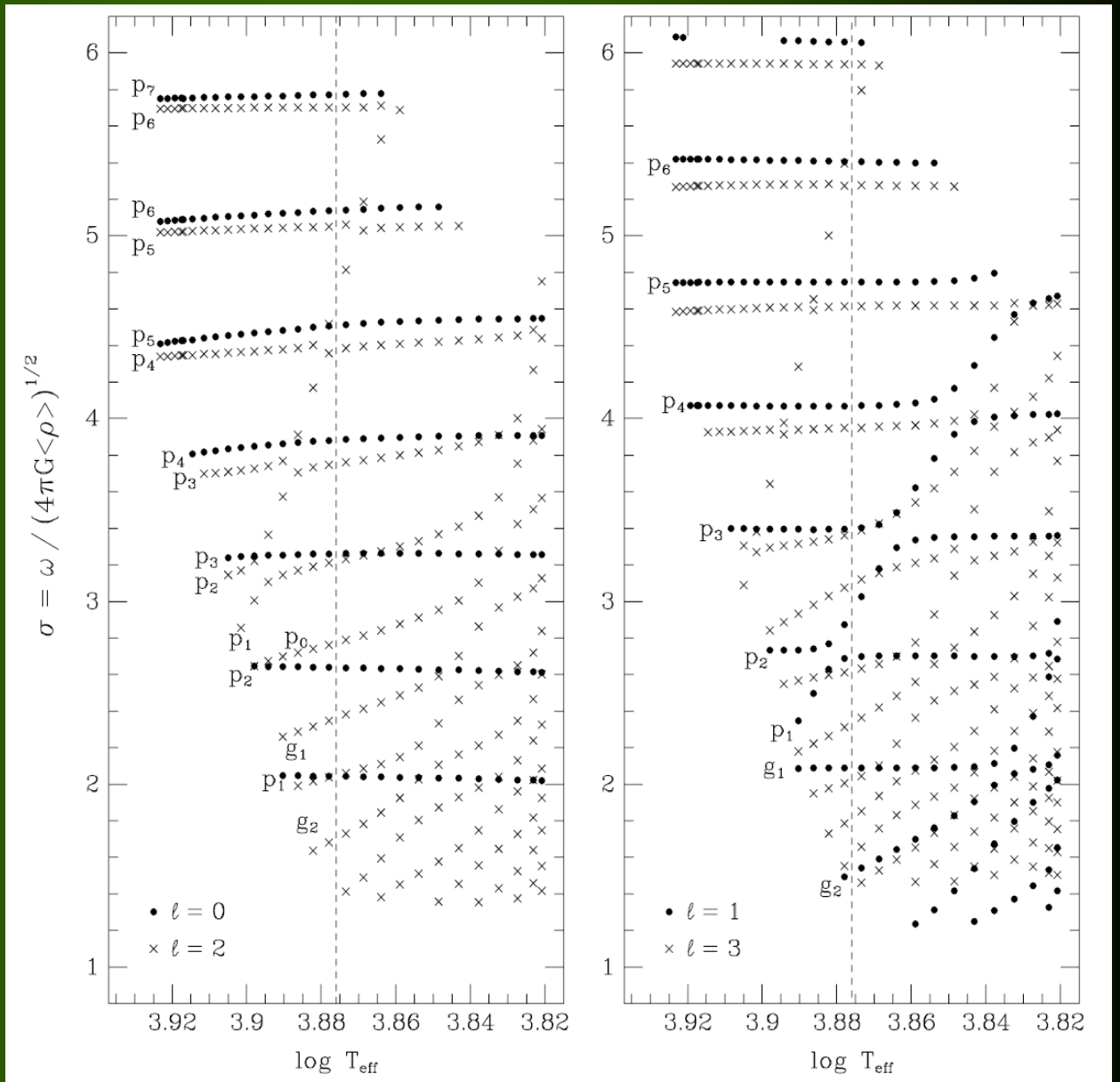
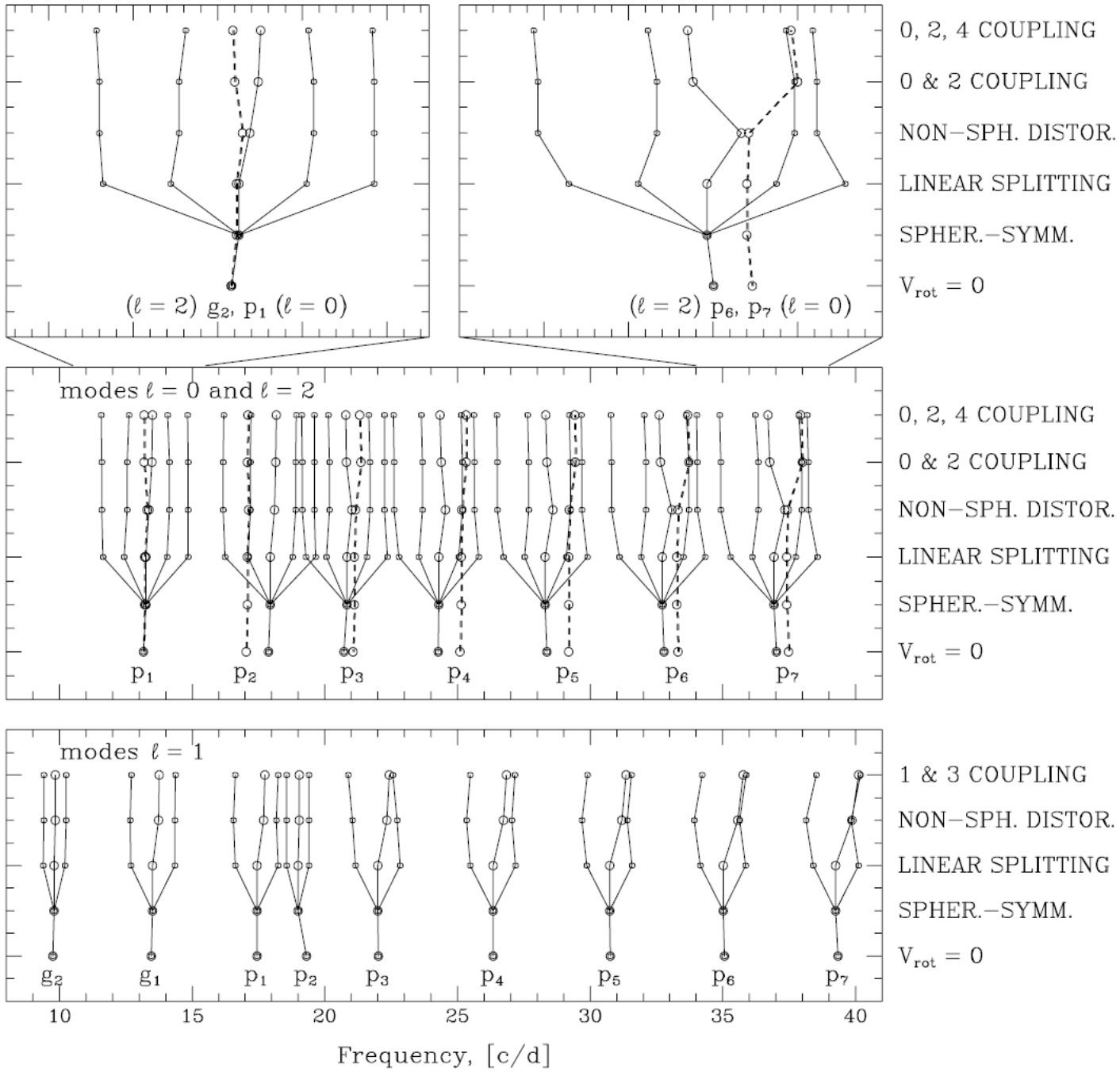
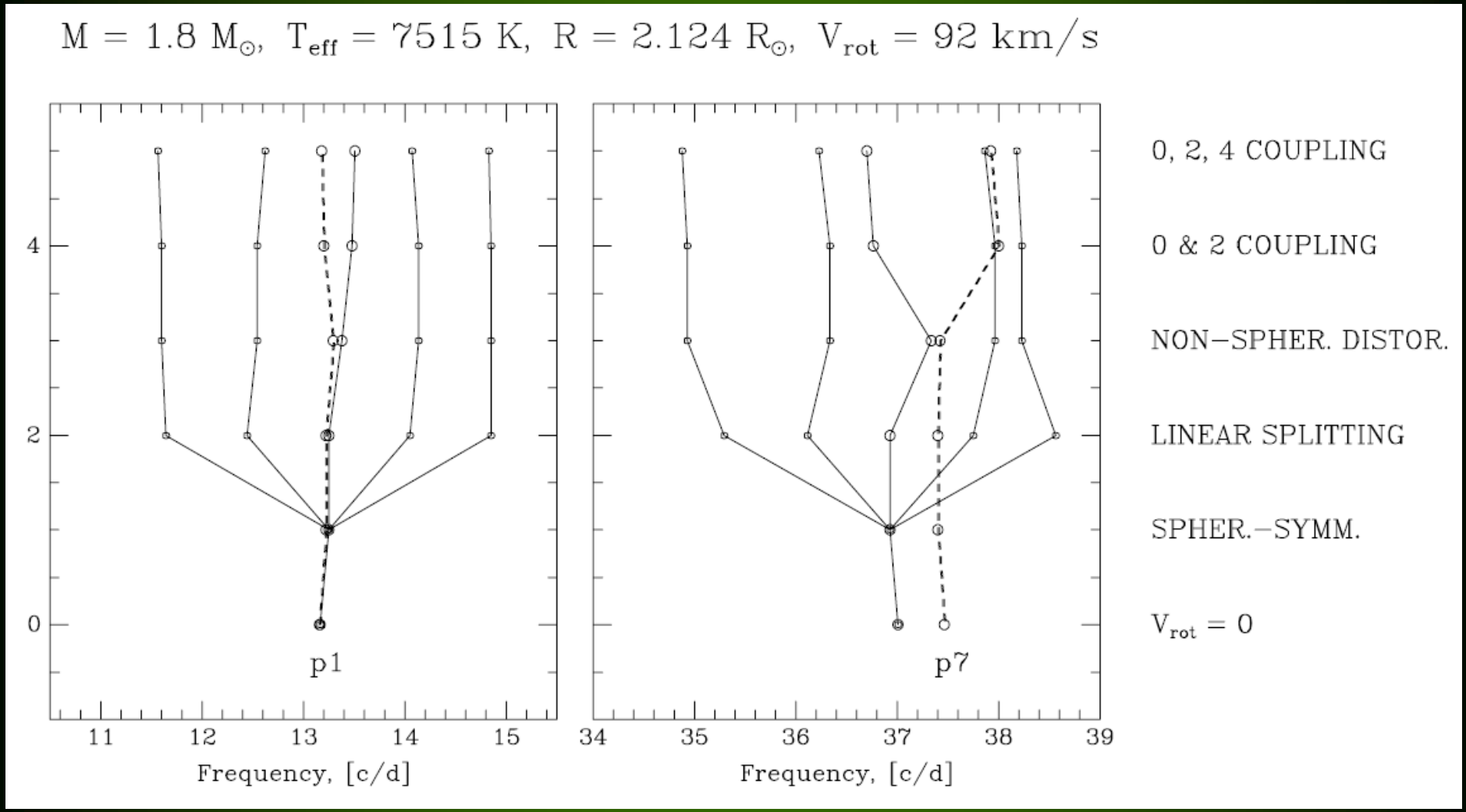


Figure 4. Evolution of frequency spectra of unstable modes with $\ell = 0, 2$ (left panel) and with $\ell = 1, 3$ (right panel) in a sequence of δ Sct models with mass of 1.8 $M_\odot$. For simplicity, only modes of $\ell = 1$ are identified in the right panel. Dotted vertical line corresponds to the model of $T_{\rm eff}$ = 7515 K. Effects of rotation on frequency spectrum of this model are presented in Figure 5.

Pamyatnykh 2003

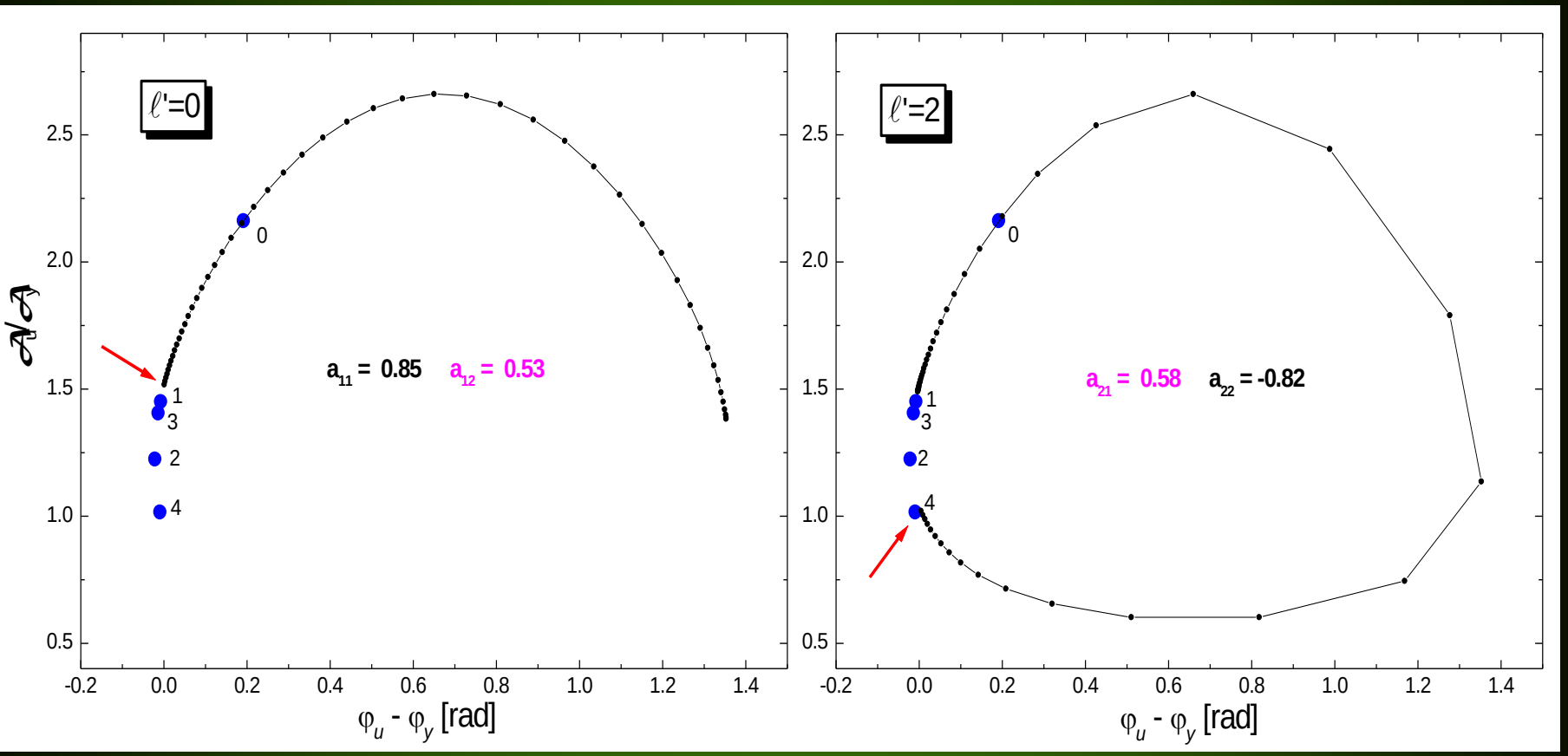

(ℓ = 2) g2, p1 (ℓ = 0)
(ℓ = 2) p6, p7 (ℓ = 0)
0, 2, 4 COUPLING
0 & 2 COUPLING
NON−SPH. DISTOR.
LINEAR SPLITTING
SPHER.−SYMM.
Vrot = 0
modes ℓ = 0 and ℓ = 2
p1 p2 p3 p4 p5 p6 p7
0, 2, 4 COUPLING
0 & 2 COUPLING
NON−SPH. DISTOR.
LINEAR SPLITTING
SPHER.−SYMM.
Vrot = 0
modes ℓ = 1
g2 g1 p1 p2 p3 p4 p5 p6 p7
1 & 3 COUPLING
NON−SPH. DISTOR.
LINEAR SPLITTING
SPHER.−SYMM.
Vrot = 0
10 15 20 25 30 35 40
Frequency, [c/d]

Effects of rotational splitting of two *l*=2 modes (including rotational coupling with close radial modes)



Pamyatnykh 2003

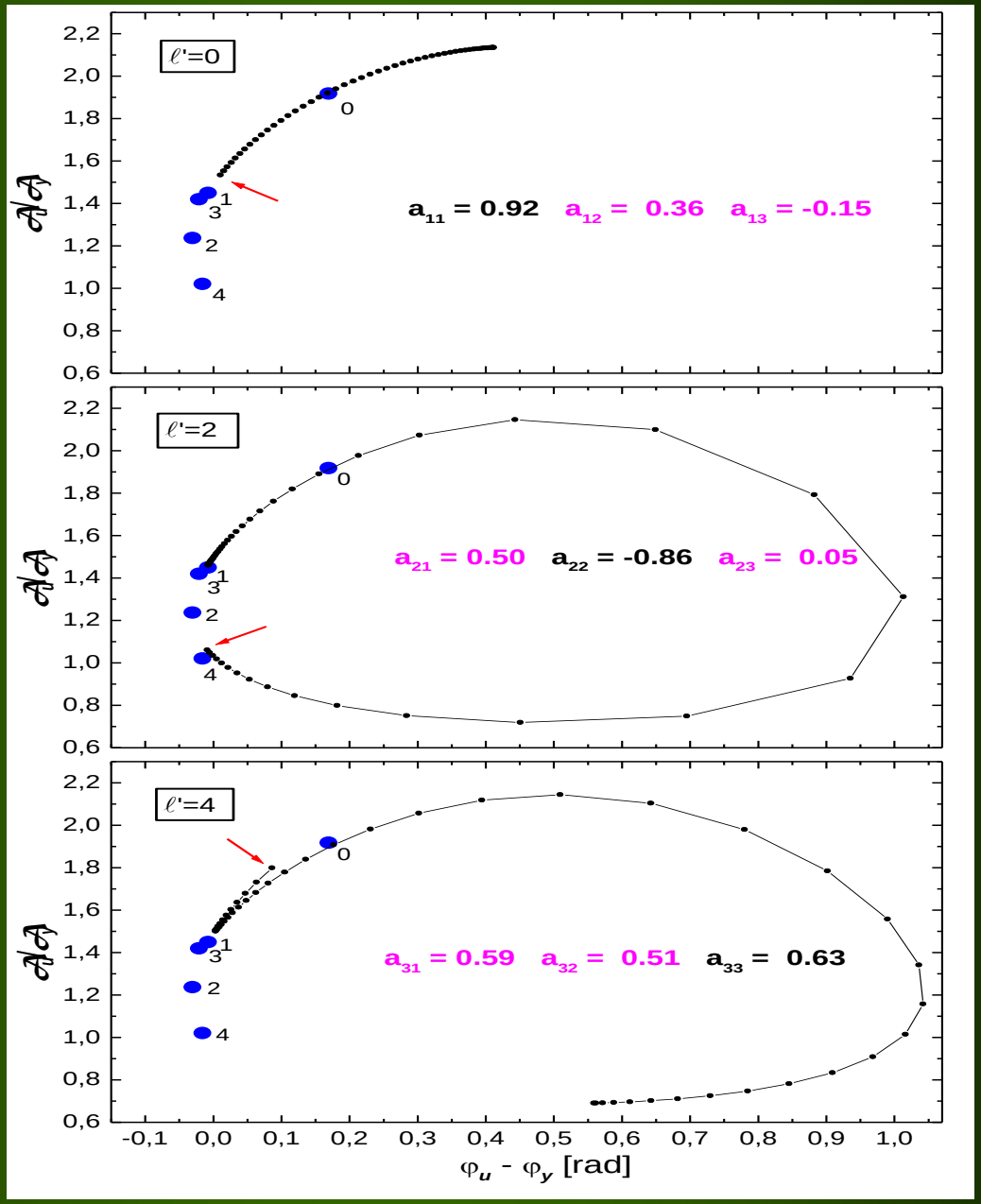
Effect of rotational mode coupling ($\ell = 0$, p_1 and $\ell = 2$, g_1 ,m=0), $\Delta\sigma_0$ =0.003, for a model of 12 $M_\odot$, log T_{eff} =4.374, V_{rot}= 100 km/s.



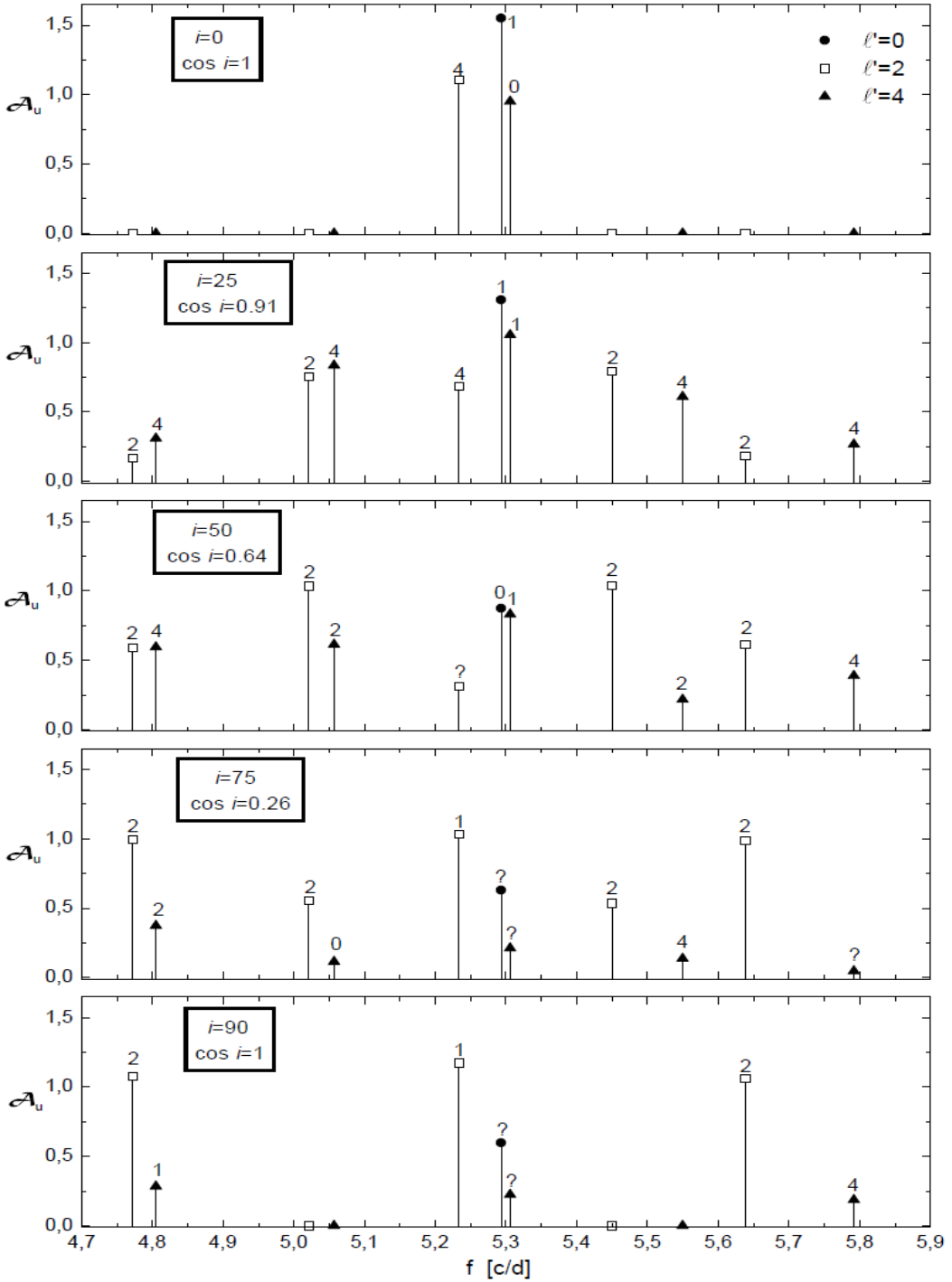
Spacing between consecutive dots is 0.02 in cos i.
Arrows correspond to observations from the polar direction, i=0º.

Daszyńska-Daszkiewicz et al. 2002, A&A 392, 151.

An example of the three mode coupling: (ℓ = 0, p_1), (ℓ =2, g_1), (ℓ =4, g_2)



Daszyńska-Daszkiewicz et al. 2002, A&A 392, 151.



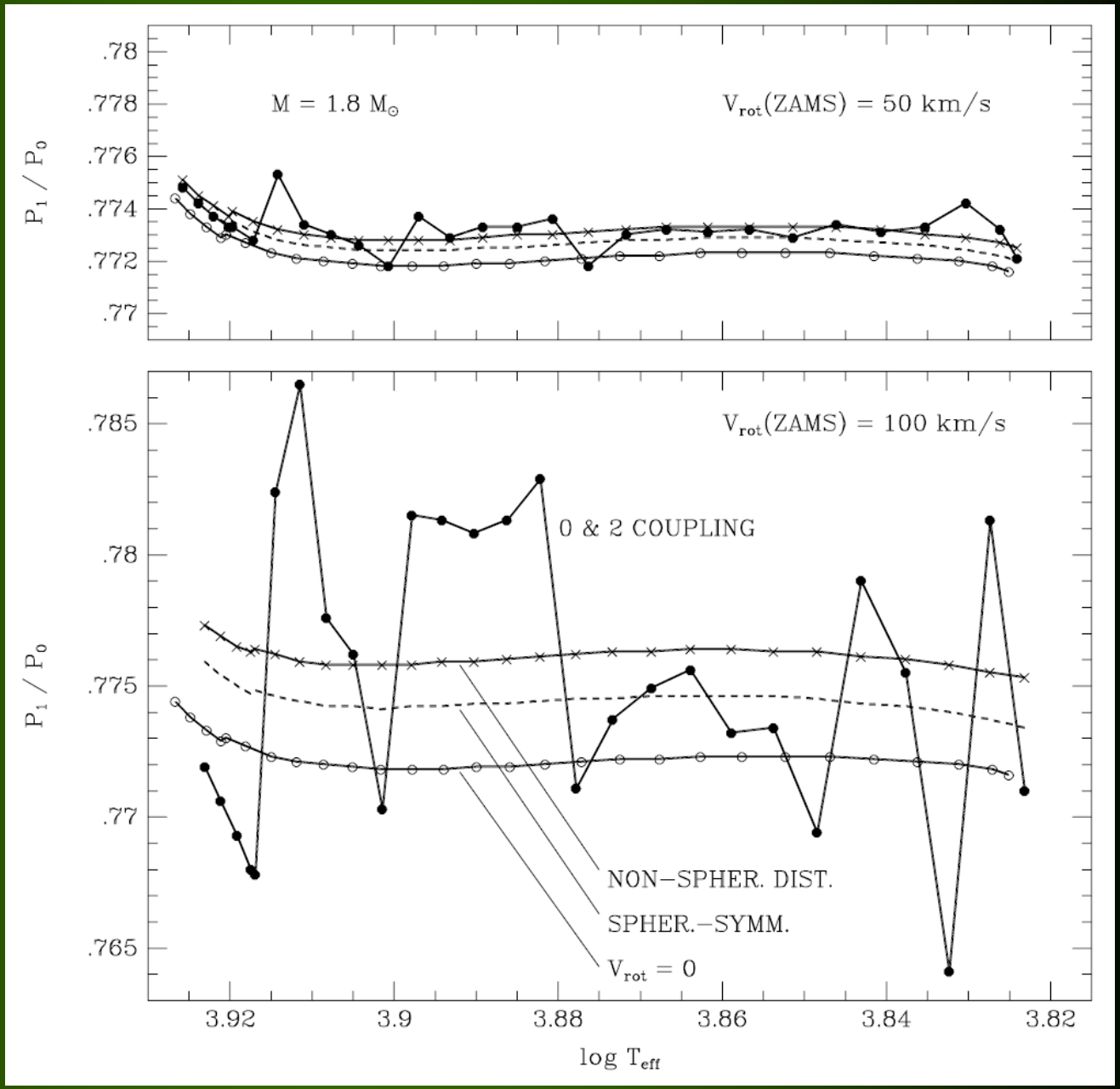
Daszyńska-Daszkiewicz et al. 2002, A&A 392, 151.

Due to rotational coupling, the higher degree modes, ℓ=4 (5), can acquire significant contribution of ℓ=0 (1).

This means that they can be detected in the ground-based observations.

The mode identification must be performed together with determination of the global stellar parameters, the rotation velocity and the inclination angle

Effect of rotational coupling on period ratio of radial modes.



Pamyatnykh 2003

SLOW (LOW-FREQUENCY) MODES

$\omega \sim \Omega$

Traditional approximation

e.g. Townsend 2003,
Daszyńska-Daszkiewicz, Dziembowski, Pamyatnykh 2007,
Shibahashi 2011, this Conference,
Aerts & Dupret, 2011, astro-ph, 31 August

An example:

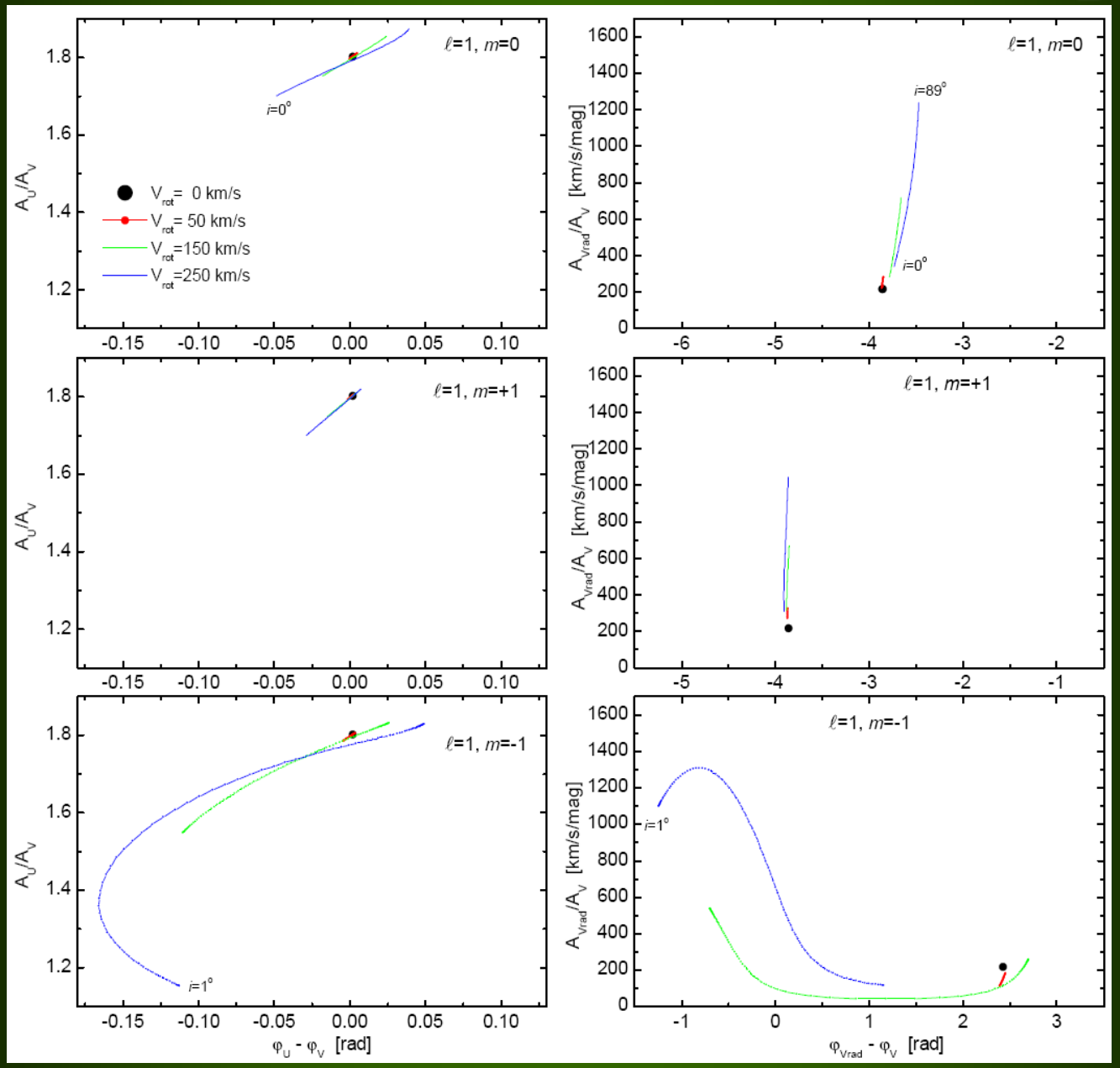
$M = 6\ M_{\odot}$, MS star

$\log T_{eff} = 4.205$ ($T_{eff} = 16\ 000$ K)

$\log L/L_{\odot} = 3.204$

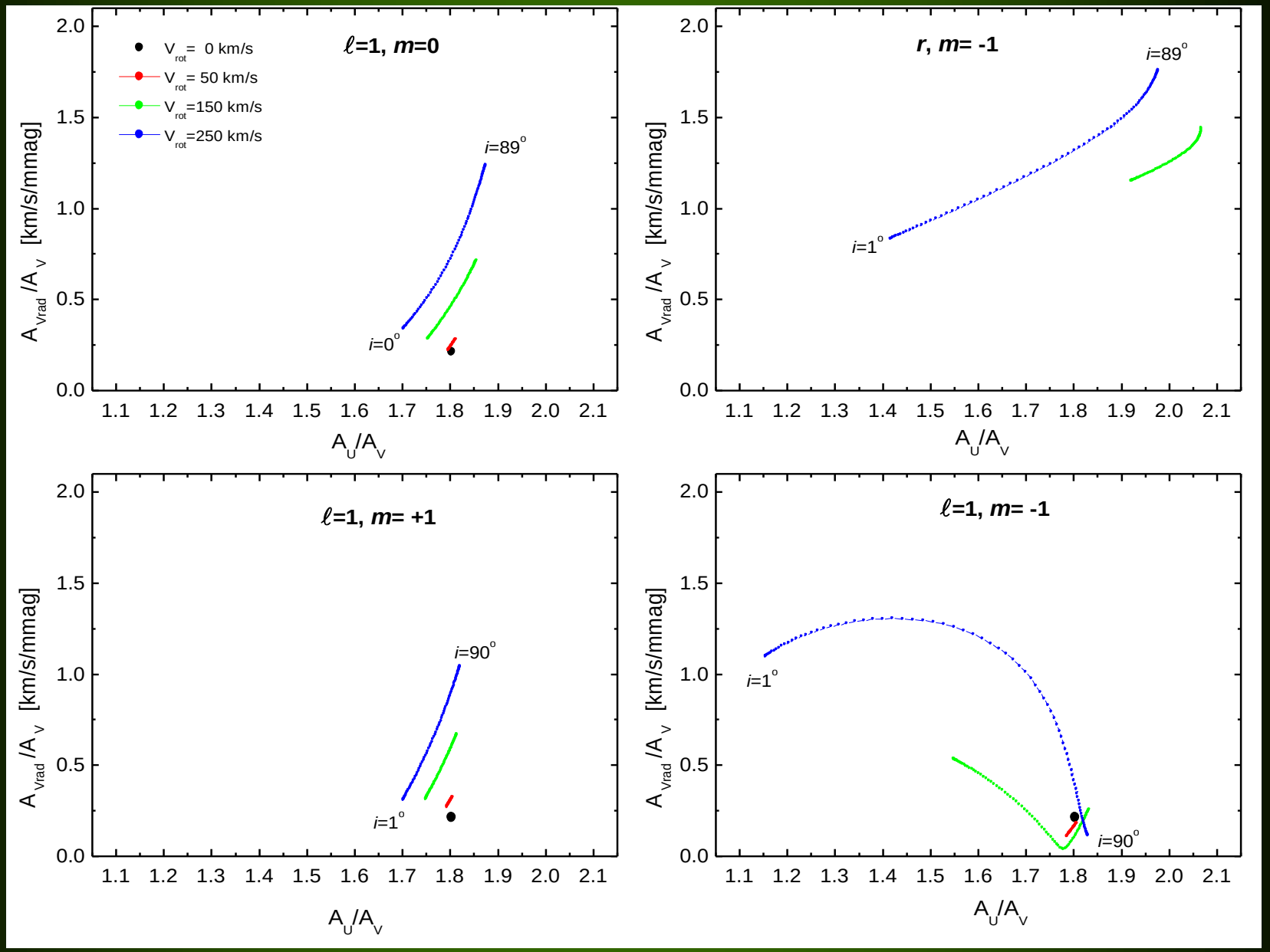
$V_{rot} = 0, 50, 150, 250$ km/s

A_U/A_V *versus* $\varphi_U - \varphi_V$ $\quad$ A_{Vrad}/A_V *versus* $\varphi_{Vrad} - \varphi_V$



due to Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2007

A_{Vrad}/A_V *versus* A_U/A_V



due to Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2007

PHOTOMETRIC COMPLEX AMPLITUDES OF SLOW MODES (ω~Ω)

$$\mathcal{A}_x(i) = \varepsilon \sum_{j=1}^{\infty} \gamma_{\ell_j}^m(s) Y_{\ell_j}^m(i,0) \left[\mathcal{D}_{\ell_j}^x f + \mathcal{E}_{\ell_j}^x \right]$$

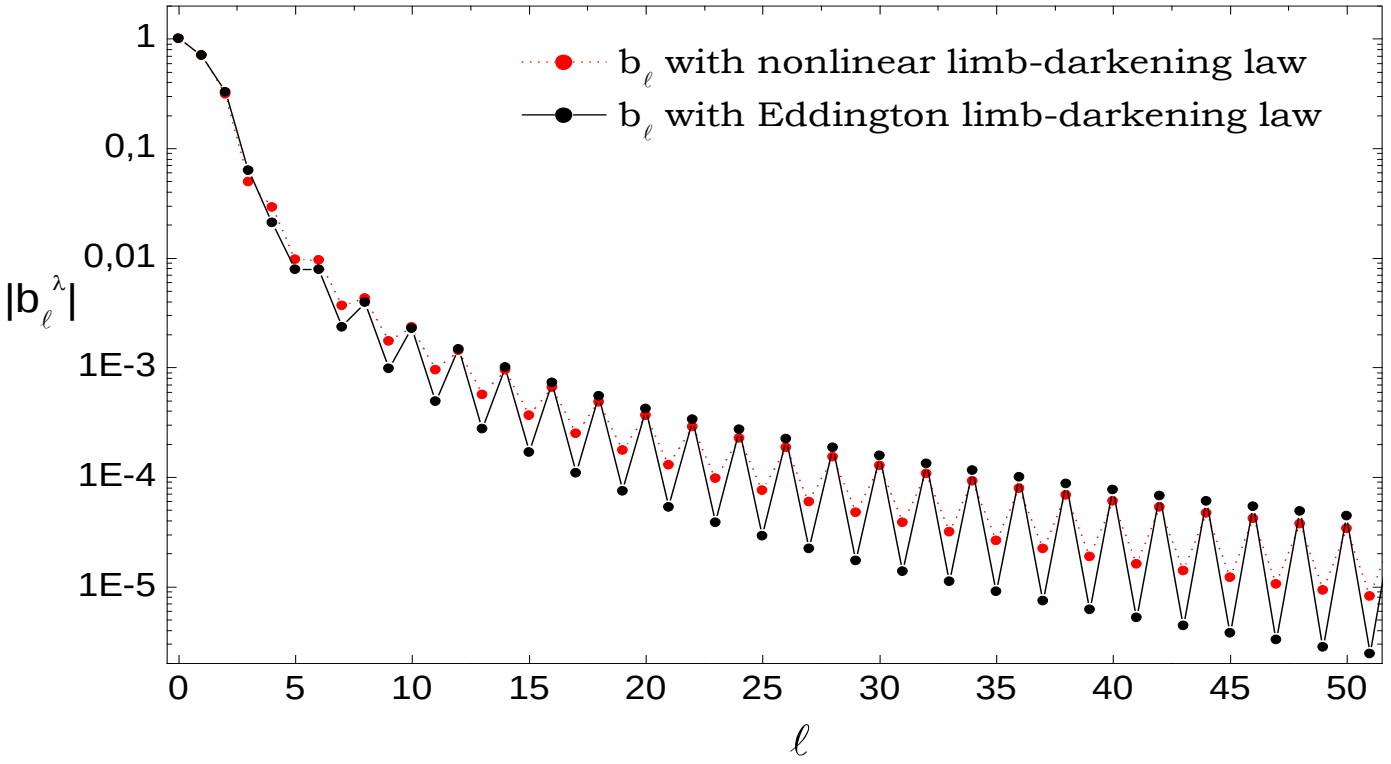
$$\ell_j = \begin{cases} |m| + 2(j-1) \\ |m| + 2(j-1) + 1 \end{cases}$$

- even-parity modes
- odd-parity modes

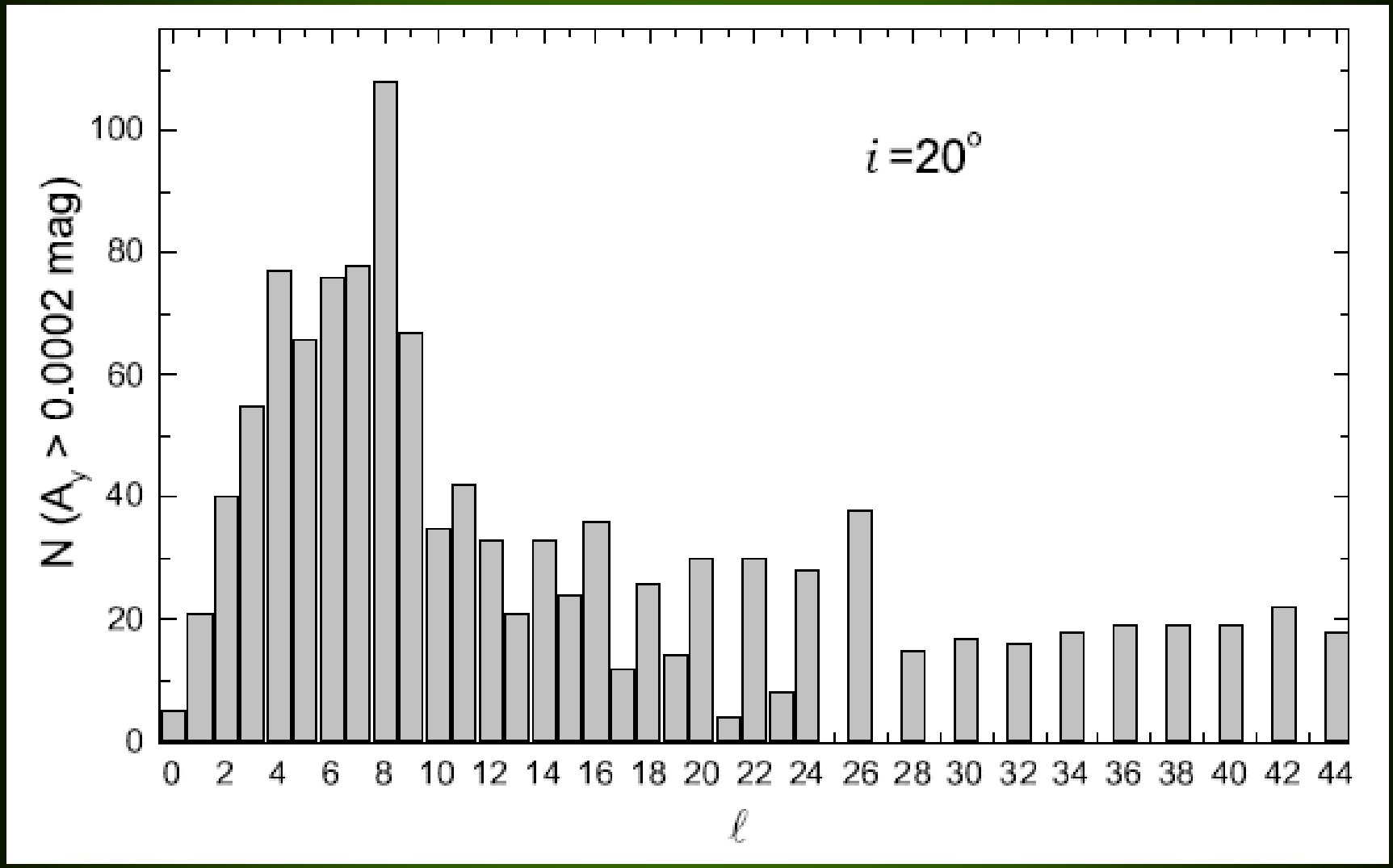
HIGH ℓ MODES

b_ℓ as a function of ℓ

$\log T_{eff} = 3.86$, $\log g = 3.9$, [m/H] = 0.0



Number of modes with photometric Ay > 0.2 mmag for different *l*.
Intrinsic amplitude of displacement is assumed to be ε = 0.005.



Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2006, Mem.S.A.It., 77, 113.

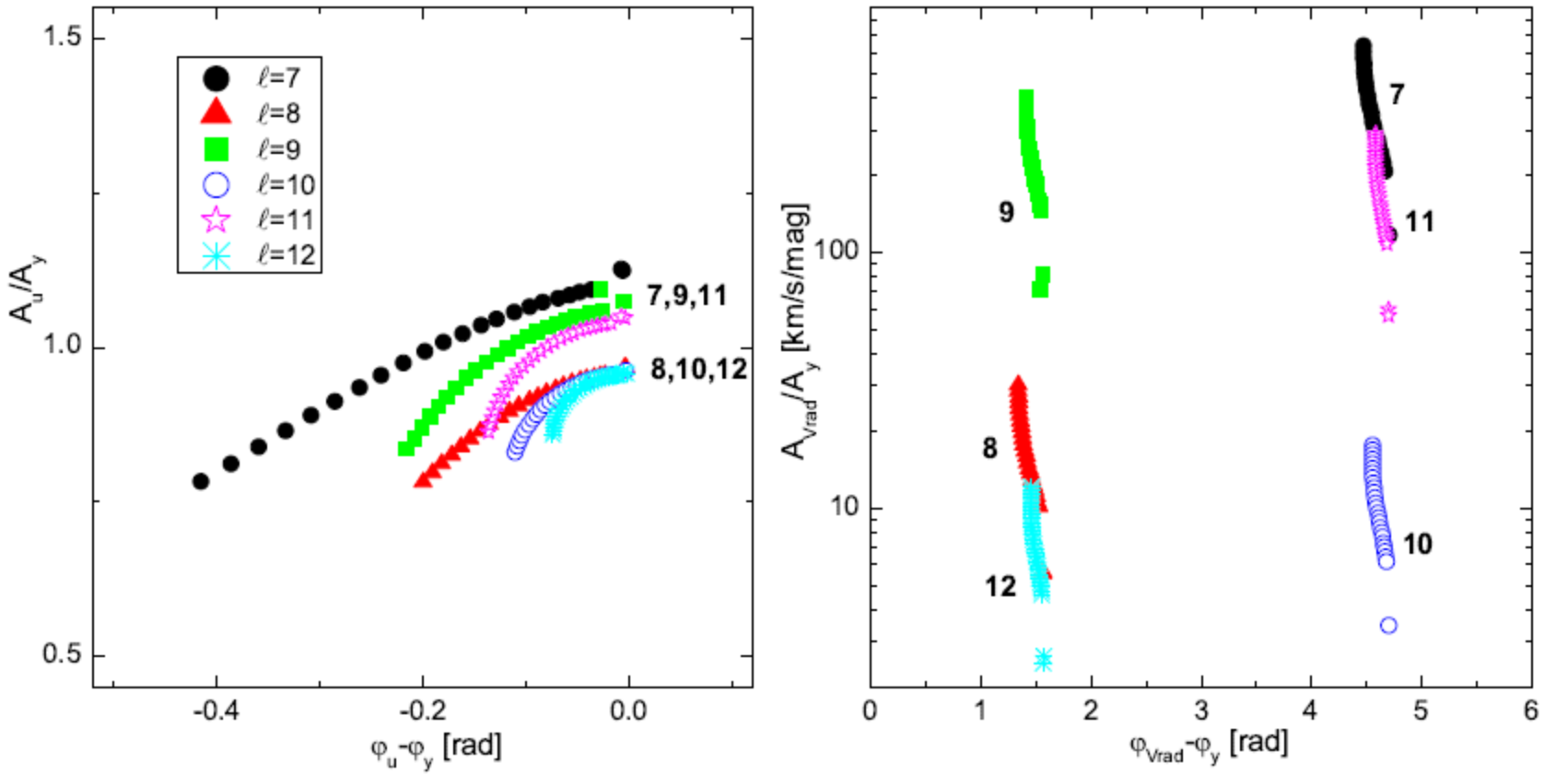


Fig. 4 Position of modes with ℓ from 7 up to 12 in the diagrams using the Strömgren *uy* passbands (left panel) and the radial velocity data and the *y* filter (right panel) for the β Cep model with $M = 8.5 M_{\odot}$ and $\log T_{\rm eff} = 4.322$. Only unstable modes are shown.

THE END

Additional plots

Khi2

The parameters of the pulsation modes are derived only from the amplitude and phase across the line. The reduced χ^2_ν is calculated from complex amplitudes in order to combine amplitude and phase information in the following way

$$\chi^2_\nu = \frac{1}{2n_\lambda - m} \sum_{i=1}^{n_\lambda} \left[\frac{(A^{\rm o}_{R,i} - A^{\rm t}_{R,i})^2}{\sigma^2_{R,i}} + \frac{(A^{\rm o}_{I,i} - A^{\rm t}_{I,i})^2}{\sigma^2_{I,i}} \right] \quad (1)$$

where n_λ is the number of pixels across the profile, m the number of free parameters, $A^{\rm o}$ and $A^{\rm t}$ denote observationally and theoretically determined values, respectively, $A_R = A_\lambda \cos\phi_\lambda$ and $A_I = A_\lambda \sin\phi_\lambda$ are the real and imaginary parts of the complex amplitude, and σ is the observational error. Equation (1) can be easily modified if the observed amplitude and phase from photometric passbands is included for the calculation of χ^2_ν.

Since the amplitude and phase of a given wavelength bin are treated as independent variables, the variances are calculated from

$$\sigma^2_{R,\lambda} = \sigma(A_\lambda)^2 \cos^2\phi_\lambda + \sigma(\phi_\lambda)^2 A^2_\lambda \sin^2\phi_\lambda \quad (2)$$

$$\sigma^2_{I,\lambda} = \sigma(A_\lambda)^2 \sin^2\phi_\lambda + \sigma(\phi_\lambda)^2 A^2_\lambda \cos^2\phi_\lambda. \quad (3)$$

Zima 2006 due to JDD

ZMIANY PRĘDKOŚCI RADIALNEJ

Po scałkowaniu dostaniemy

$$V_{rad} = -\mathrm{i}\varepsilon\omega R N_l^0 d_{\ell m0}(i)(u_\ell + \alpha_H v_\ell)\mathrm{e}^{-\mathrm{i}\omega t}$$

$$u_\ell = \int_0^1 h\mu^2 P_\ell(\mu) d\mu,$$

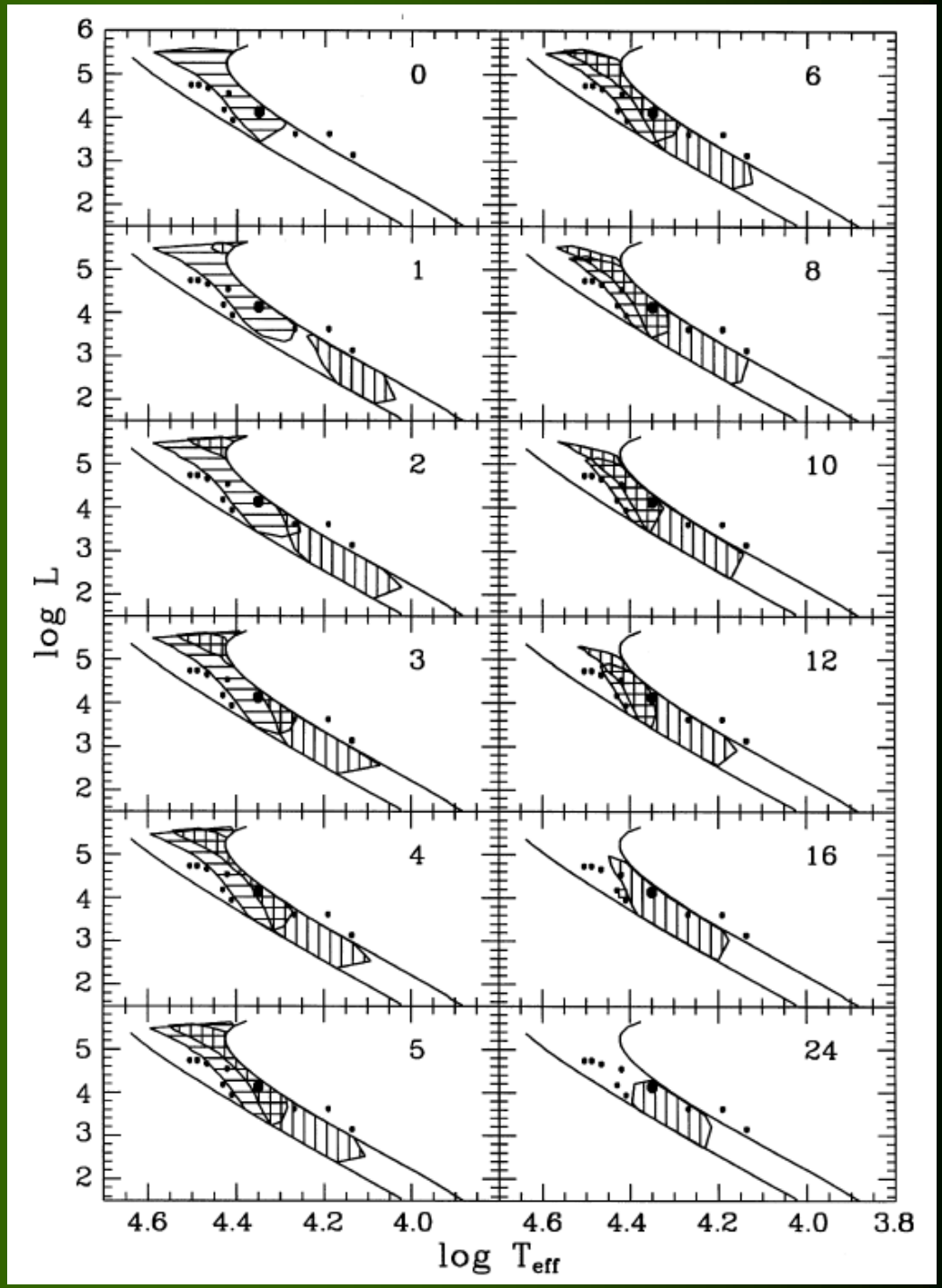
$$v_\ell = \ell \int_0^1 h(P_{\ell-1} - \mu P_\ell)\mu d\mu,$$

$$u_\ell^\lambda = \frac{1}{2\ell+1}[(\ell+1)b_{\ell+1}^\lambda + \ell b_{\ell-1}^\lambda]$$

$$v_\ell^\lambda = \frac{\ell(\ell+1)}{2\ell+1}[b_{\ell-1}^\lambda - b_{\ell+1}^\lambda]$$

Dziembowski (1977) AcA 27, 203

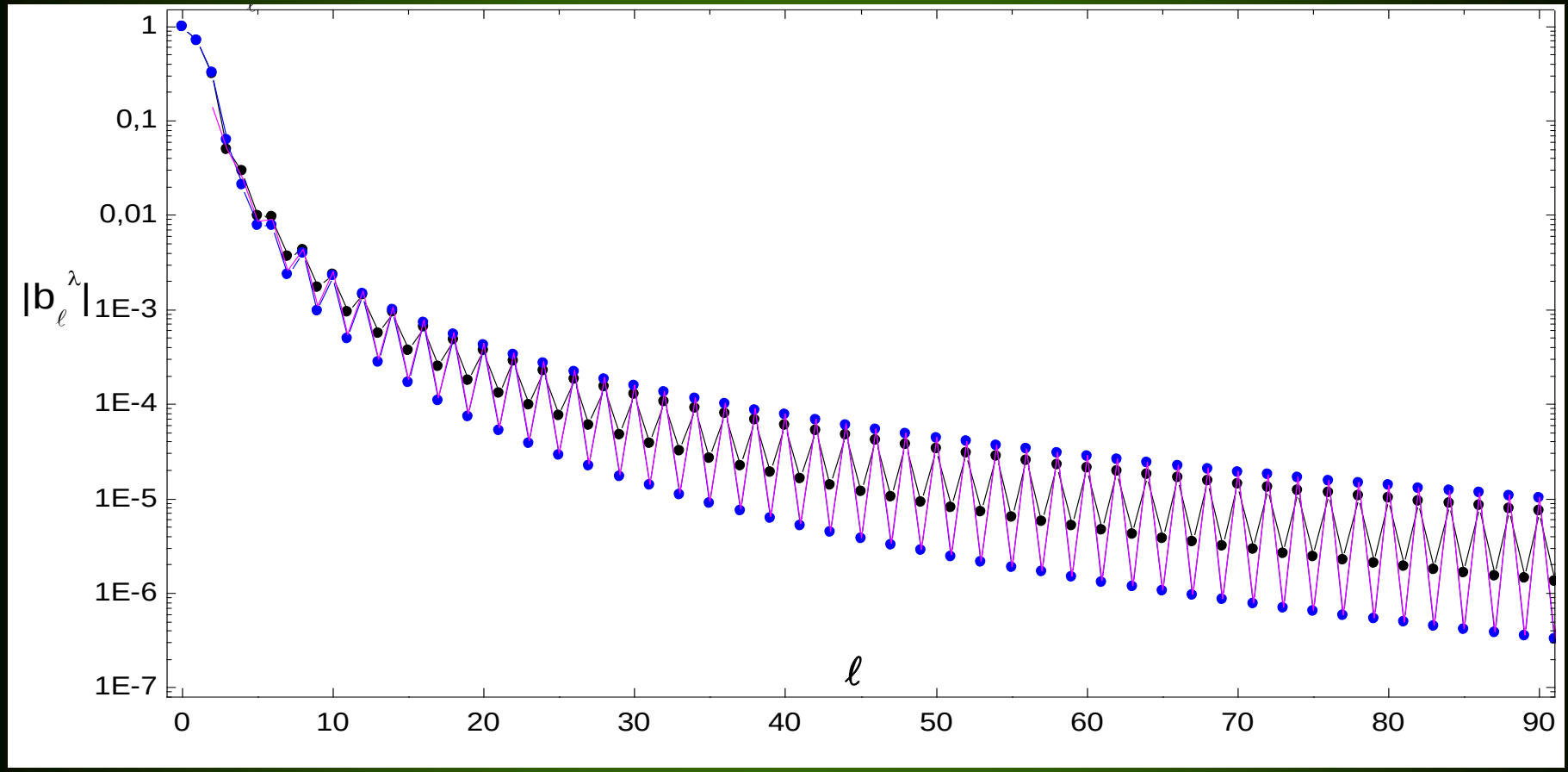
Short-period (β Cep) and long-period (SPB) instability strips for mode degree *l* from 0 to 24.



Balona & Dziembowski 1999

High degree modes

b_ℓ^λ *in passband y, nonlinear* $h_\lambda(\mu)$ *(Claret)*
b_ℓ^λ *- Eddington* $h(\mu)$ *(*$h(\mu)$ =1.5μ+1*)*



Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh 2006, Mem.S.A.It., 77, 113.

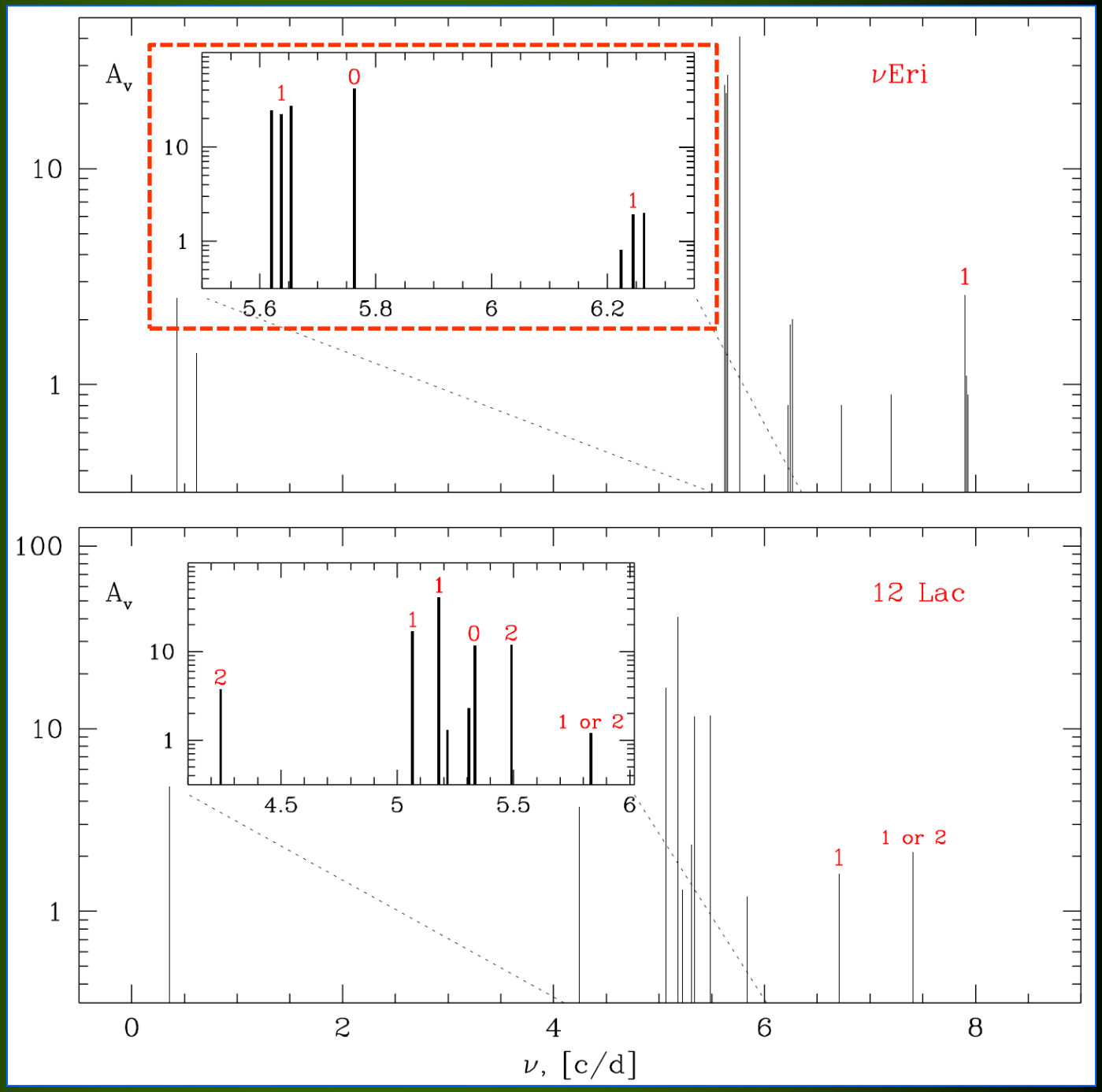
ν Eridani

Testing rotation in interiors

(5 slides from lecture 01)

Oscillation spectra of ν Eri and 12 Lac

Two rotationally splitted triplets of $l = 1$ modes (g1 and p1)

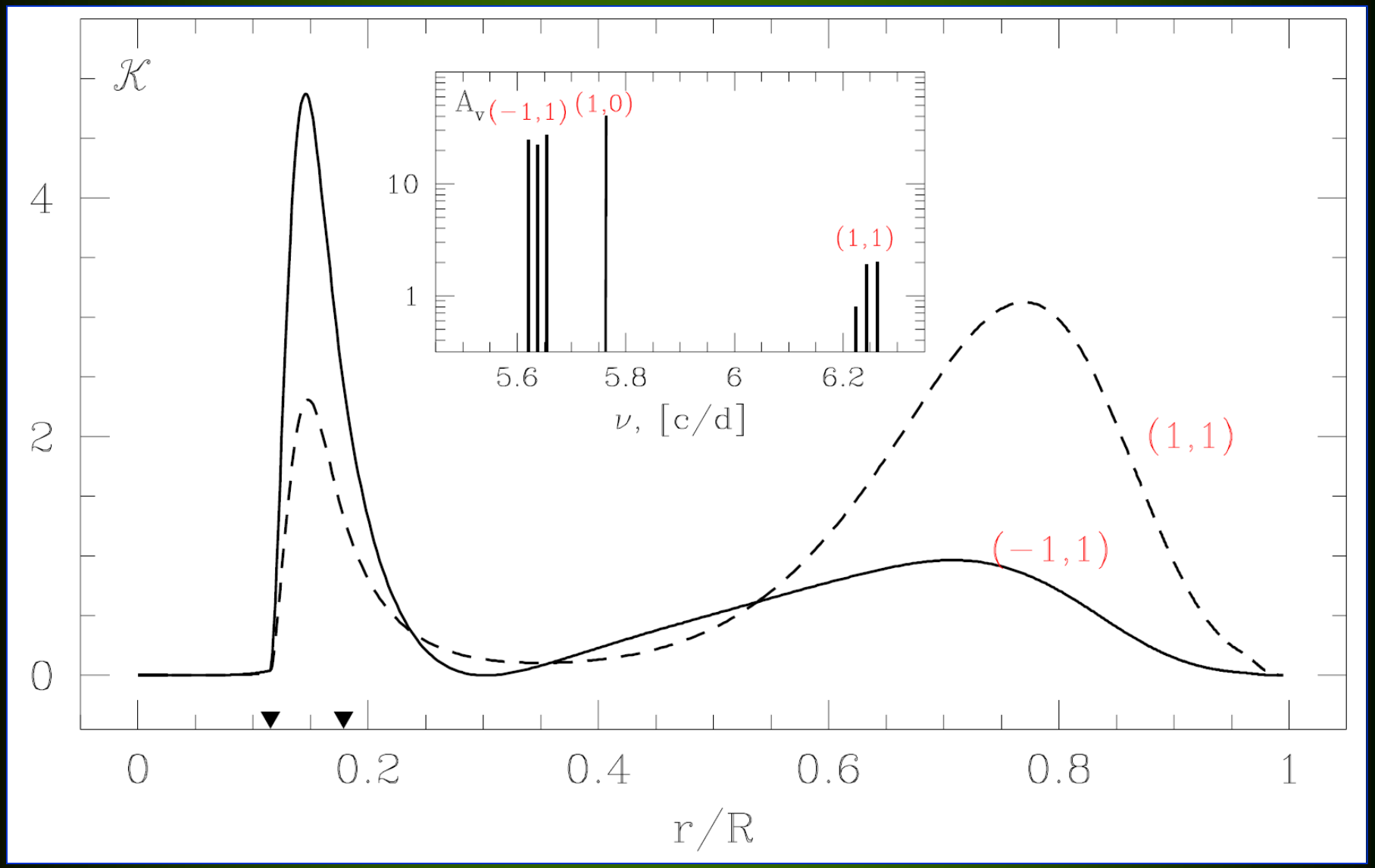


Dziembowski & Pamyatnykh 2008

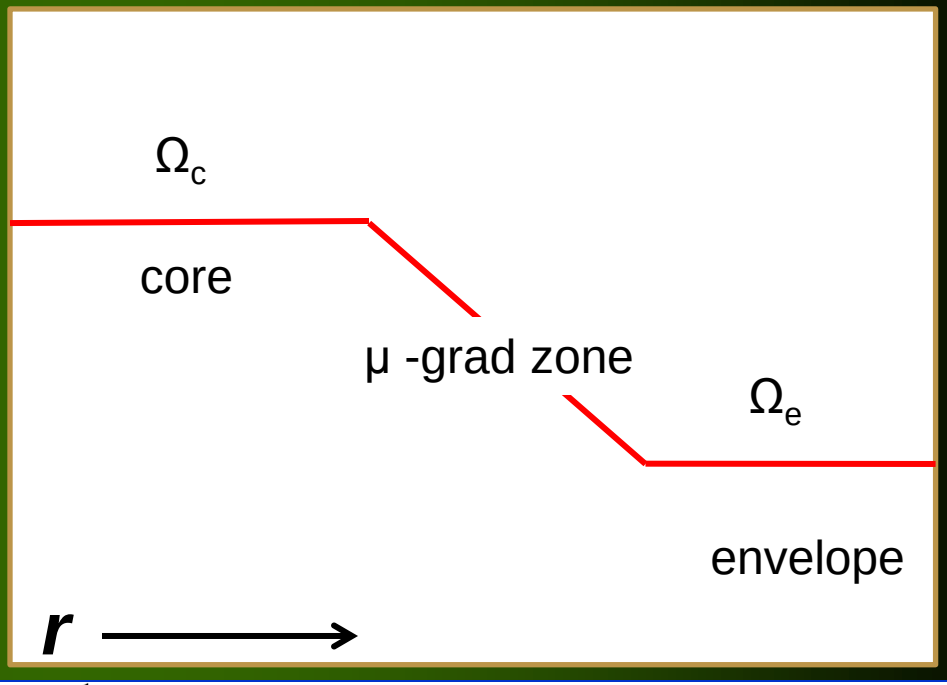
$$S \equiv 0.5(\nu_+ - \nu_-) = \int_0^1 dx \mathcal{K}(x)\Omega/2\pi$$

ν Eri: the splitting kernels for two normal modes
***l* = 1, g1 (solid line), *l* = 1, p1 (dashed line).**
Triangles mark boundaries of the μ-gradient zone.

S_g=0.017, S_p=0.020 c/d



Simple rotation law



Results:

$V_{rot} = 6$ km / s

$\Omega_{core} = 5 * \Omega_{envelope}$

Table 1. The O-C (in cd^{-1}) for the $\ell = 1$, p_2 mode frequencies and the differences between observed effective temperature and that of the seismic model, $\Delta \log T_{eff} = \log T_{eff,obs} - \log T_{eff,cal}$. The observational uncertainty in effective temperature is $\Delta \log T_{eff} = 0.011$. Models with $\alpha_{ov} > 0$ were calculated with $w = 8$. V_{rot} is given in km s^{-1}.

κ	Mixture	α_{over}	OC	$\Delta \log T_{eff}$	Ω_c/Ω_e	V_{rot}
OP	A04	0.0	−0.127	−0.0103	5.55	5.93
OP	GN93	0.0	−0.151	−0.0075	5.36	5.95
OPAL	A04	0.0	−0.188	−0.0044	5.36	5.99
OP	A04	0.1	−0.085	−0.0159	5.82	5.91
OP	A04	0.2	−0.034	−0.0244	5.78	5.93

Dziembowski, Pamyatnykh 2008

Summary of results for ν Eridani

- ν Eri has 14 independent modes, the strongest one is the radial fundamental mode, most of the others are *l* = 1 modes.
- Peaks at 0.432 and 0.614 c/d correspond to high-order g-modes (SPB-type pulsations). ν Eri – a hybrid star.
- Rotational splitting of two *l*=1 modes (g1 and p1) can be used to infer information on internal rotation. The core rotates approximately 5 times faster than the envelope.
- Overshooting from the convective core seems to be ineffective in ν Eri, but the estimate critically depends on the Teff determination.
- For standard opacities modes at lowest and highest observed frequencies are stable. An additional opacity enchancement around Z bump region may solve the problem.
- A new way of probing opacity