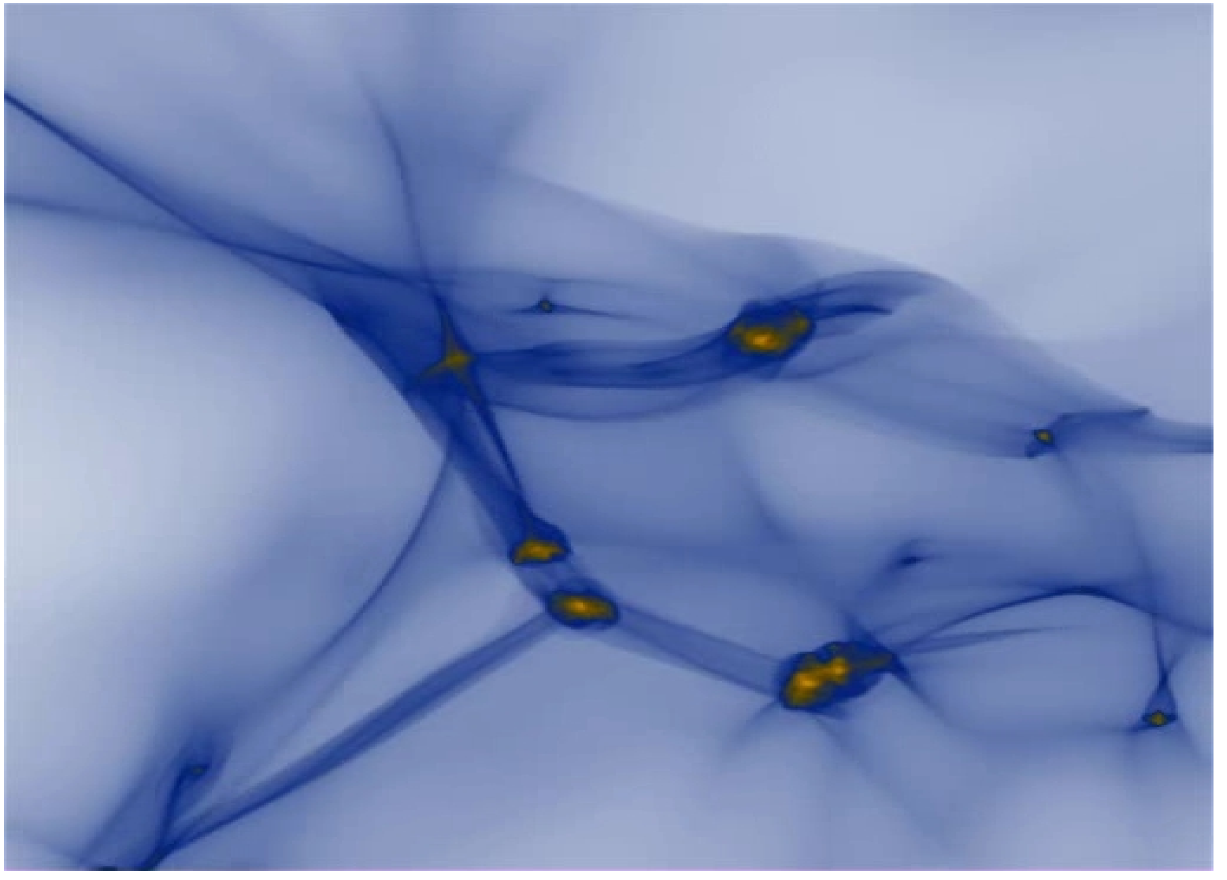
Cosmological Structure Formation in the Continuous Limit



Angulo, Hahn & Abell 2013

Raul E. Angulo

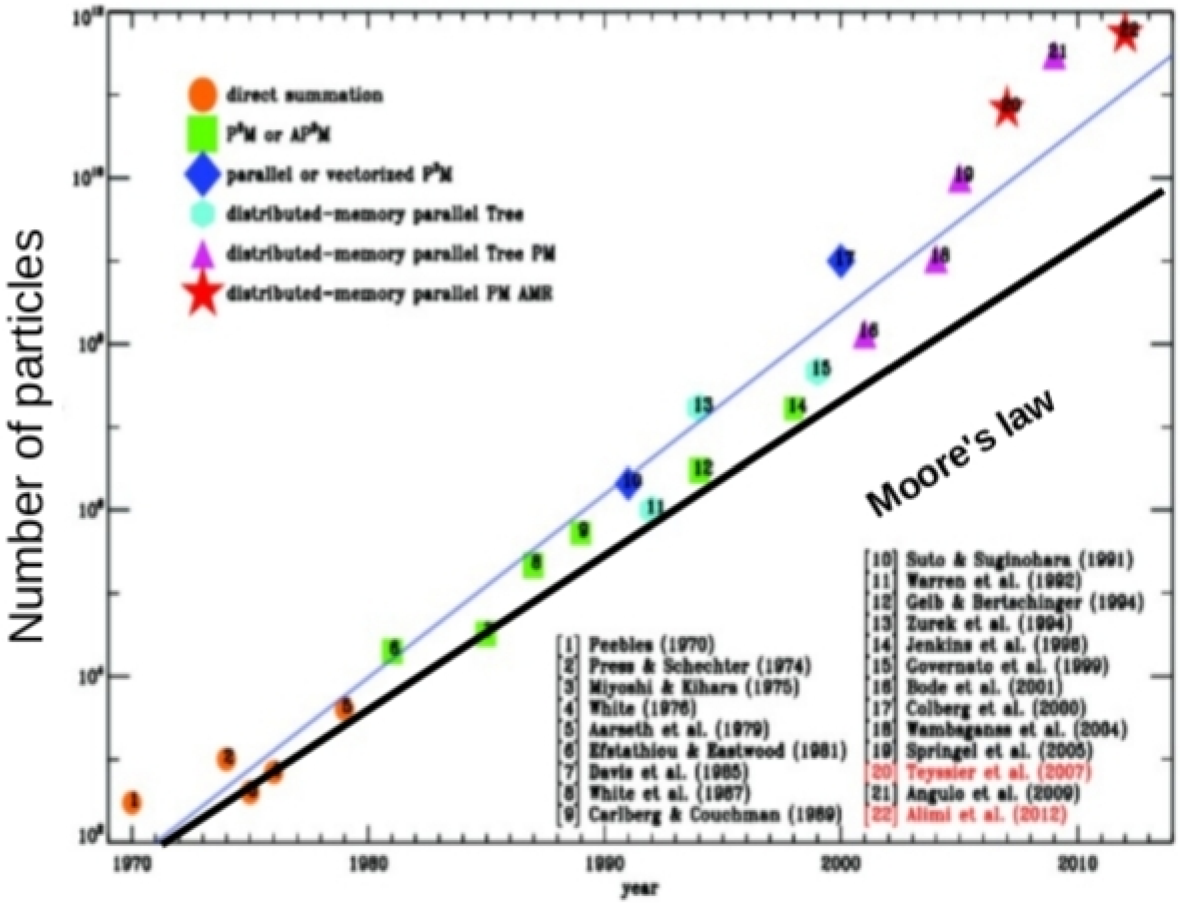
with O. Hahn, T, Abel, D. Powell, R. Kaehler, S. Hilbert.

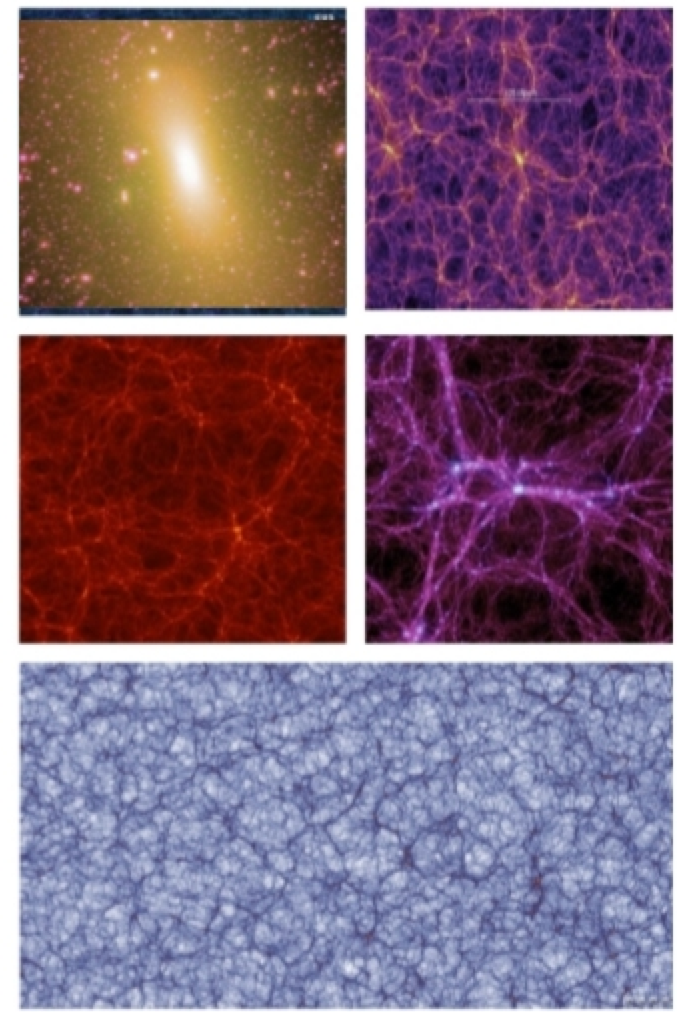
Warsaw
August 2015

Exponential growth of computing power over 40 years

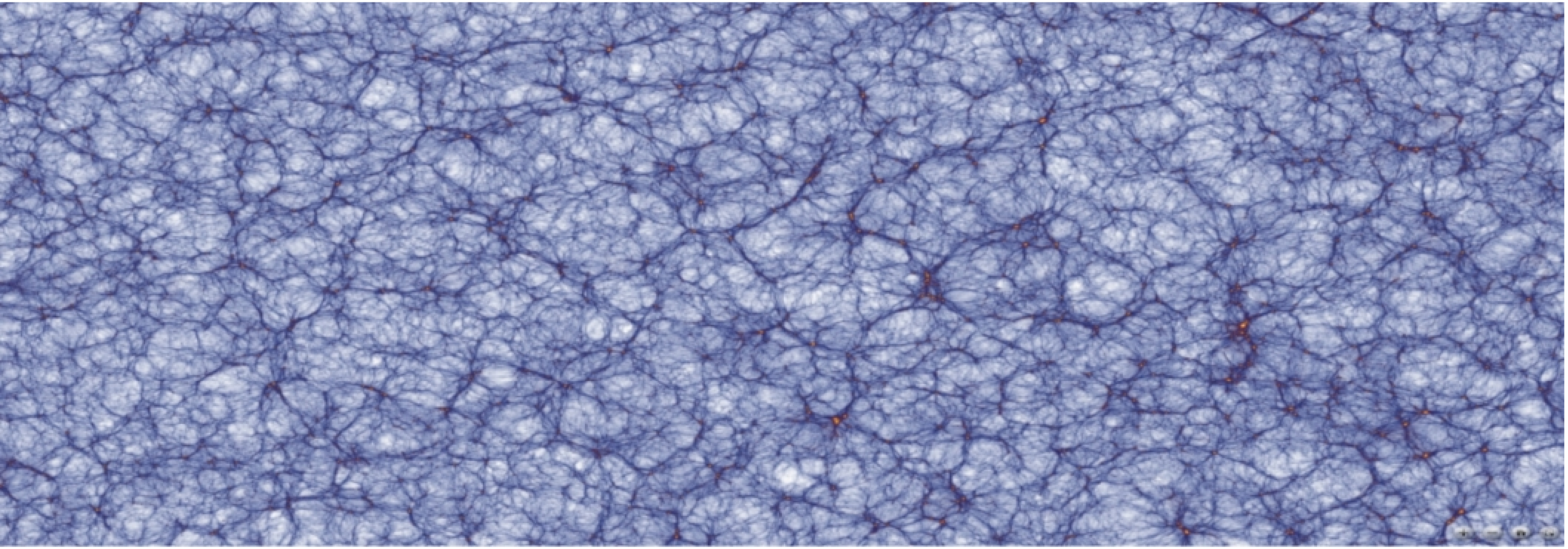
10 trillion particle N-body simulations are expected by 2020

Springel et al 2005, Alimi et al 2012





1e8 times more particles than Davis et al (1985). Challenges in the raw execution time, scalability, memory requirements and data handling.



Large-scale N-body simulations aim to predict:

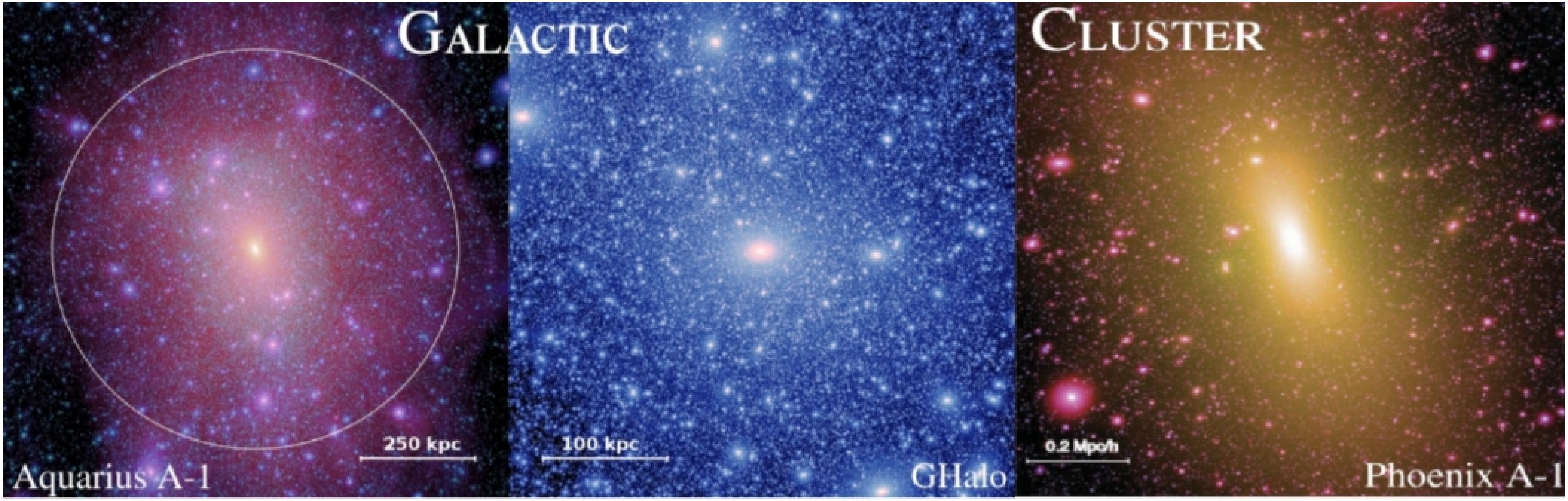
→ The nonlinear state of mass

→ The velocity field

→ Abundance and properties of collapsed DM structures

→ The places of galaxy formation

BAO & Galaxy Clustering
Abundance of Clusters
Weak Gravitational Lensing
Redshift-Space Distortions



Springle+ 2008 Stadel+ 2009 Gao+ 2012

Zoom-In N-body simulations aim to predict:

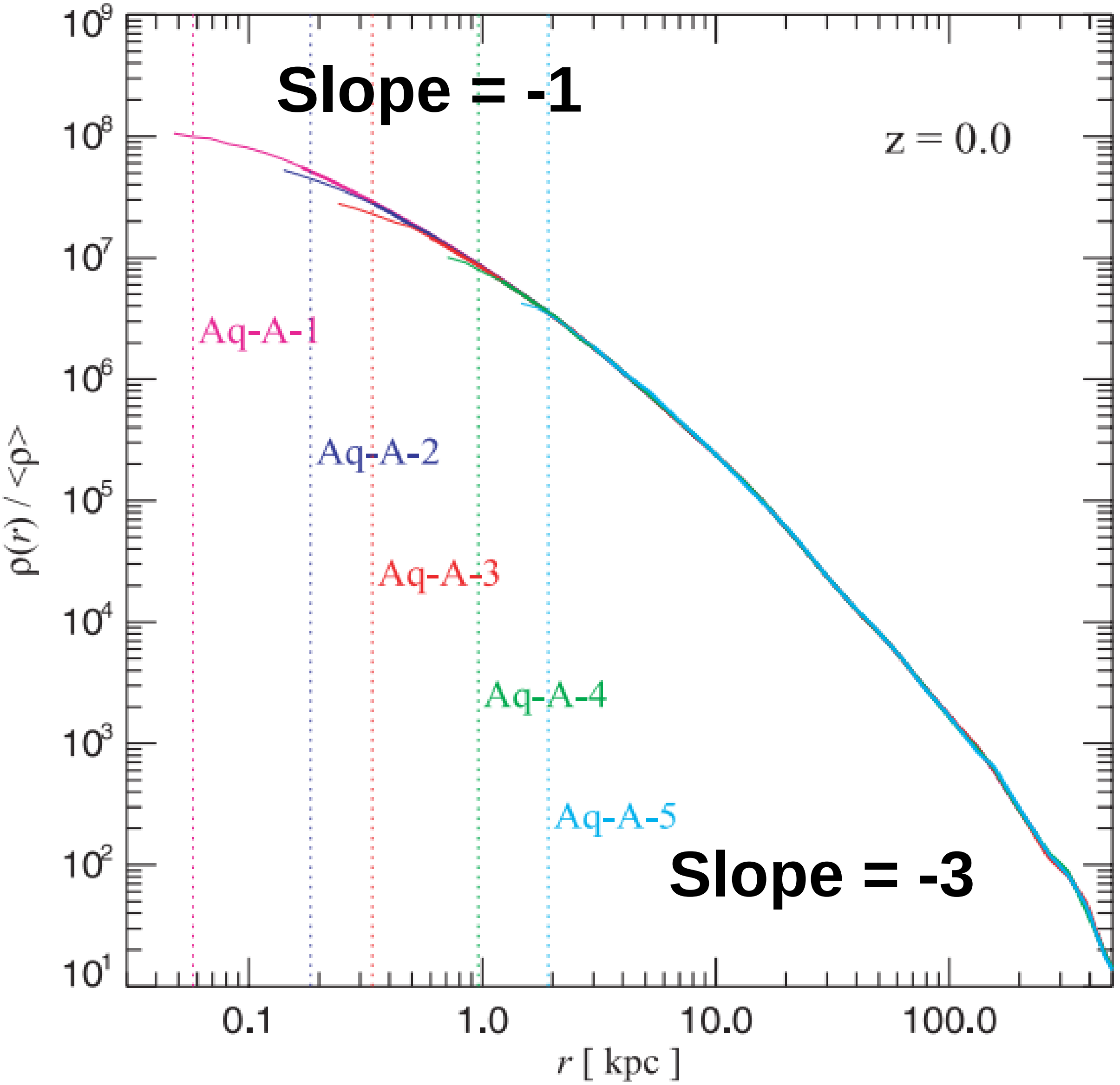
→ Halo density and velocity profiles

→ Substructure mass function

→ Substructure spatial distribution

Direct Detection
Indirect Detection
Astrophysical Probes

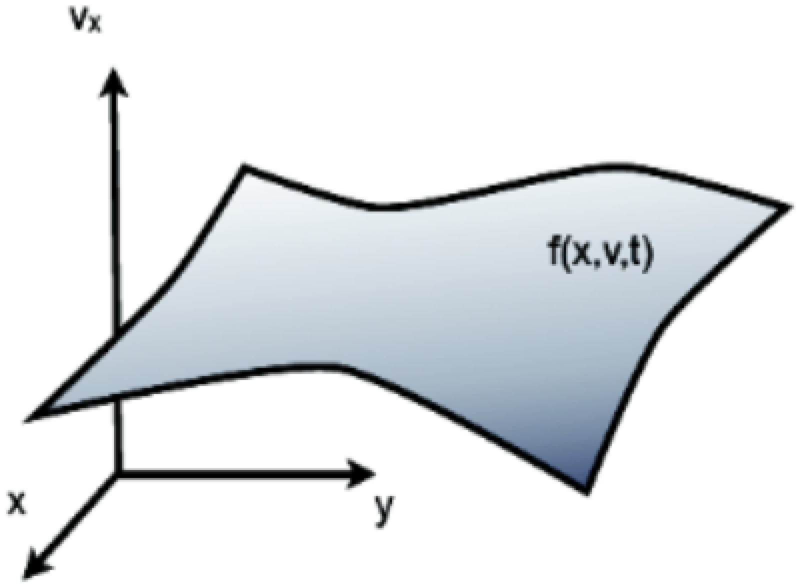
The density profile of Dark Matter halos



Springel et al 2008

Simulating structure formation in the Universe

Most of the mass in the Universe is in the form of an unknown elementary particle: the Cold Dark Matter



Properties of CDM

→ Almost no extent in the velocity direction

→ Interacts mainly gravitationally

→ Tiny primordial fluctuations

...but simulating trillions of micro-physical CDM particles is impossible

CDM forms a "sheet": A continuous 3D surface embedded in a 6D space

The Vlassov-Poisson Equation

$$0 = \frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\mathbf{v}}{a^2} \cdot \frac{\partial f}{\partial \mathbf{v}} - \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \Phi}{\partial \mathbf{x}}$$

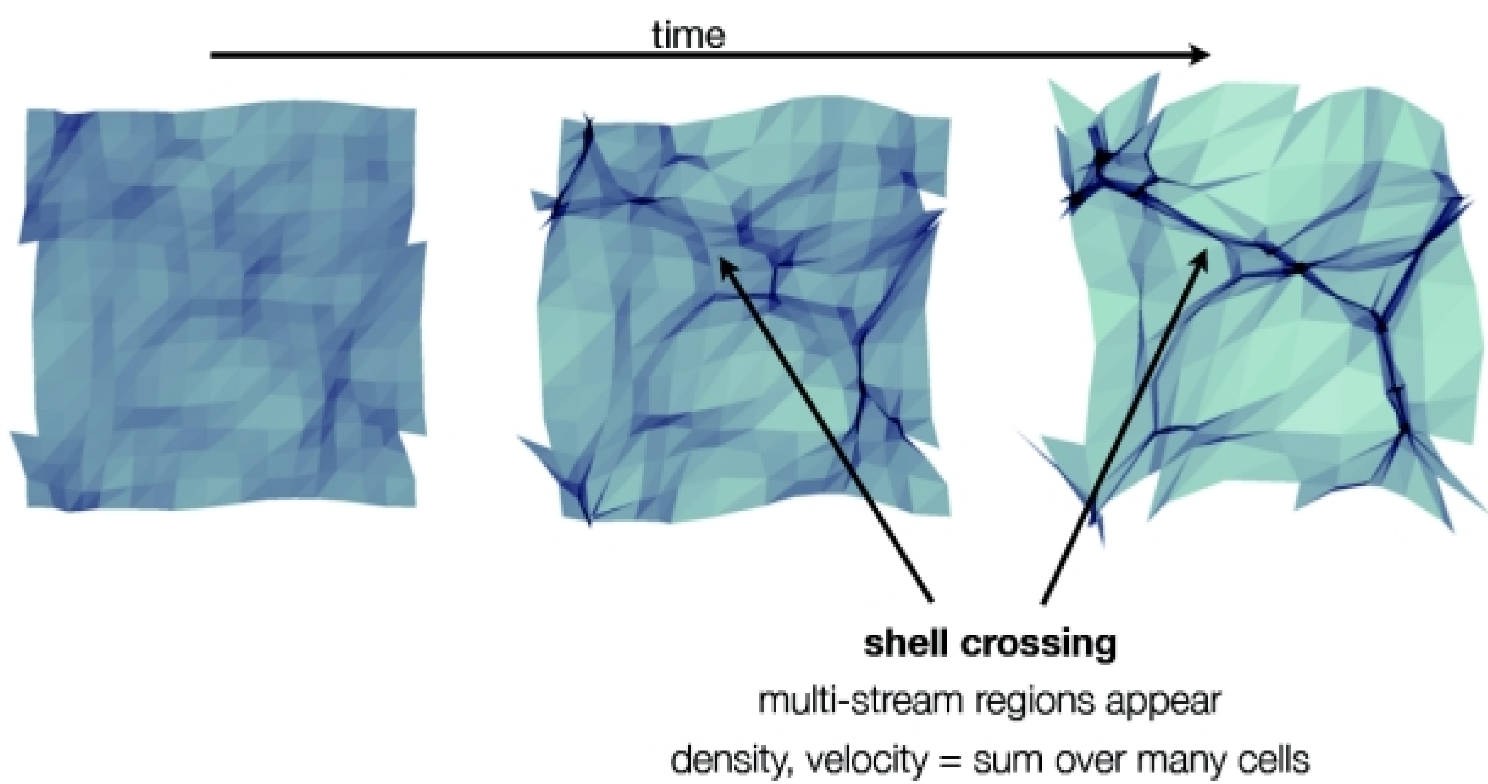
$$\nabla^2 \Phi = \frac{4\pi G}{a} \int f \mathrm{d}^3 v.$$

CDM Sheet Properties

→ phase-space is conserved along characteristics

→ It can never tear

→ It can never intersect



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CDM Sheet Properties

- → phase-space is conserved along characteristics
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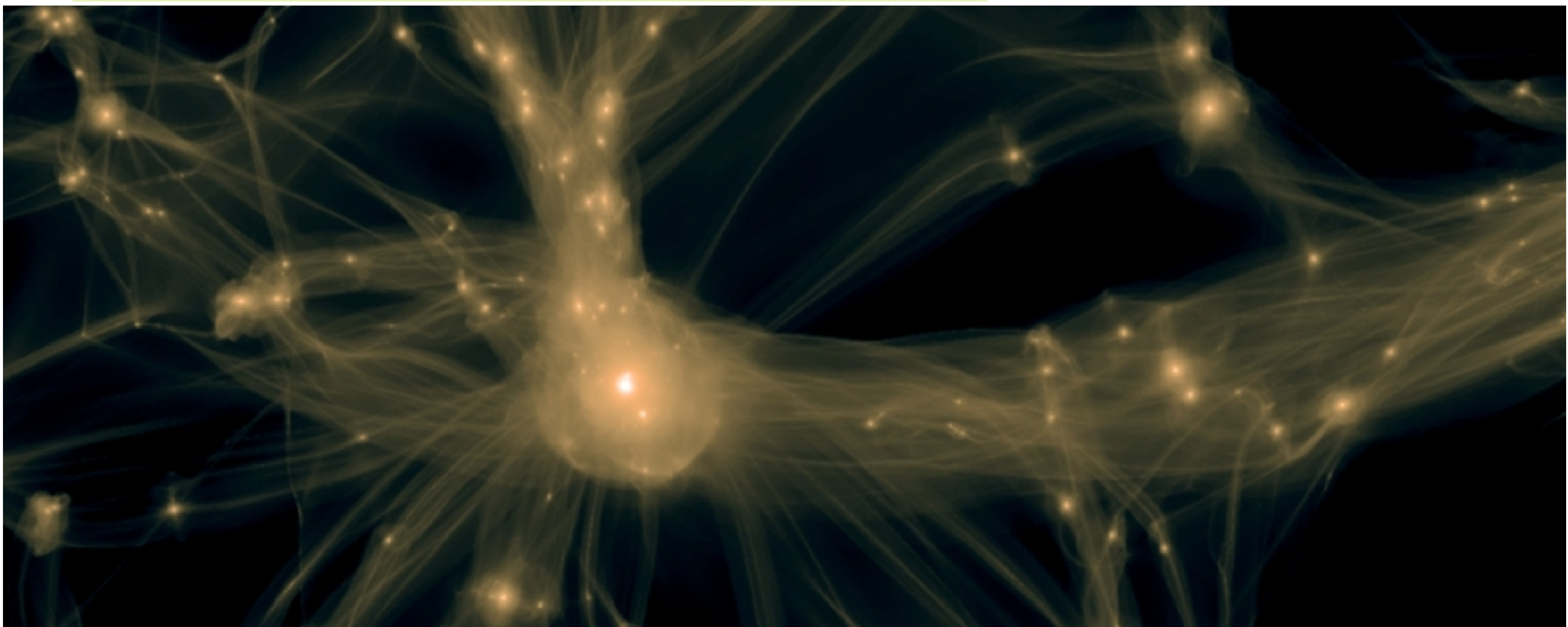
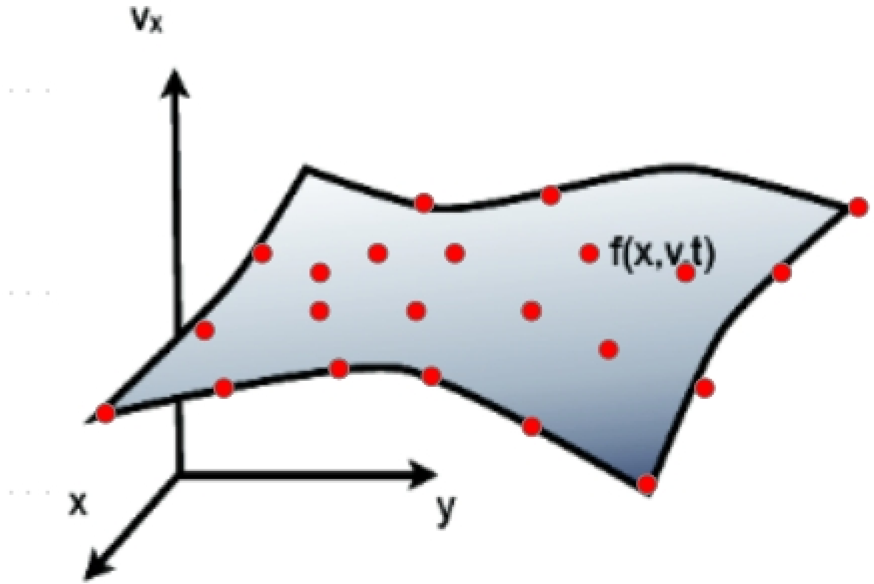


Standard approach to solving the VP equation:

Montecarlo Sampling and coarse graining the CDM distribution function

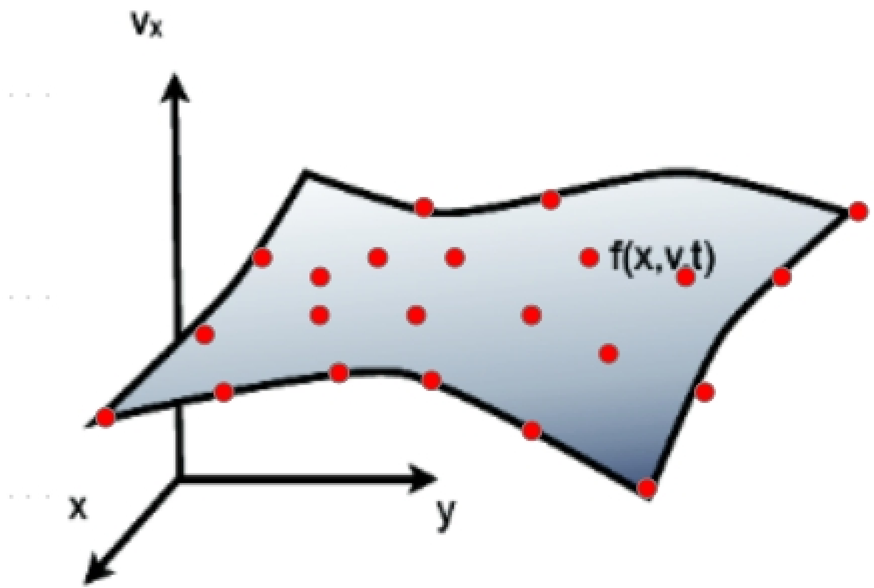


$$\frac{\mathrm{d}^2\mathbf{x}_i}{\mathrm{d}^2 t} = \nabla_i \Phi(\mathbf{x}_i),$$

$$\Phi(\mathbf{x}) = -G \sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \varepsilon^2]}$$

Standard approach to solving the VP equation:

Montecarlo Sampling and coarse graining the CDM distribution function

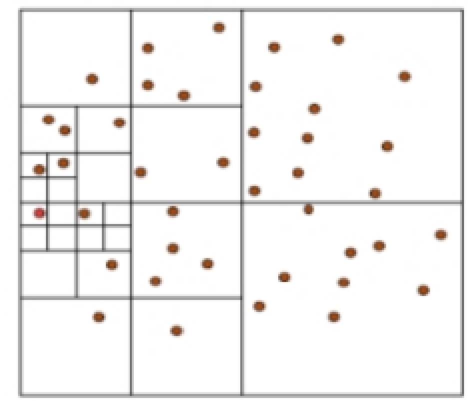


$$\frac{d^2\mathbf{x}_i}{d^2t} = \nabla_i \Phi(\mathbf{x}_i),$$

$$\Phi(\mathbf{x}) = -G \sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \varepsilon^2]}$$

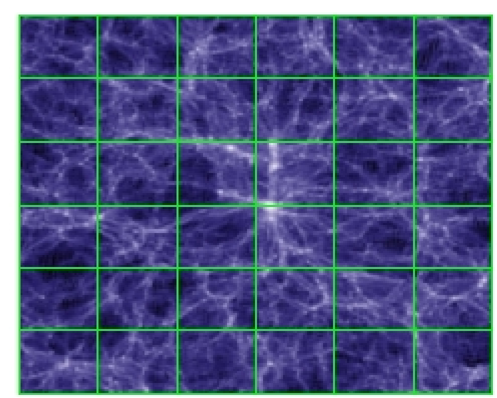
Tree Algorithms

Multipole decomposition



Particle-Mesh

Poisson equation



The evolution of the fine and coarse grained distribution functions are NOT equivalent.

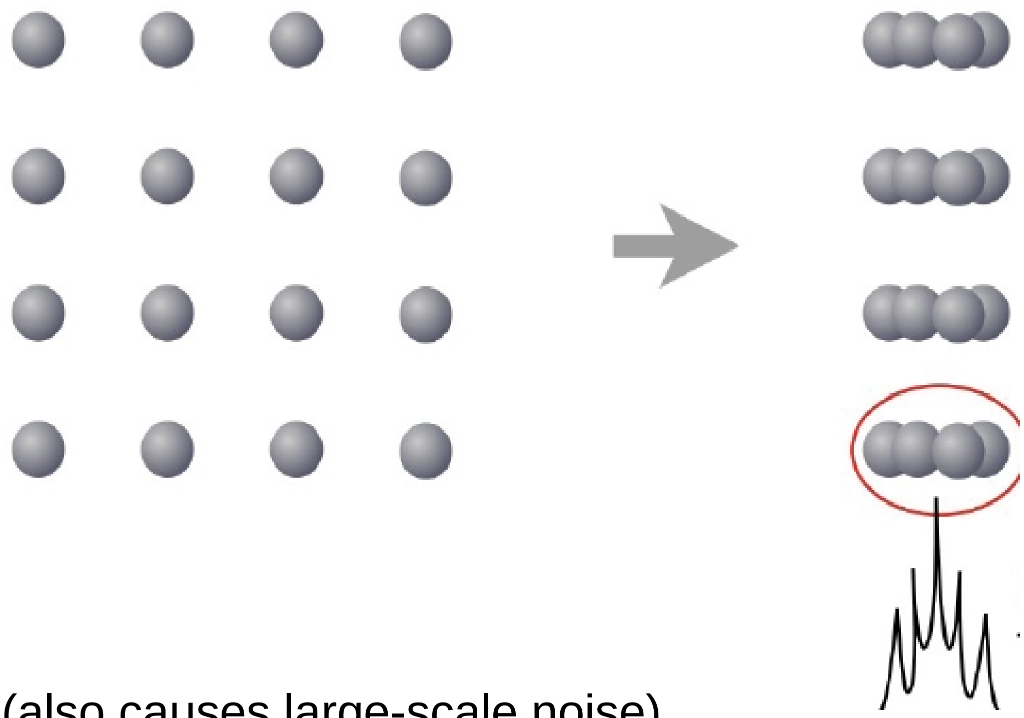
Collisionless Relaxation

Phase Mixing
Chaotic Mixing
Violent Relaxation
Landau Damping

Softening Length

It prevents forces to diverge, which would lead to large-angle scattering events

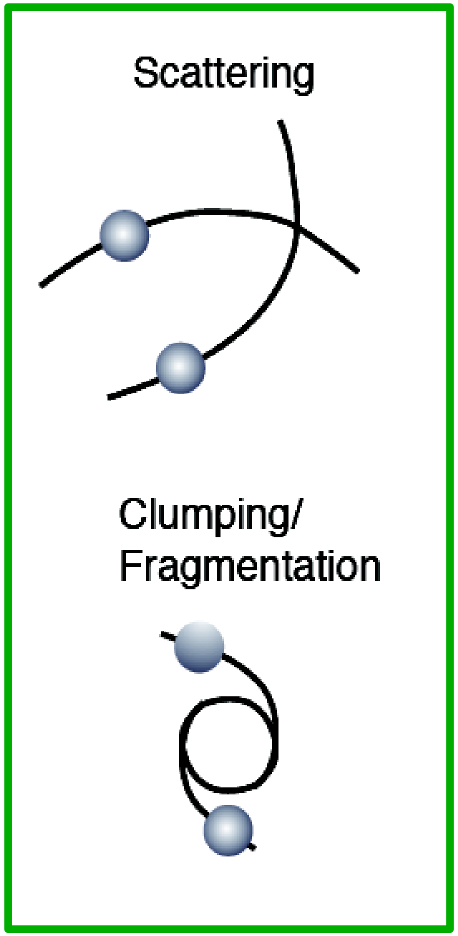
Anisotropic compression in triaxial collapse



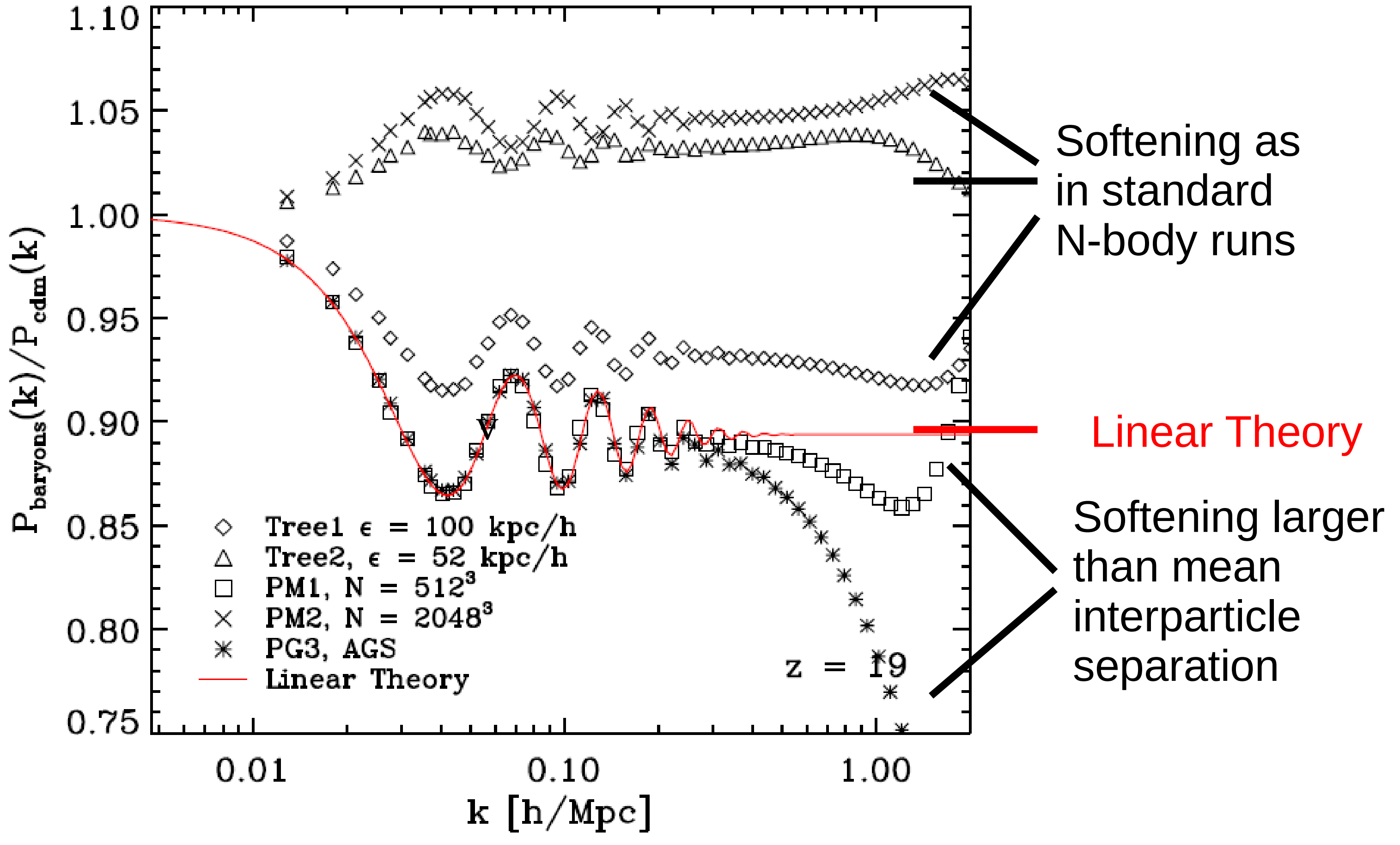
(also causes large-scale noise)

Two examples where the monte-carlo approach to collisionless dynamics (aka N-body) fails:

Two fluids with distinct primordial power spectra

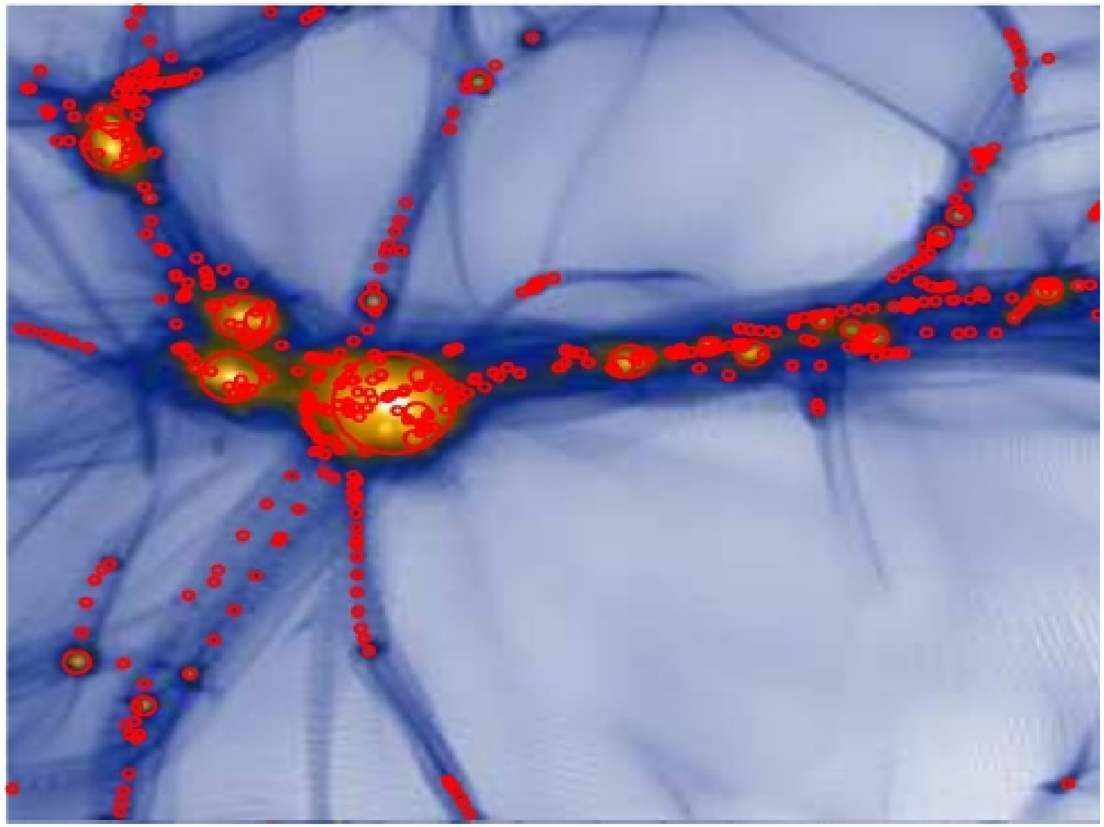


Angulo et al 2013a



Two examples where the monte-carlo approach to collisionless dynamics (aka N-body) fails:

1) Two fluids with distinct primordial power spectra
2) Artificial fragmentation in WDM cosmologies

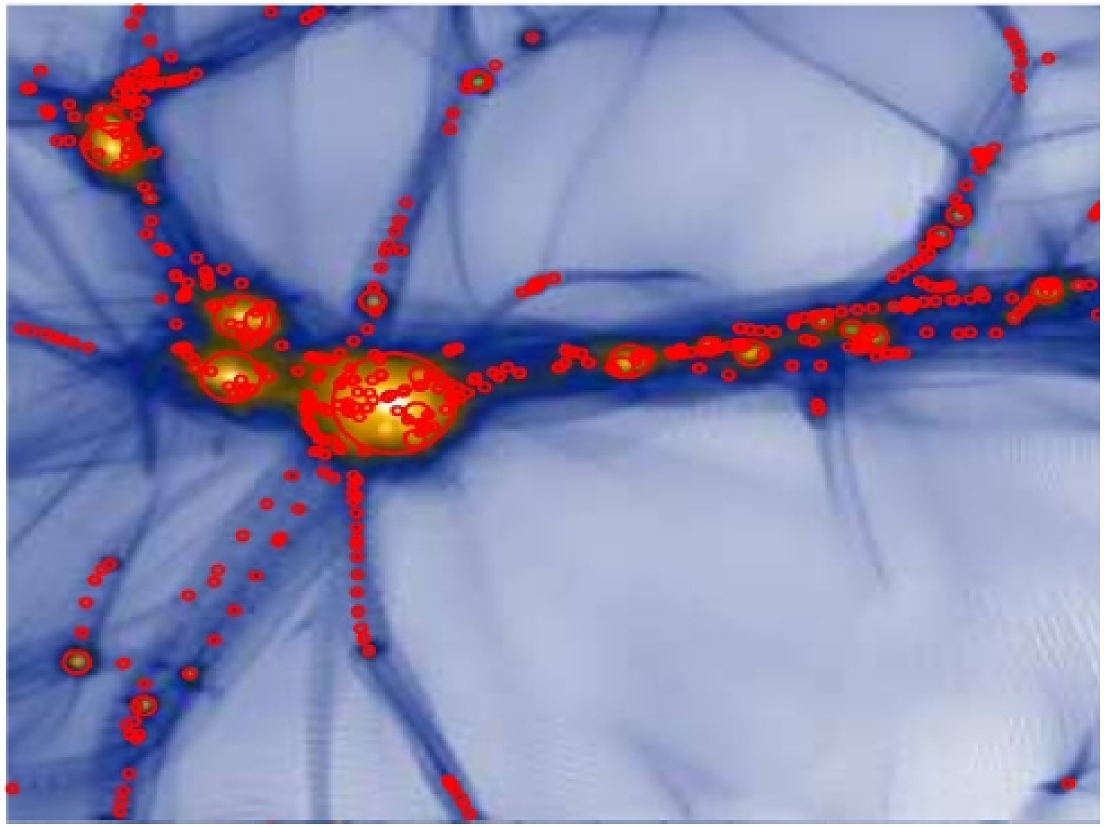


Two examples where the monte-carlo approach to collisionless dynamics (aka N-body) fails:

1) Two fluids with distinct primordial power spectra
2) Artificial fragmentation in WDM cosmologies

Two competing requirements
ii) Need a large smoothing
to reduce noise/biases.
ii) Need a small smoothing
to resolve nonlinear structure

How robust are the "well established" predictions of N-body simulations (e.g. NFW profile) for CDM and WDM?

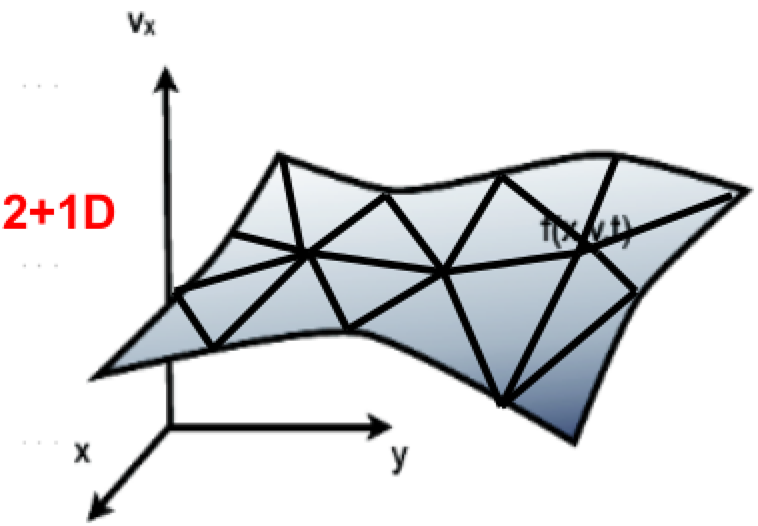


An alternative approach:

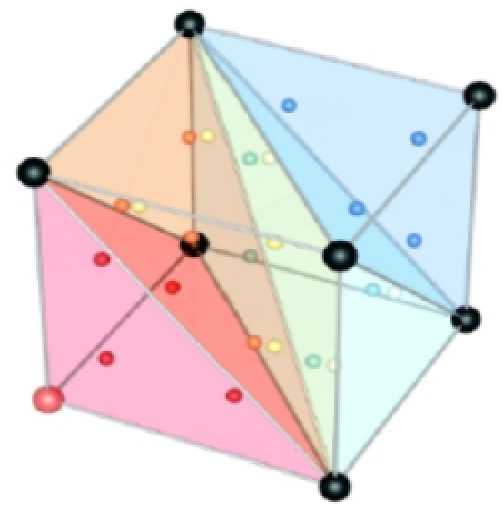
Discretization of the DM fluid using phase-space element methods

A tessellation of a finite number of mesh-generating points in Lagrangian space allows to continuously map the deformation of the dark matter sheet

2+1D

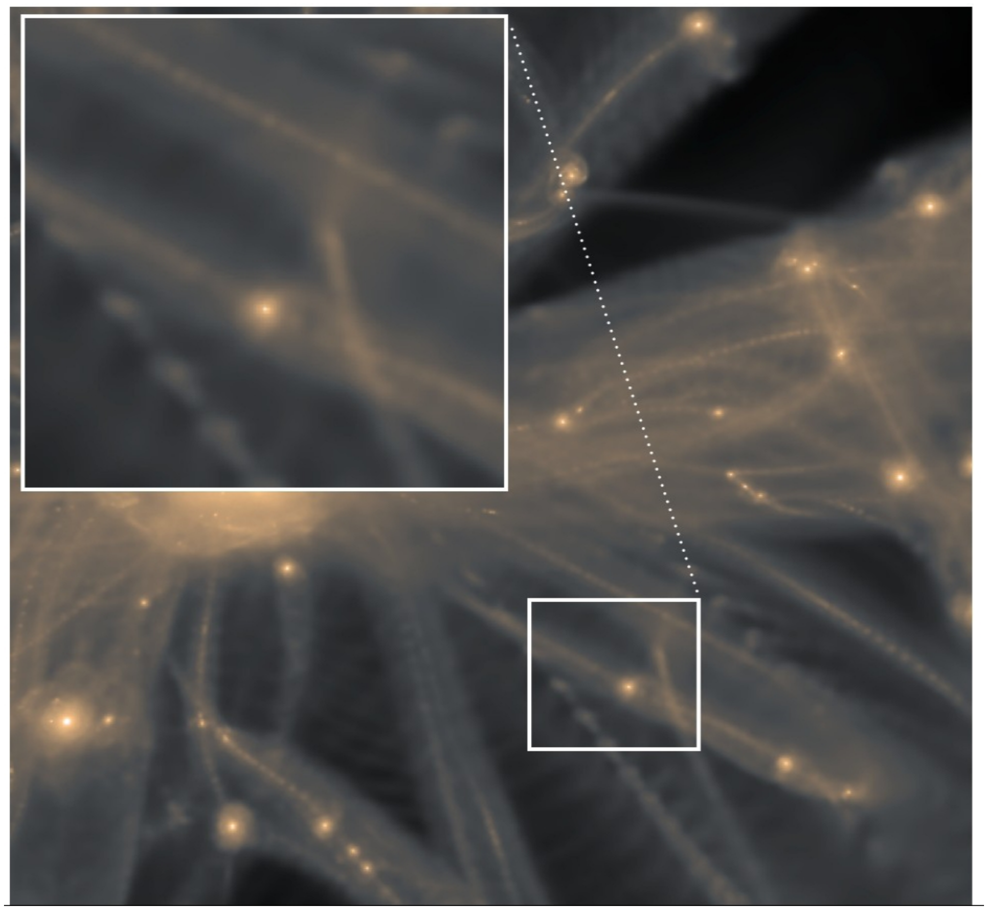


3D

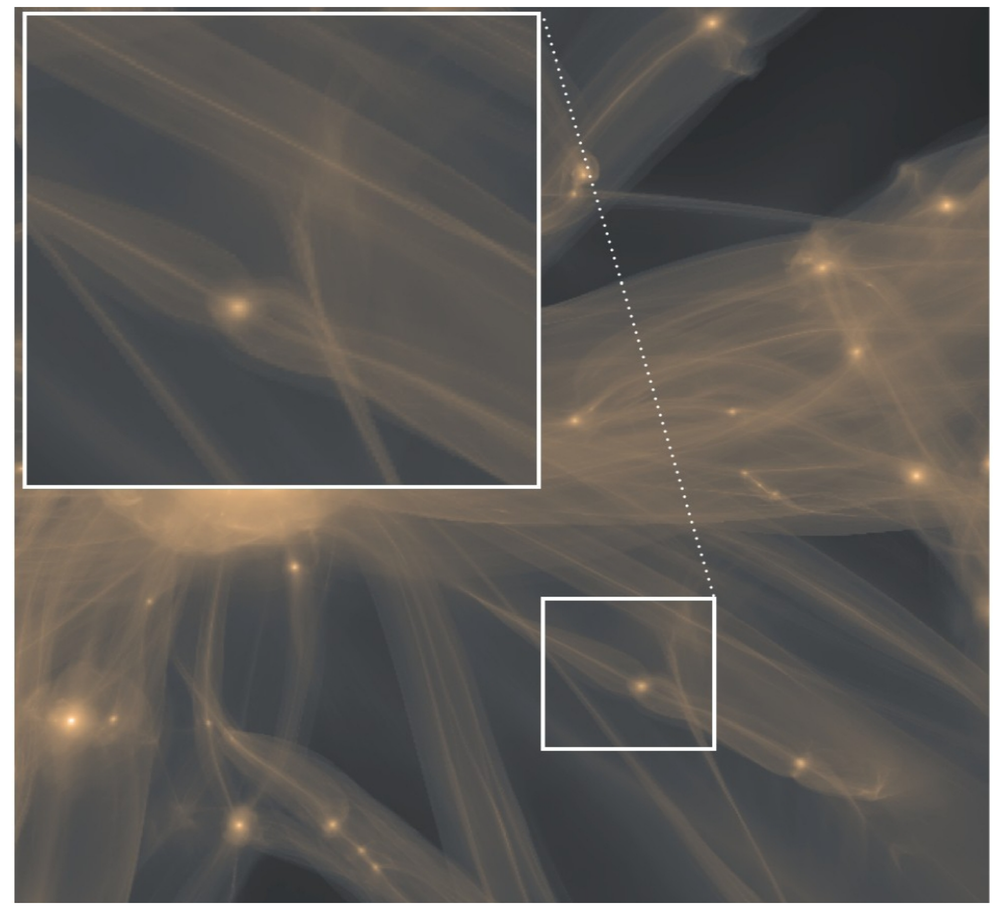


(Abel+ 2012, Shandarin+ 2012, Kaehler+ 2013, Hahn+ 2013, Angulo+ 2013, Hahn & Angulo 2014)

Two representations of the same N-body data



Adaptive-kernel smoothing

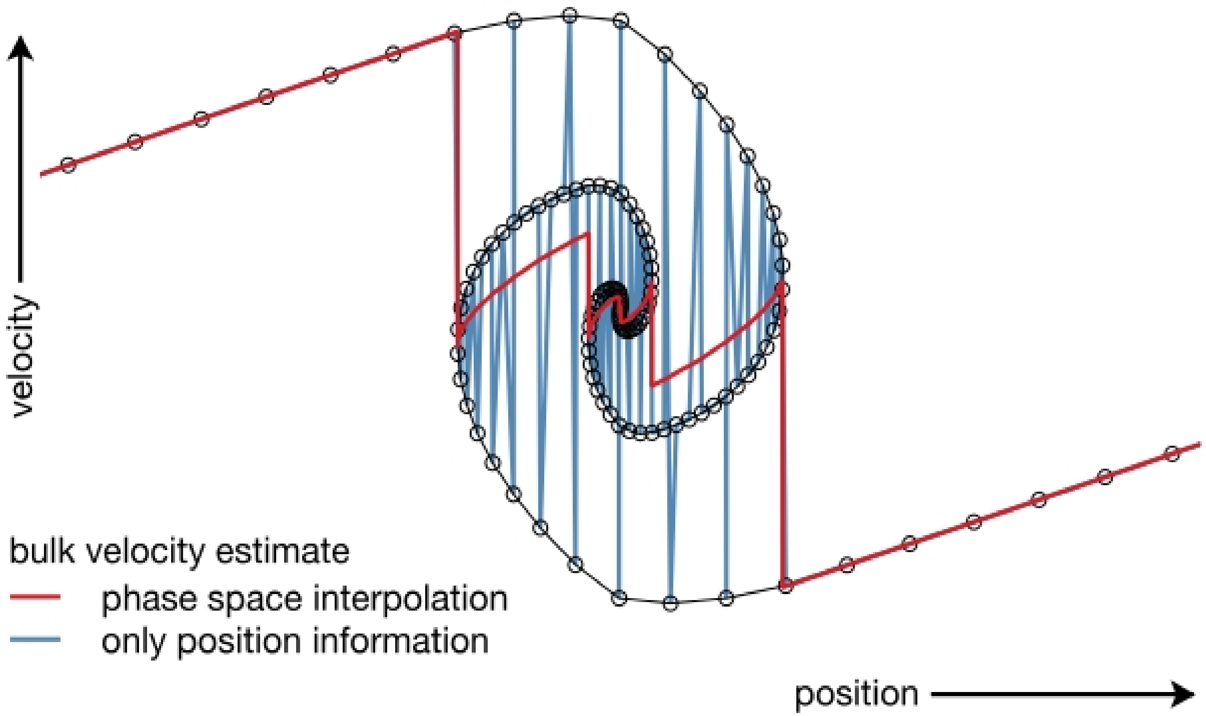


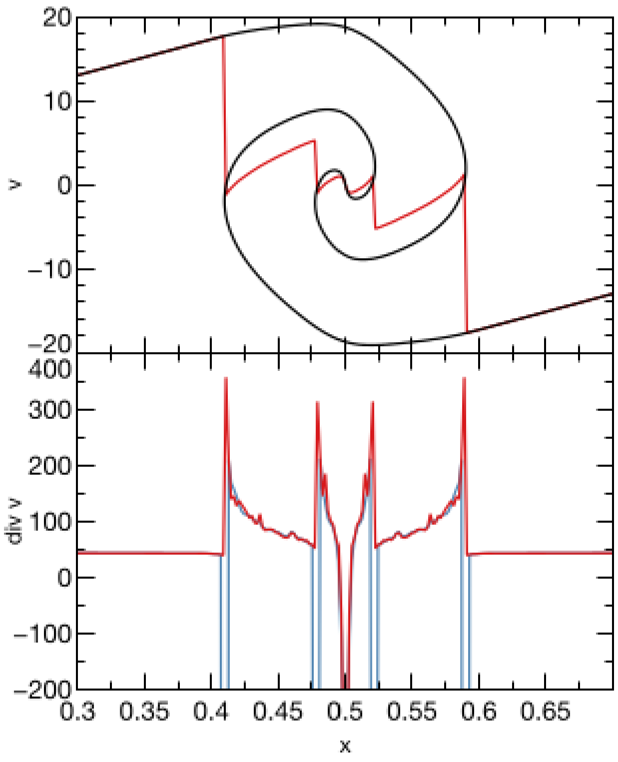
Full rendering of tetrahedra

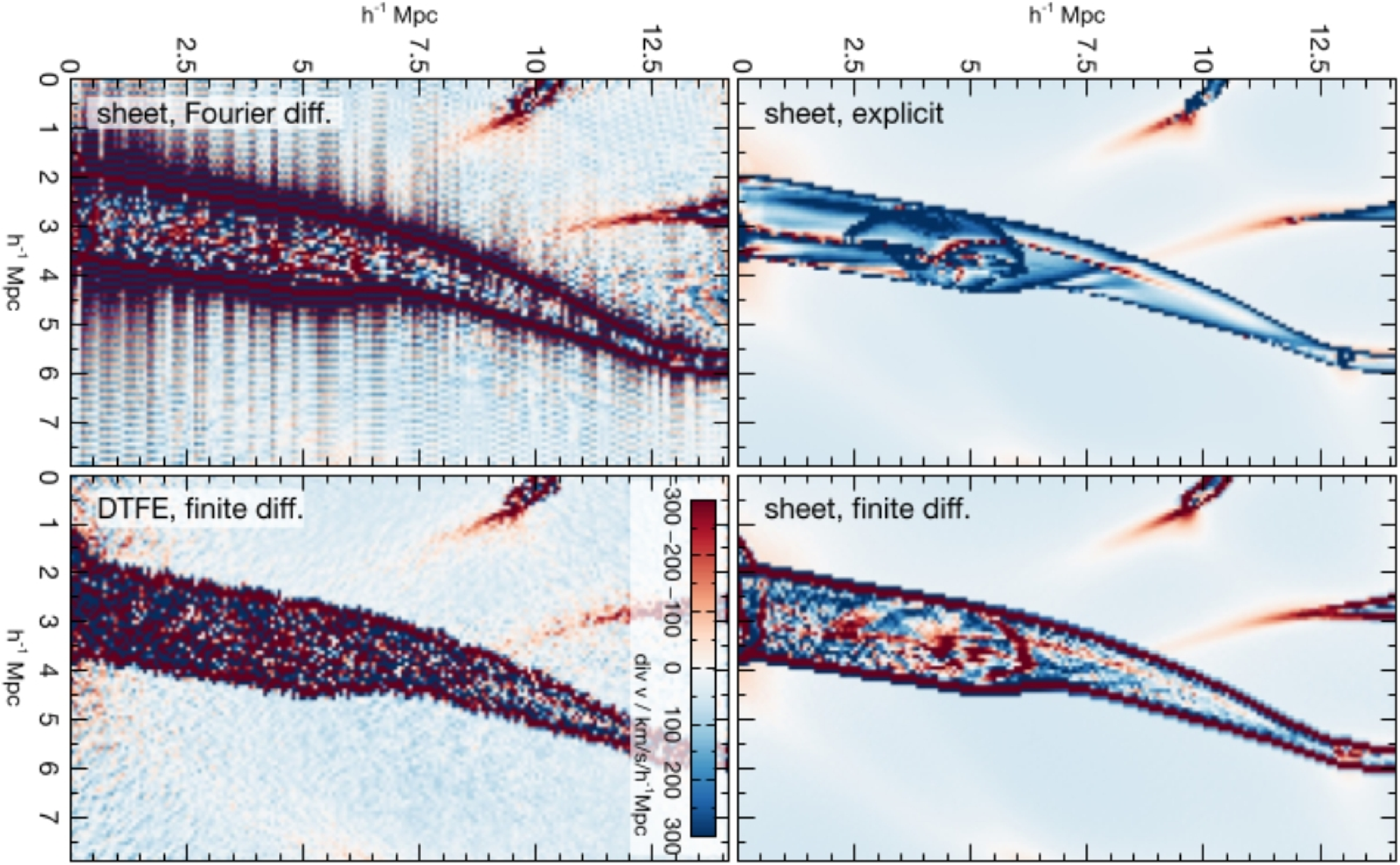
Kaehler et al (2012)

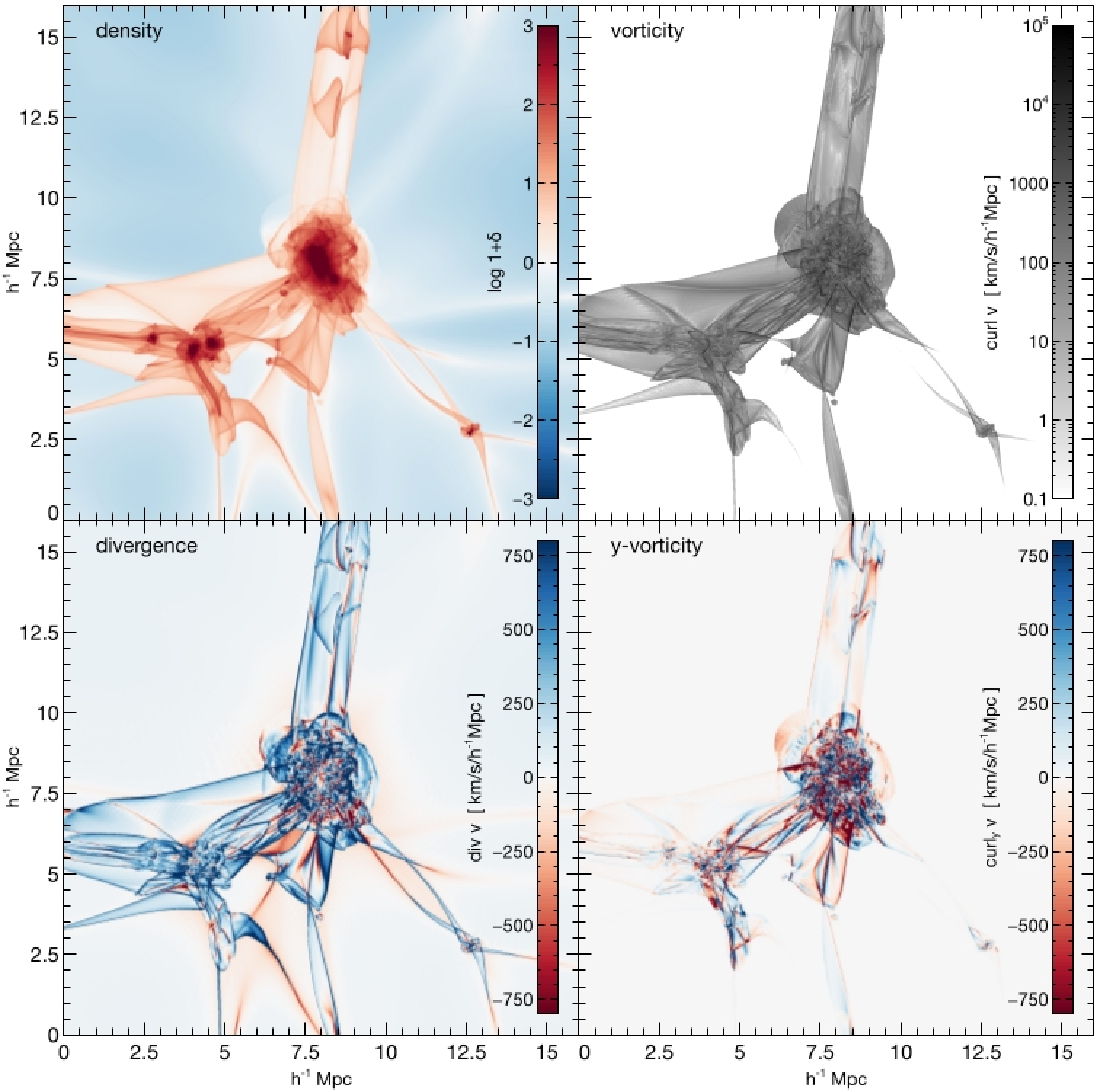
The cosmic velocity fields

Hahn, Angulo, Abel 2014






sheet, Fourier diff.
sheet, explicit
DTFE, finite diff.
sheet, finite diff.
h⁻¹ Mpc
div v / km/s/h⁻¹Mpc
−300 −200 −100 0 100 200 300

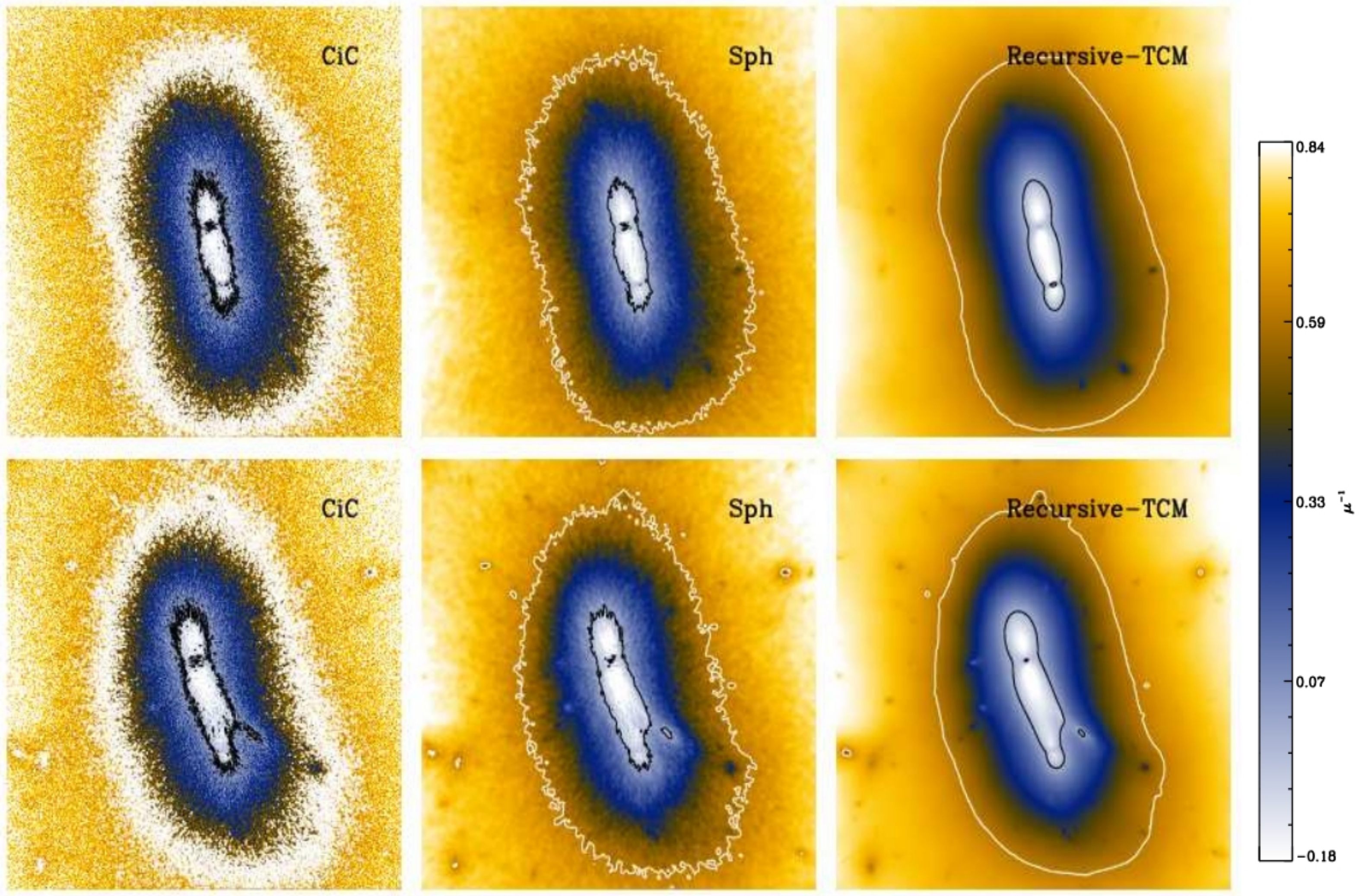

density
vorticity
divergence
y-vorticity
log 1+δ
curl v [km/s/h⁻¹Mpc]
div v [km/s/h⁻¹Mpc]
curl_y v [km/s/h⁻¹Mpc]
h⁻¹ Mpc

Noiseless gravitational simulations

Angulo, Chen, Abel & Hilbert 2014

CDM

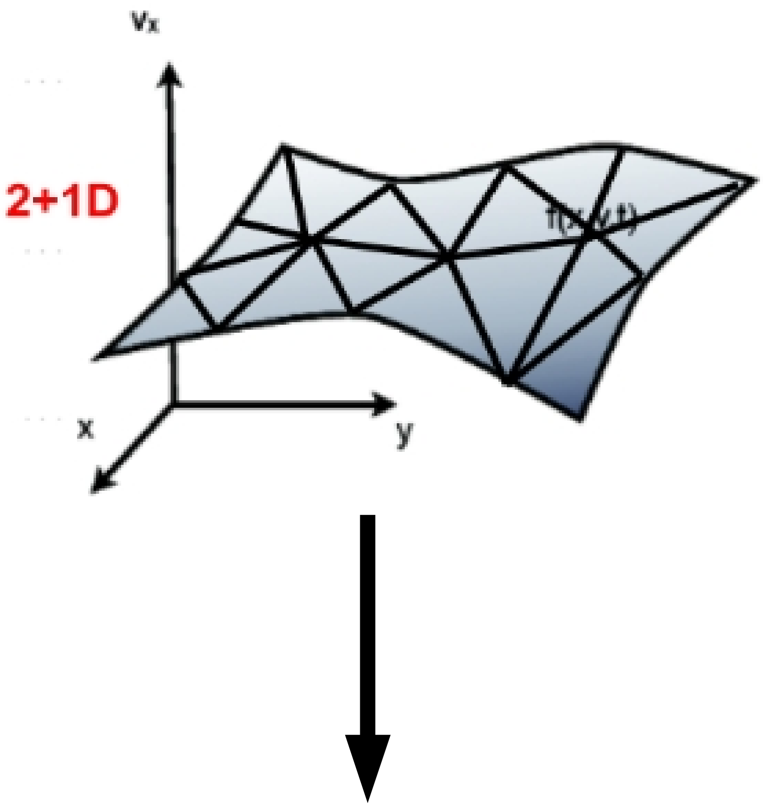
WDM



WDM structure formation without fragments

Angulo, Hahn, Abel 2013b

New sheet-based simulation code with reduced collisionality

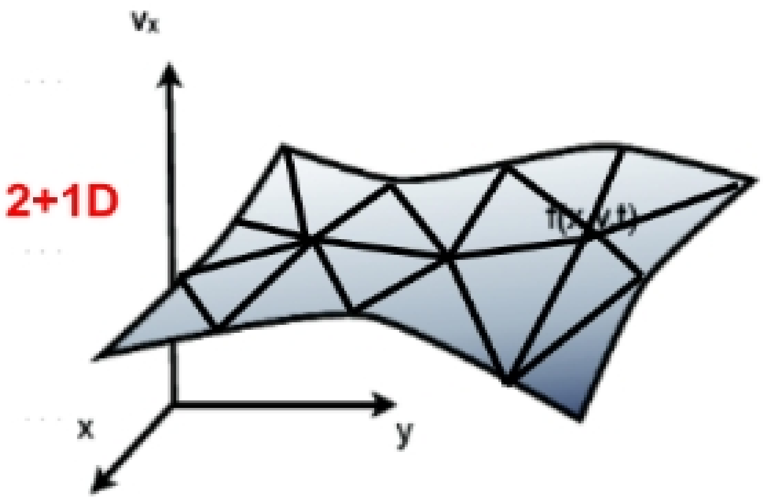


$$\nabla^2 \Phi = \frac{4\pi G}{a} \int f \, \mathrm{d}^3 v$$

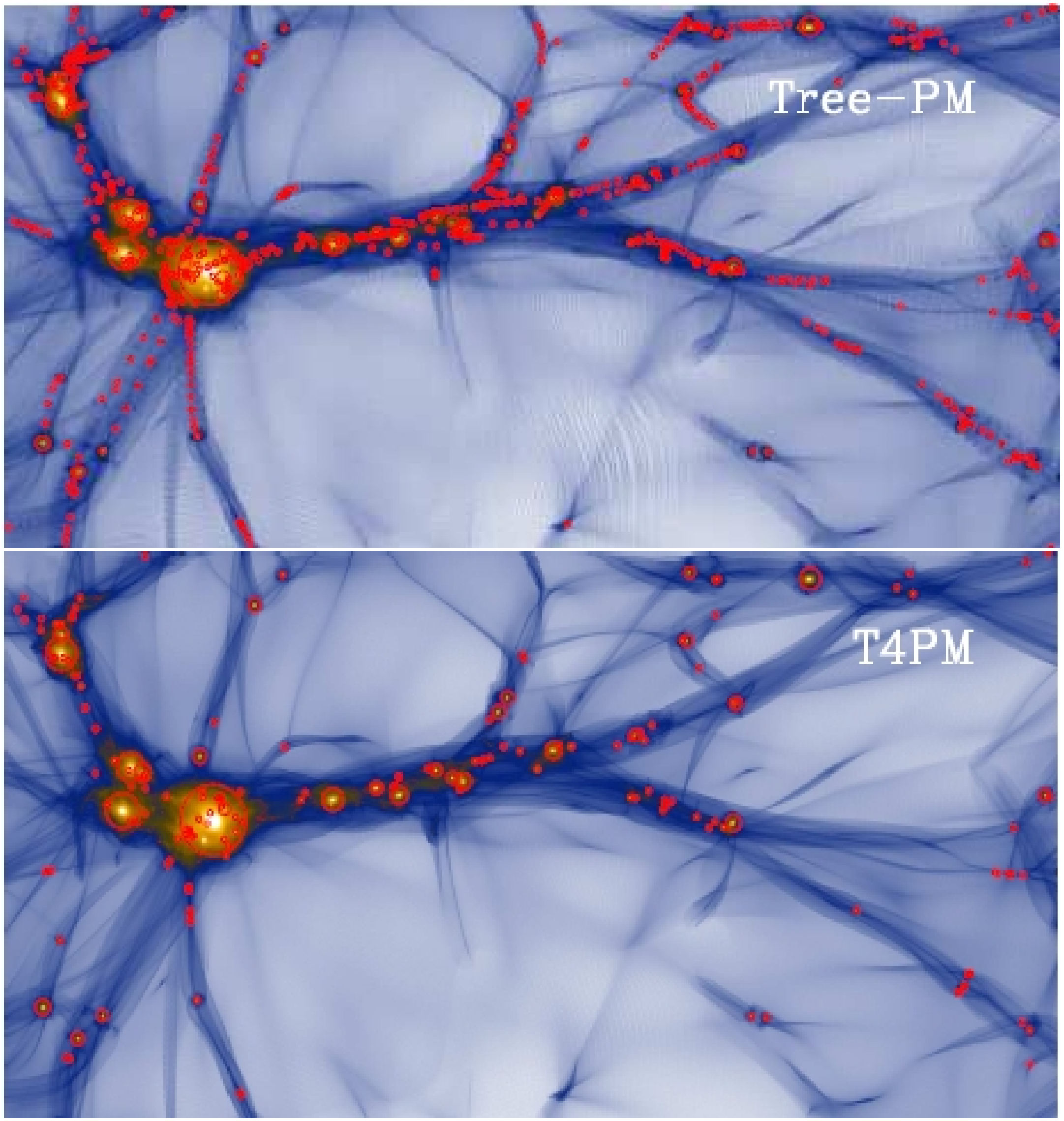
WDM structure formation without fragments

Angulo, Hahn, Abel 2013b

New sheet-based simulation code with reduced collisionality



$$\nabla^2\Phi = \frac{4\pi G}{a}\int f \mathrm{d}^3 v$$



Structure Formation in WDM

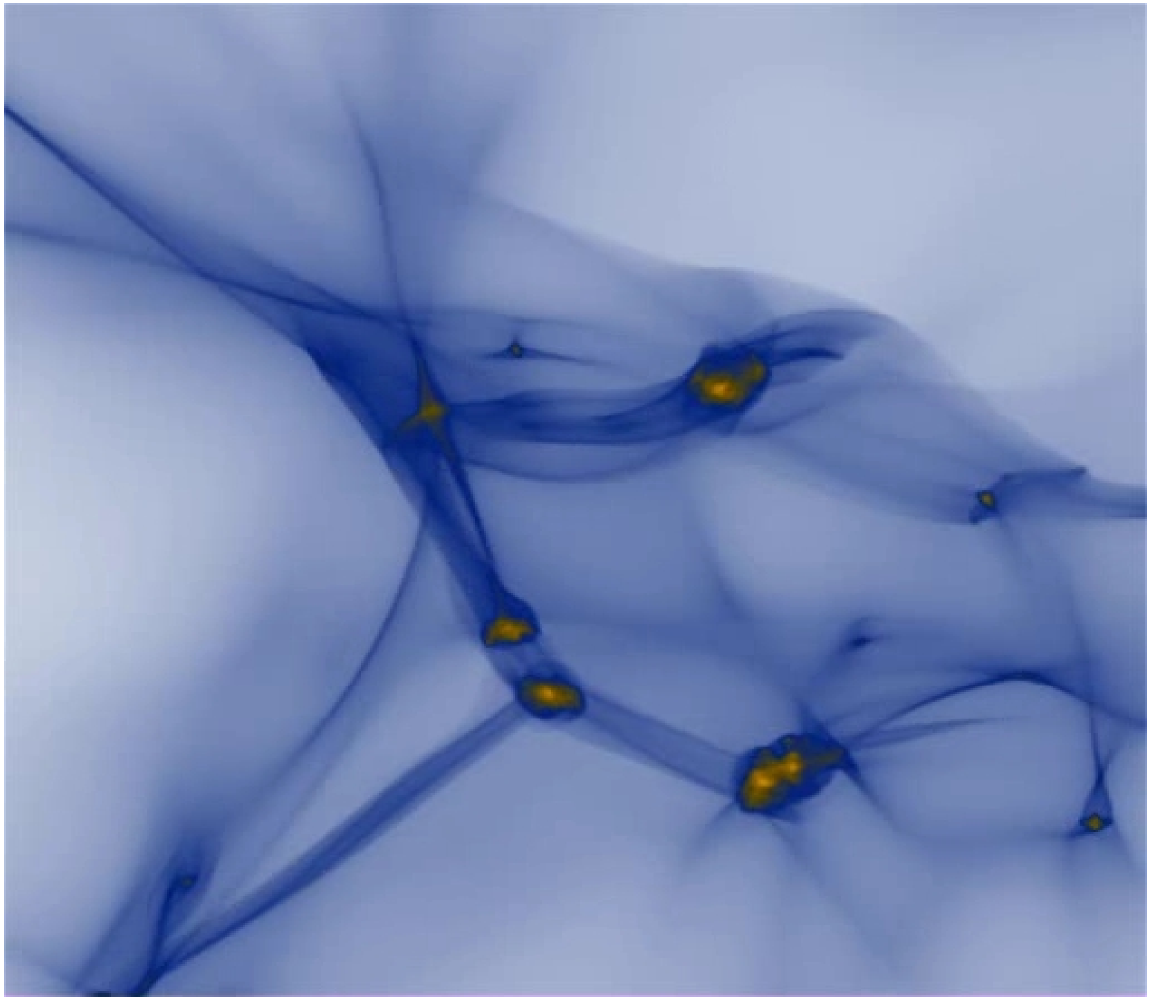
Angulo, Hahn, Abel 2013b

No bottom-up formation

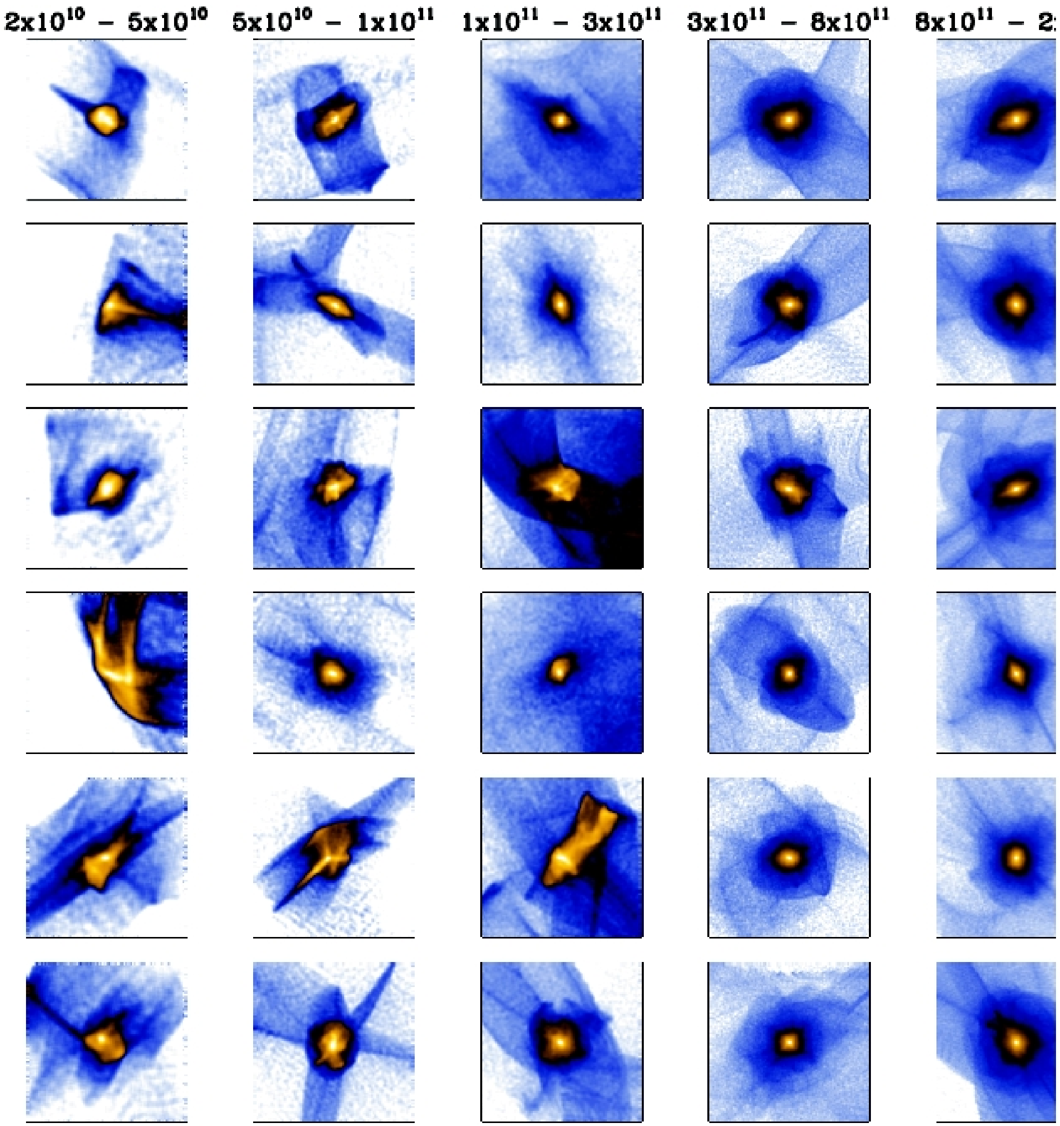
Genuine 1D and 2D structures

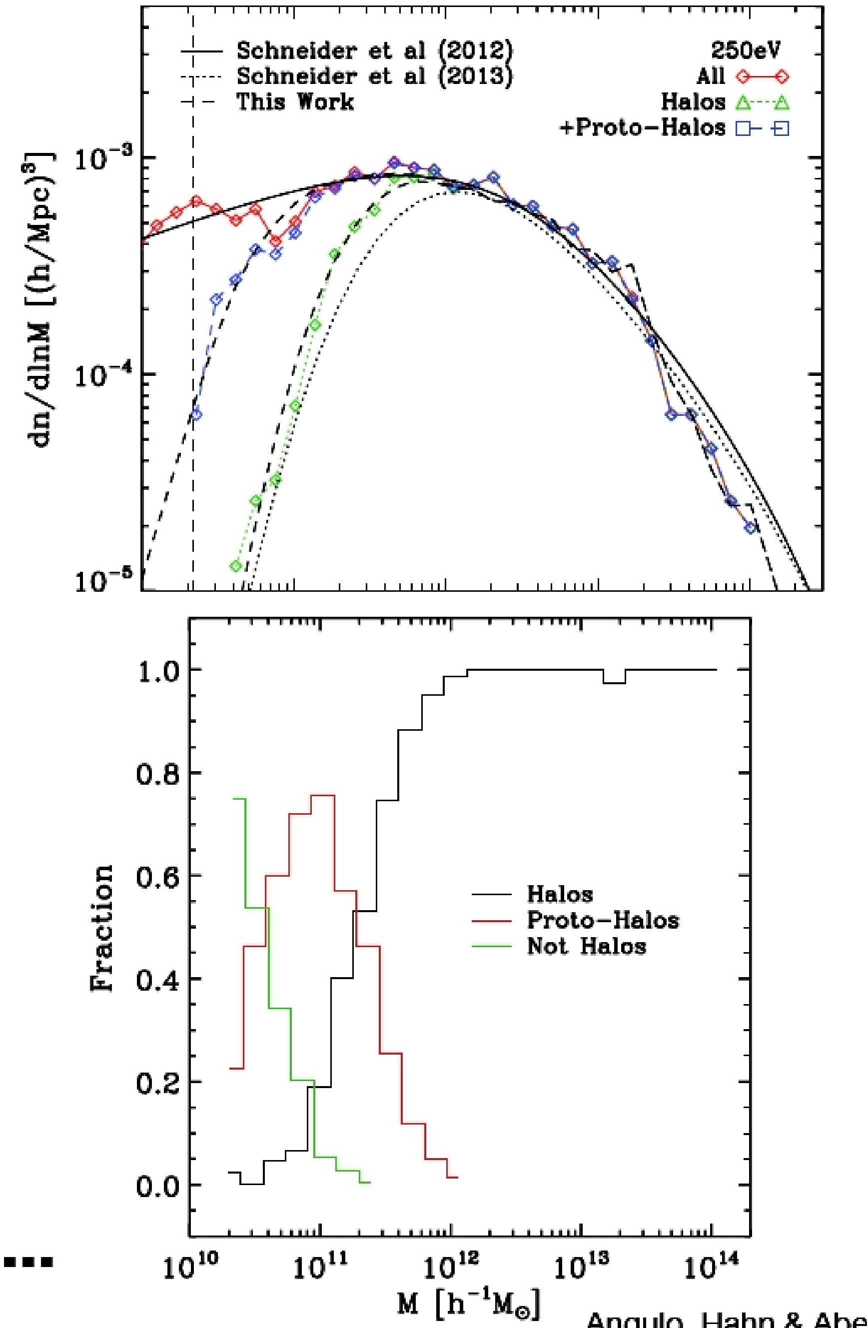
Halos have no progenitors beyond some time

Very rapid mass growth during formation



WDM structures at different masses...



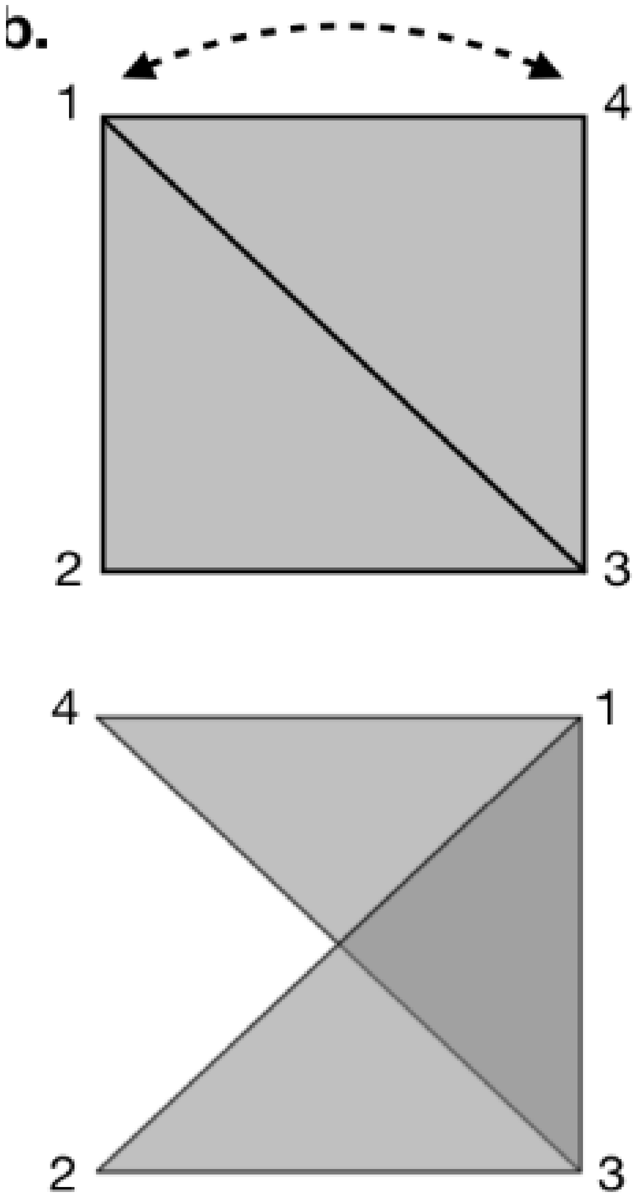


Are at different stages of formation...

Angulo, Hahn & Abel 2013

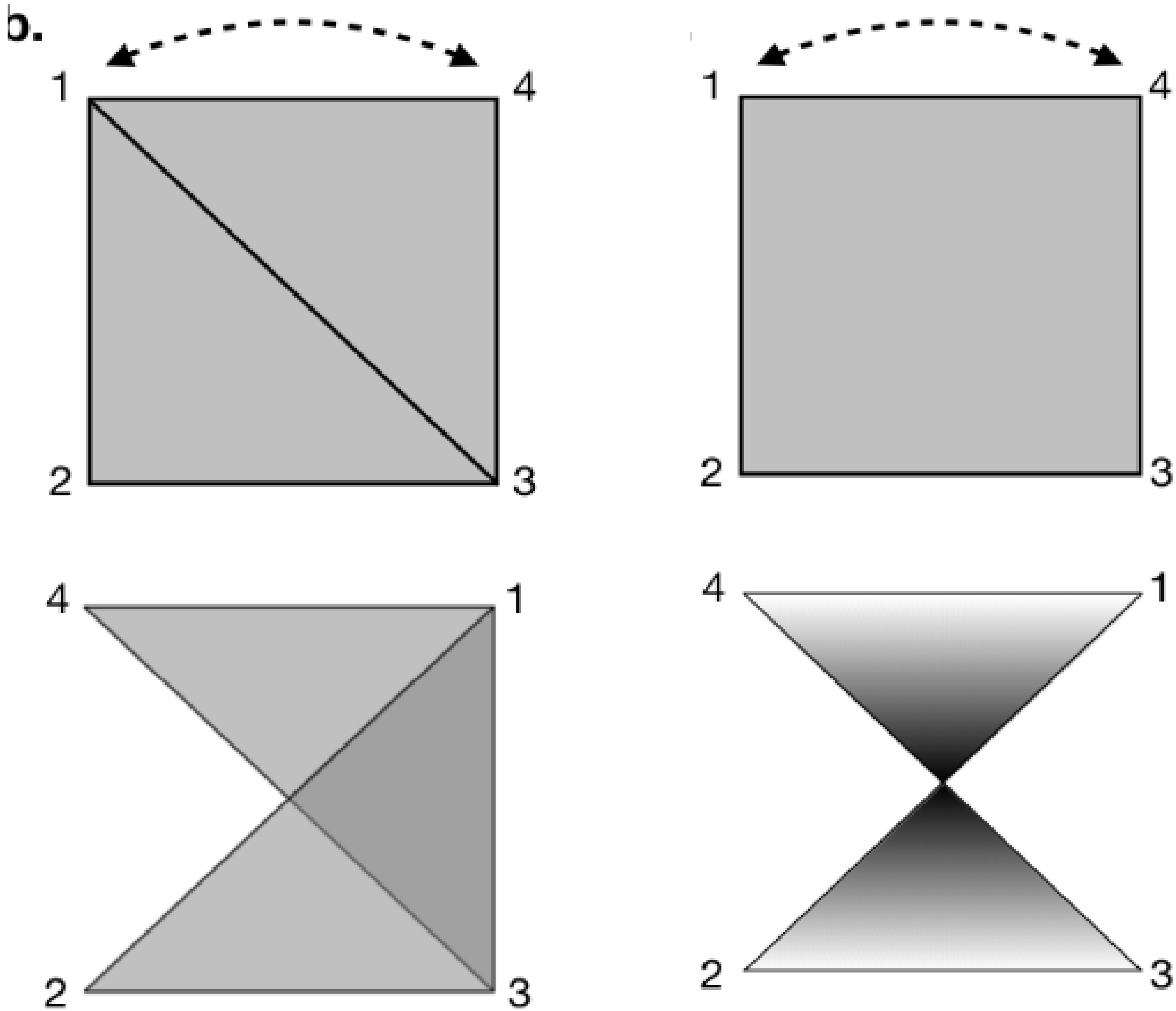
A general class of Lagrangian phase-space methods

Hahn & Angulo 2015



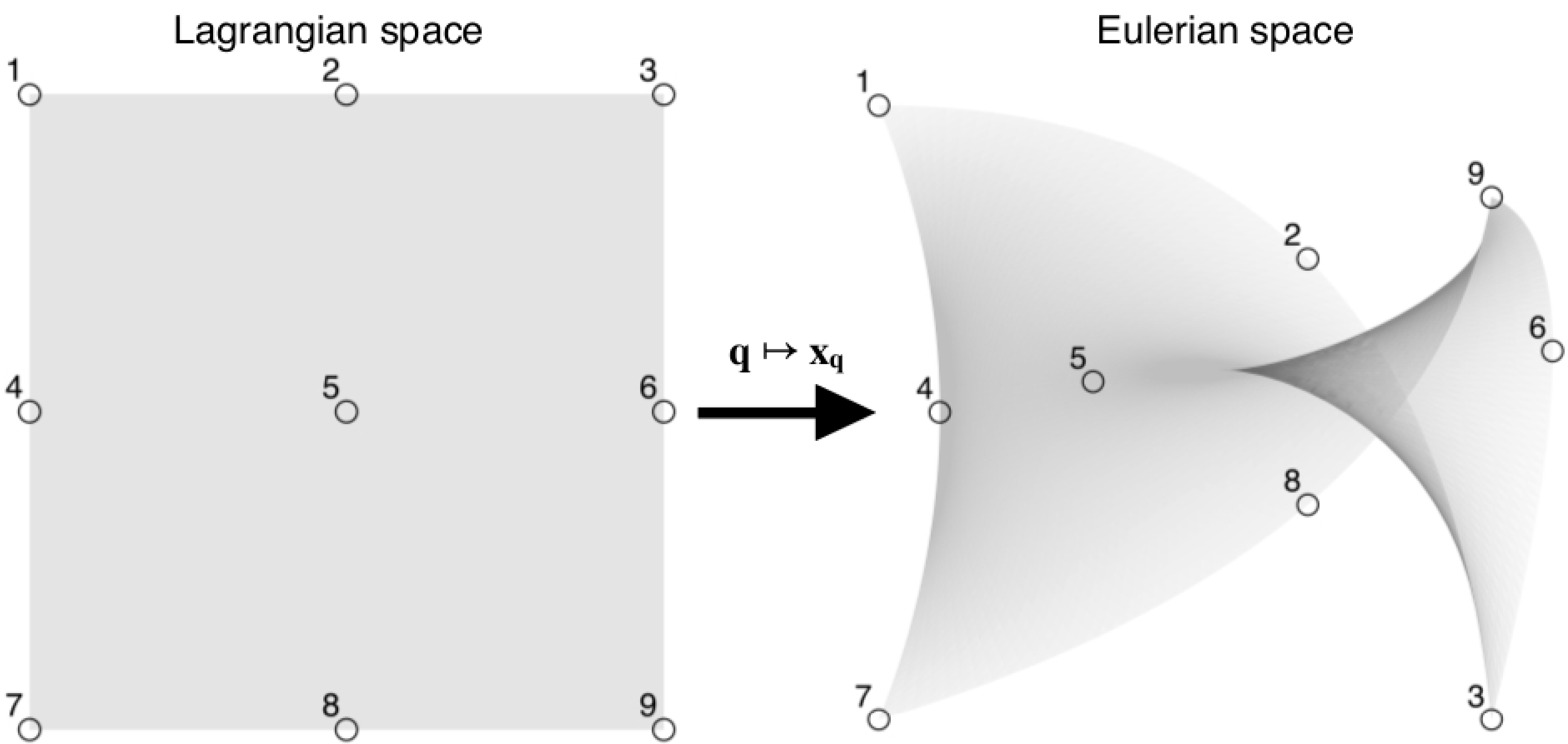
A general class of Lagrangian phase-space methods

Hahn & Angulo 2015



A general class of Lagrangian phase-space methods

Numerical simulations solve for this nonlinear mapping

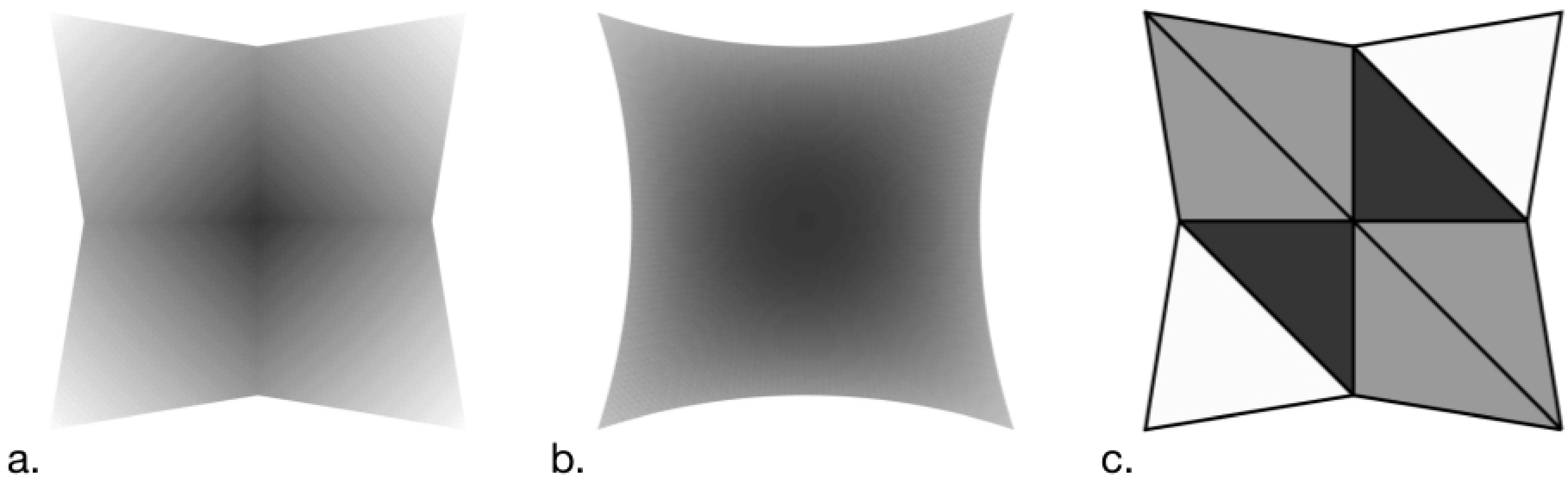


$$P_k = \{\pi(\mathbf{q}) \mid \pi(\mathbf{q}) = \sum_{\alpha,\beta,\gamma=0}^{k} a_{\alpha\beta\gamma}\, q_0^\alpha q_1^\beta q_2^\gamma\},$$

A general class of Lagrangian phase-space methods

Hahn & Angulo 2015

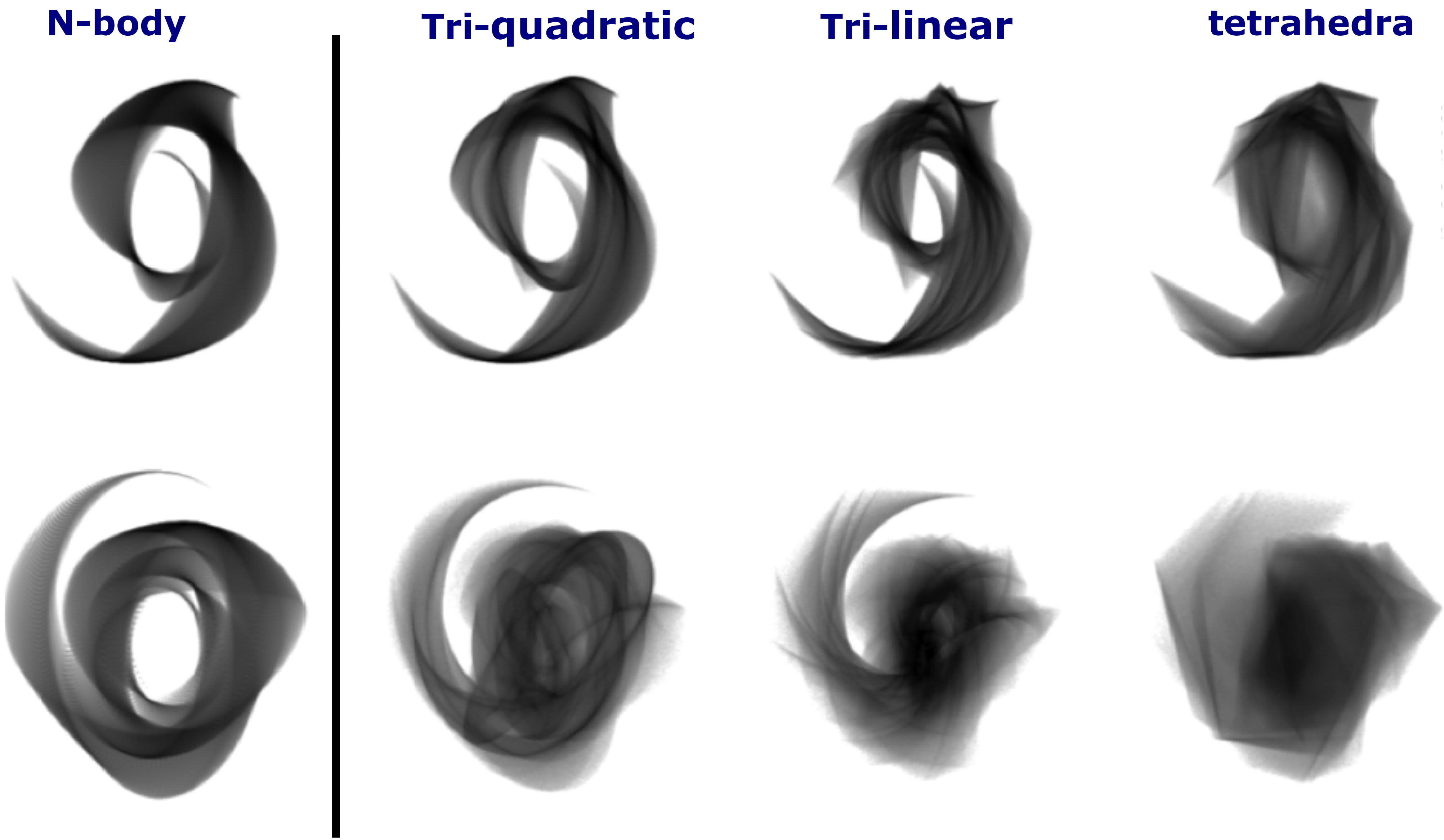
Tri-linear **Tri-quadratic** **tetrahedra**



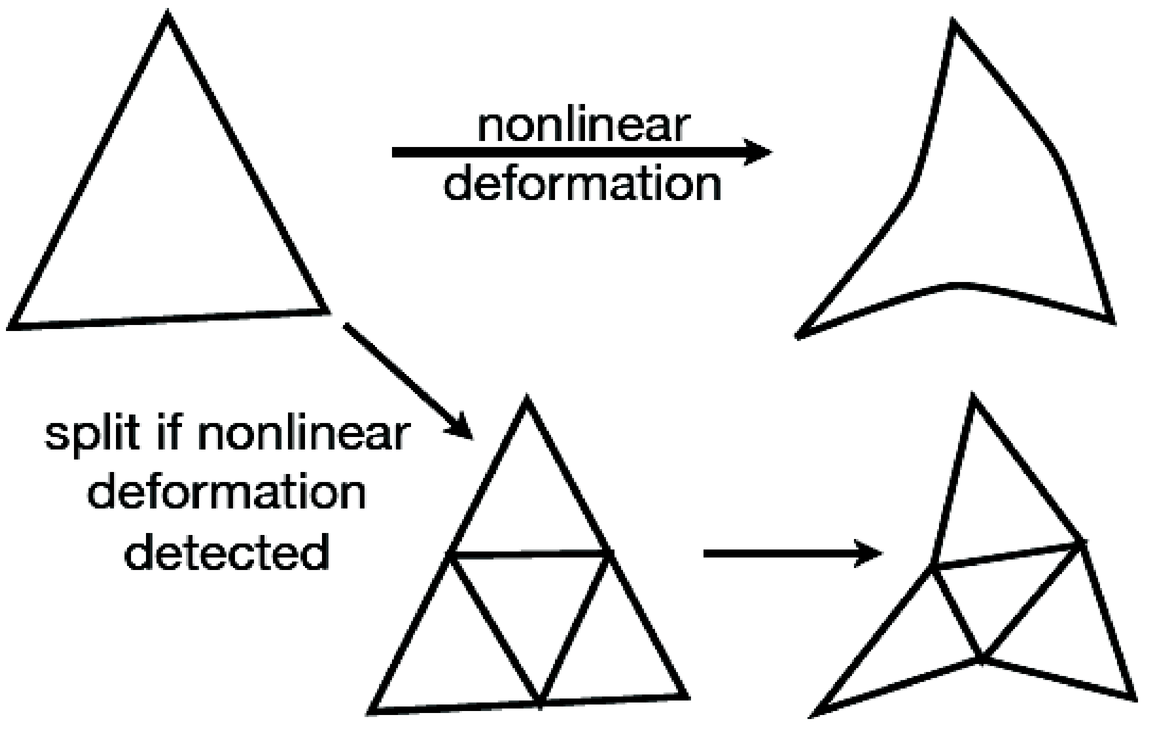
$$P_k = \{\pi(\mathbf{q}) \mid \pi(\mathbf{q}) = \sum_{\alpha,\beta,\gamma=0}^{k} a_{\alpha\beta\gamma}\, q_0^{\alpha} q_1^{\beta} q_2^{\gamma}\},$$

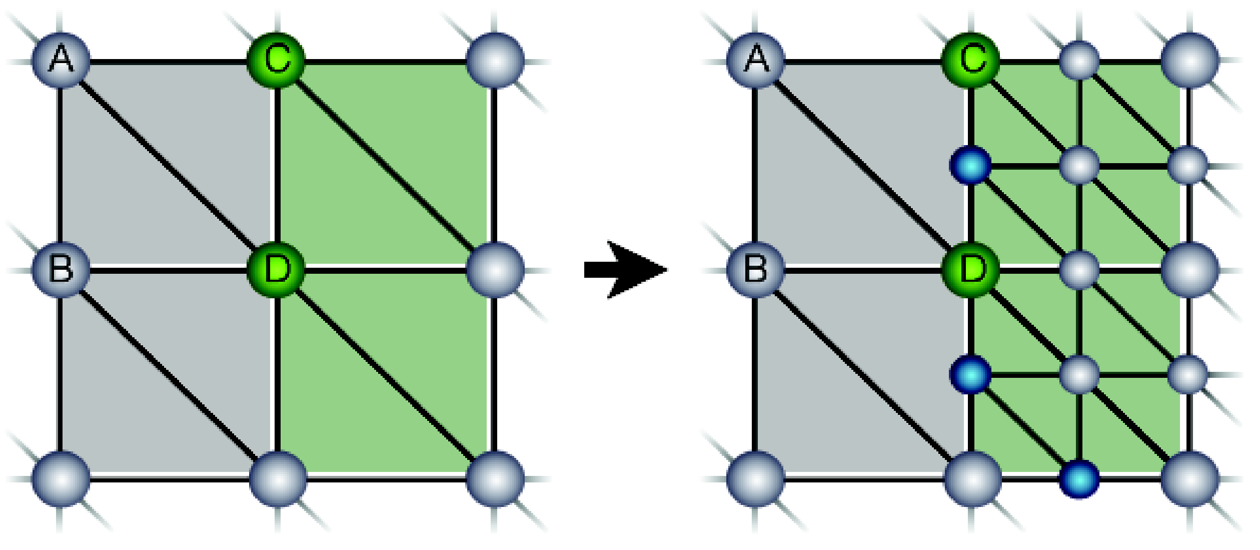
Uniform cube orbiting on a static Plummer potential

Piecewise Largrangian maps do not always conserve energy



Adaptive refinement of Lagrangian Elements



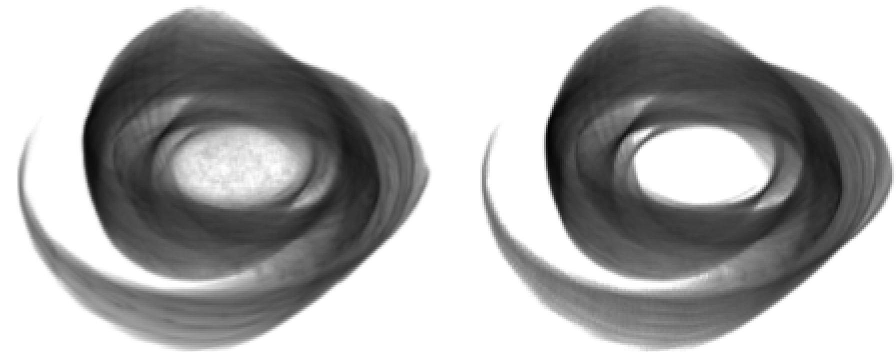


Adaptive refinement of Lagrangian Elements

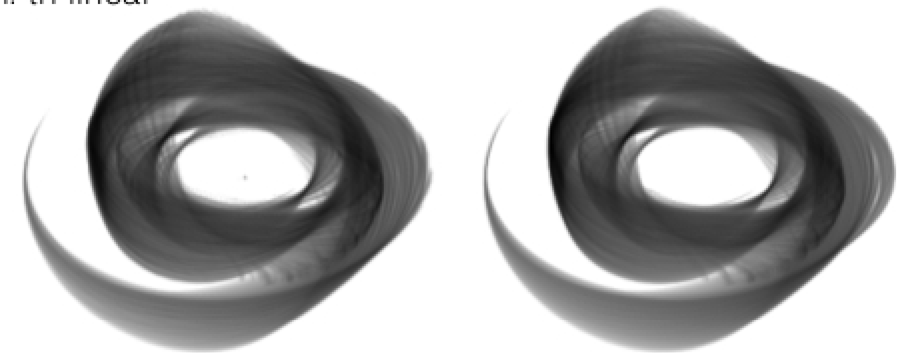
t=12

Θ=0.1 Θ=0.05

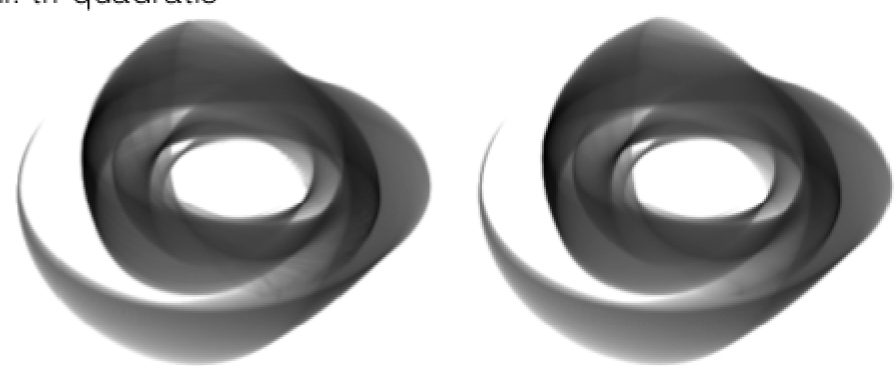
i. tetrahedra



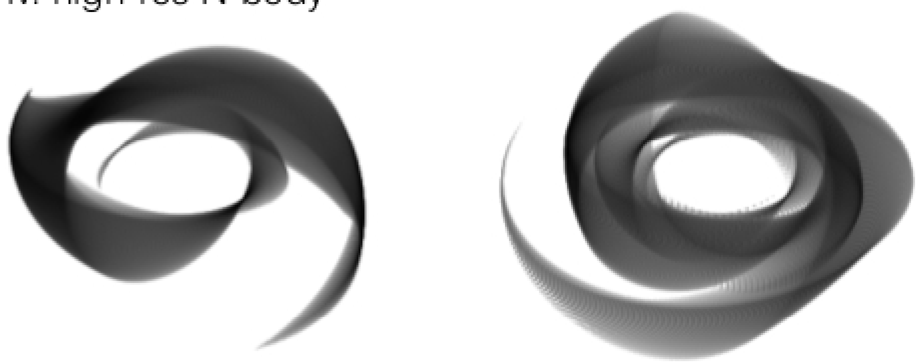
ii. tri-linear



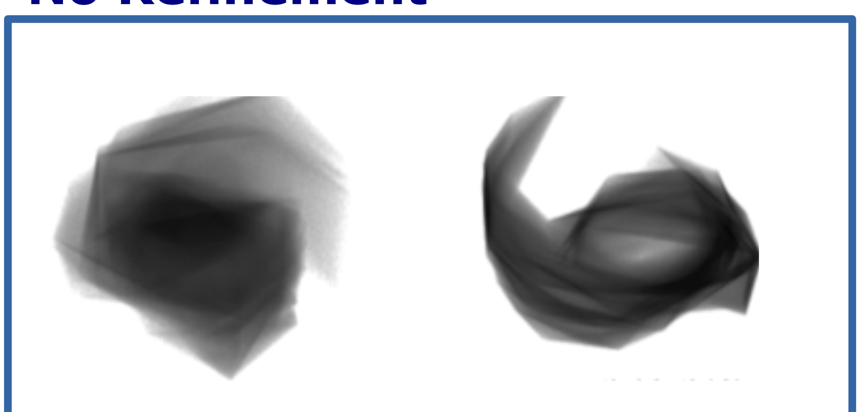
iii. tri-quadratic



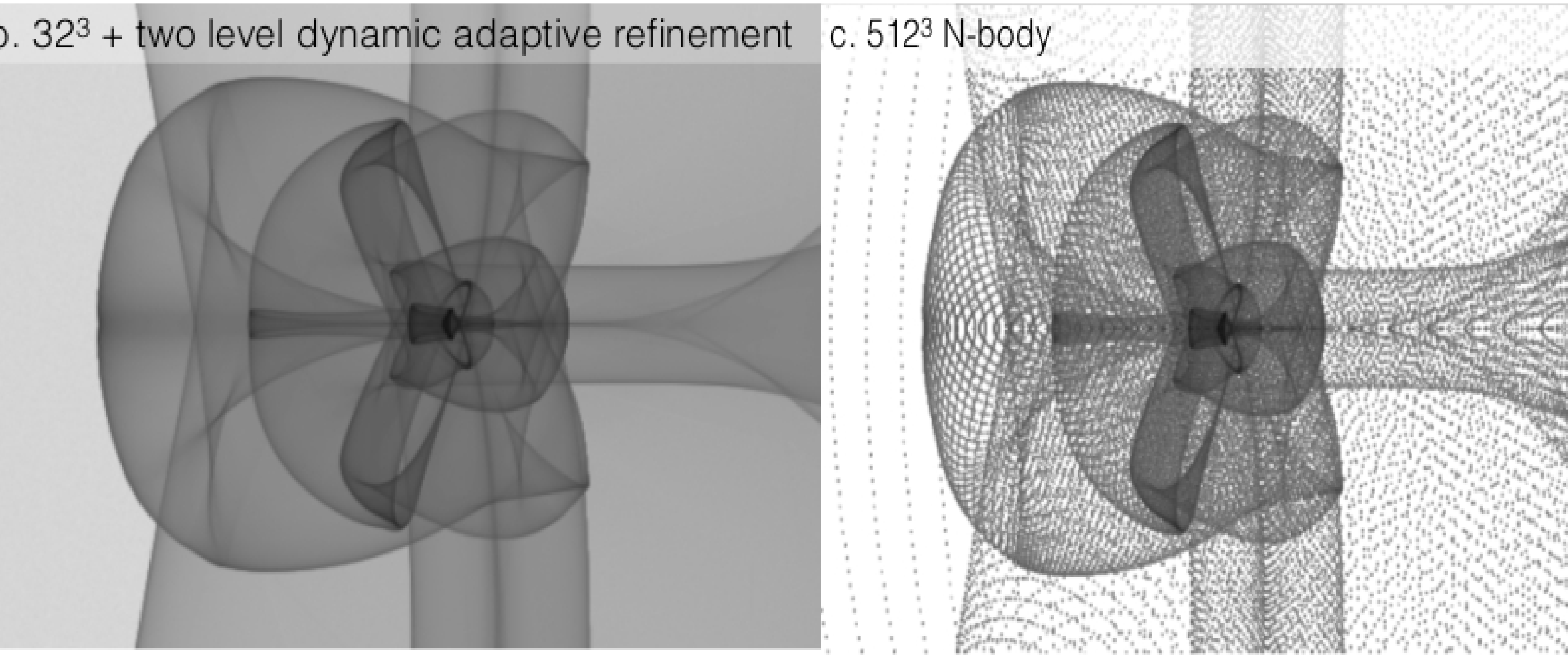
iv. high-res N-body



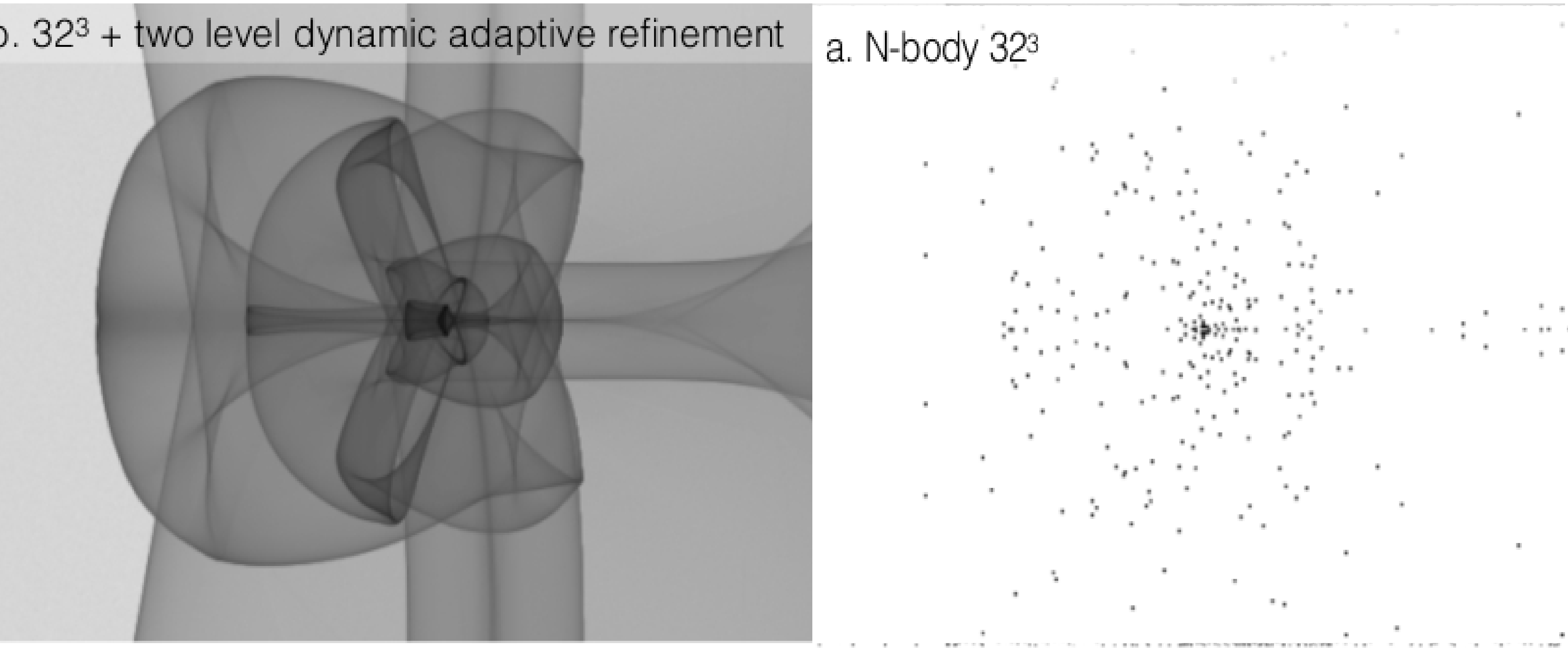
No Refinement

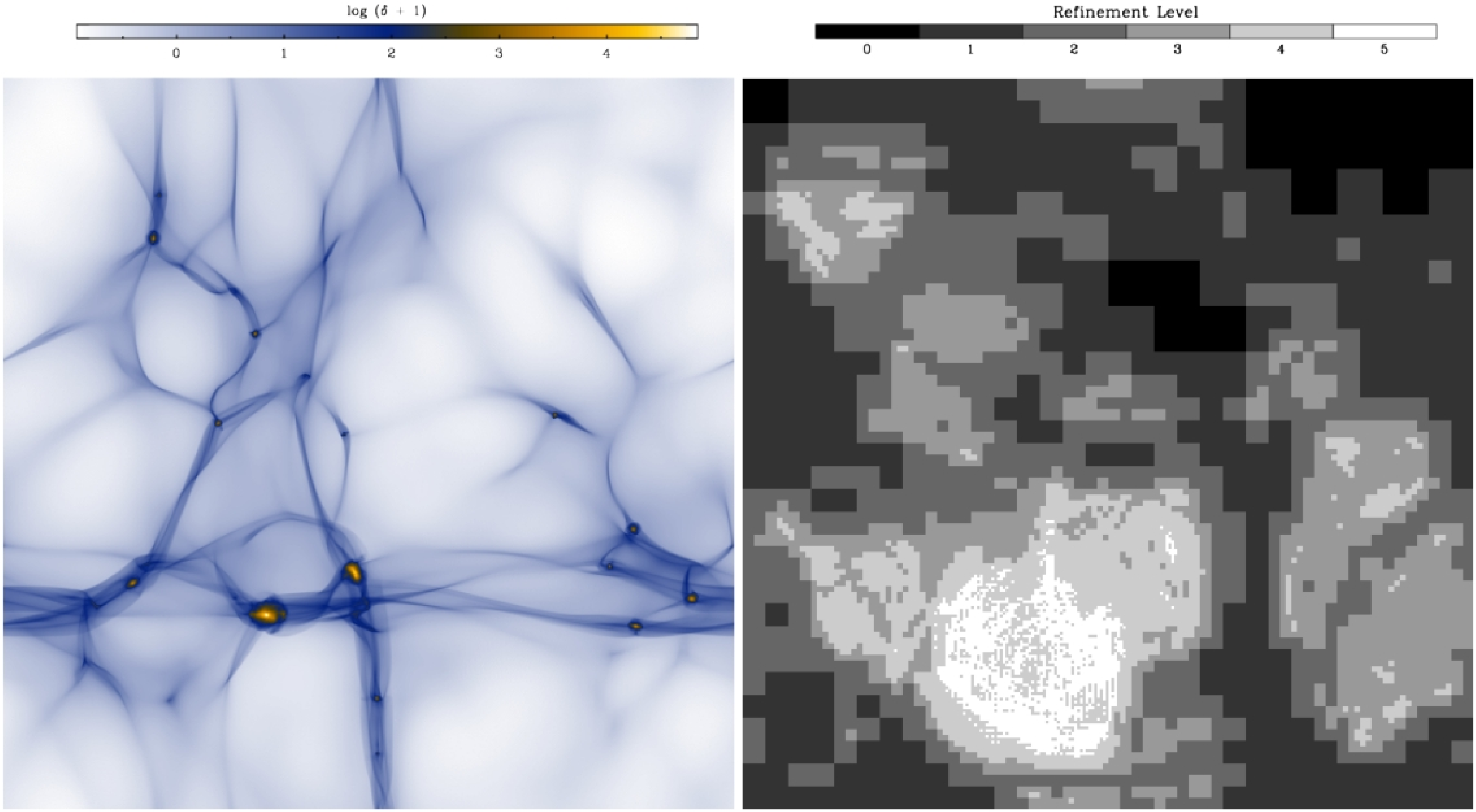


Two sine waves under self gravtiy

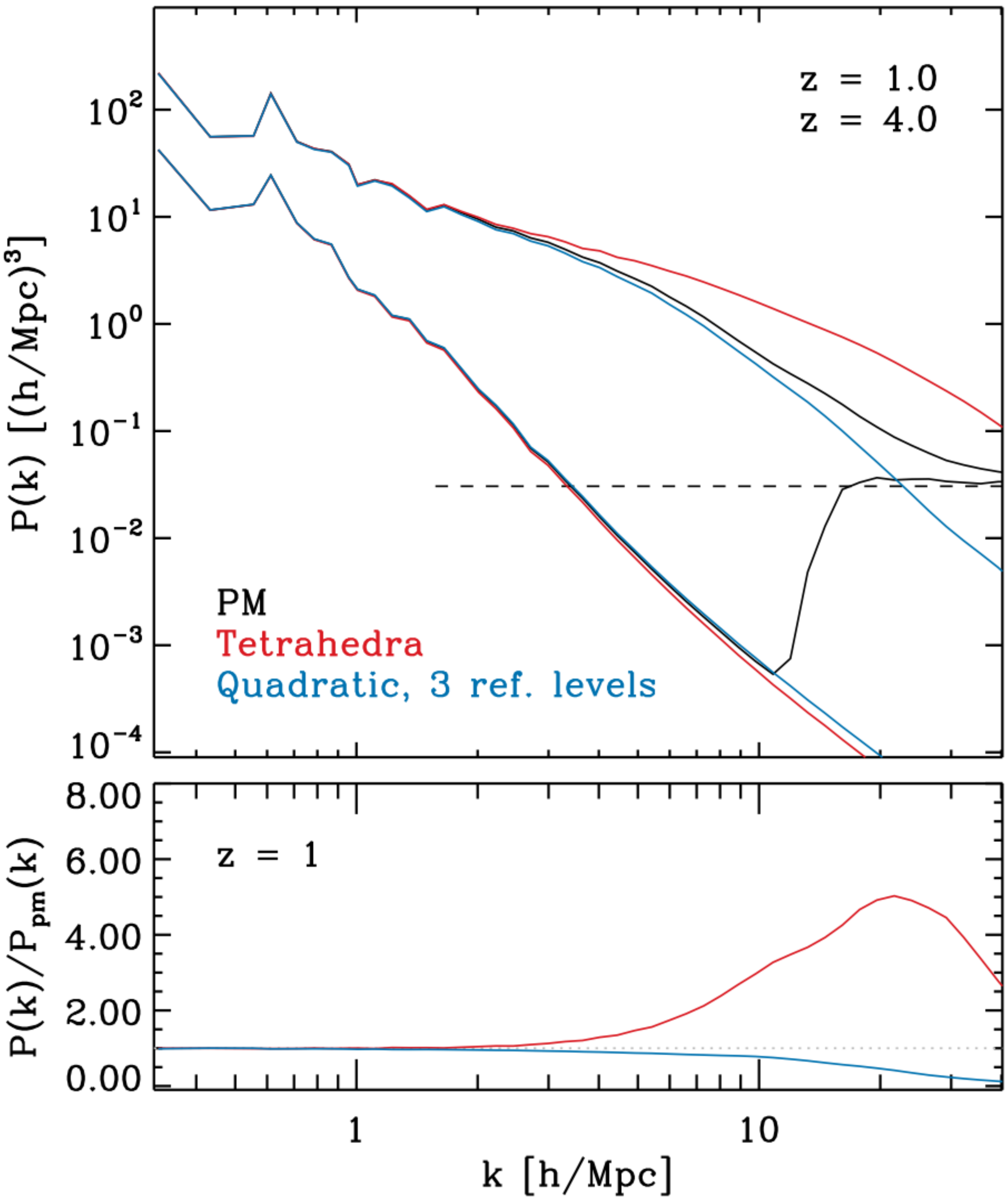


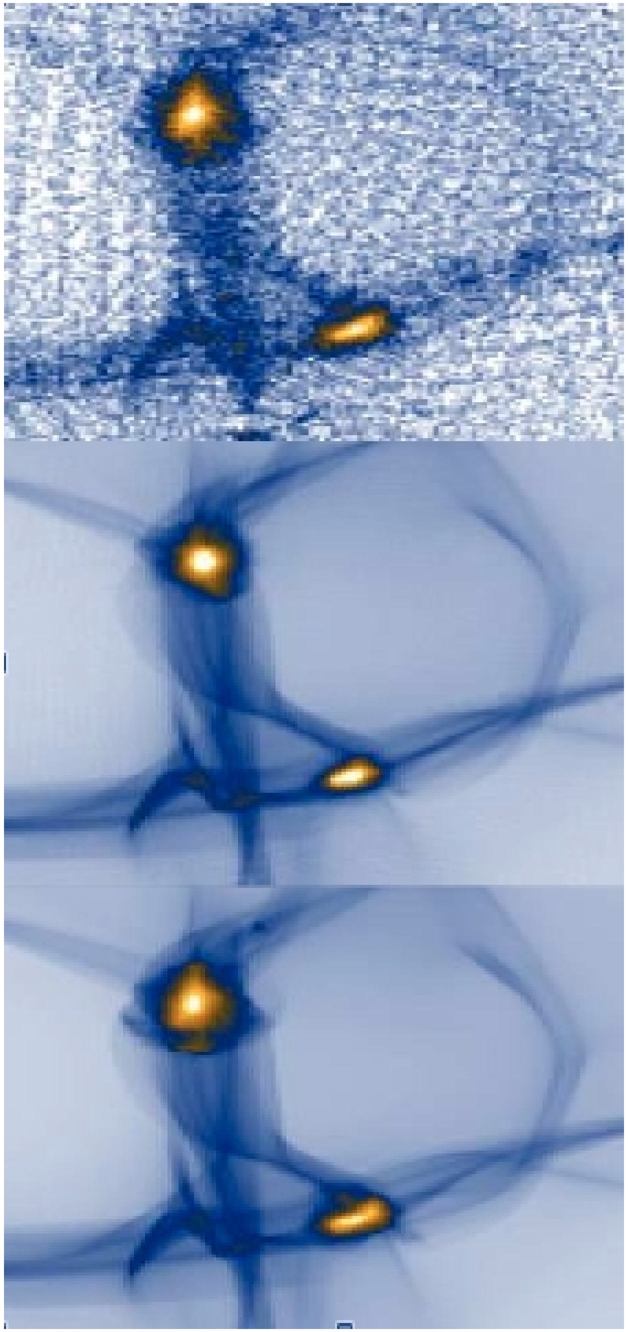
Two sine waves under self gravtiy





20 Mpc/h box
64^3 particles
512^3 PM

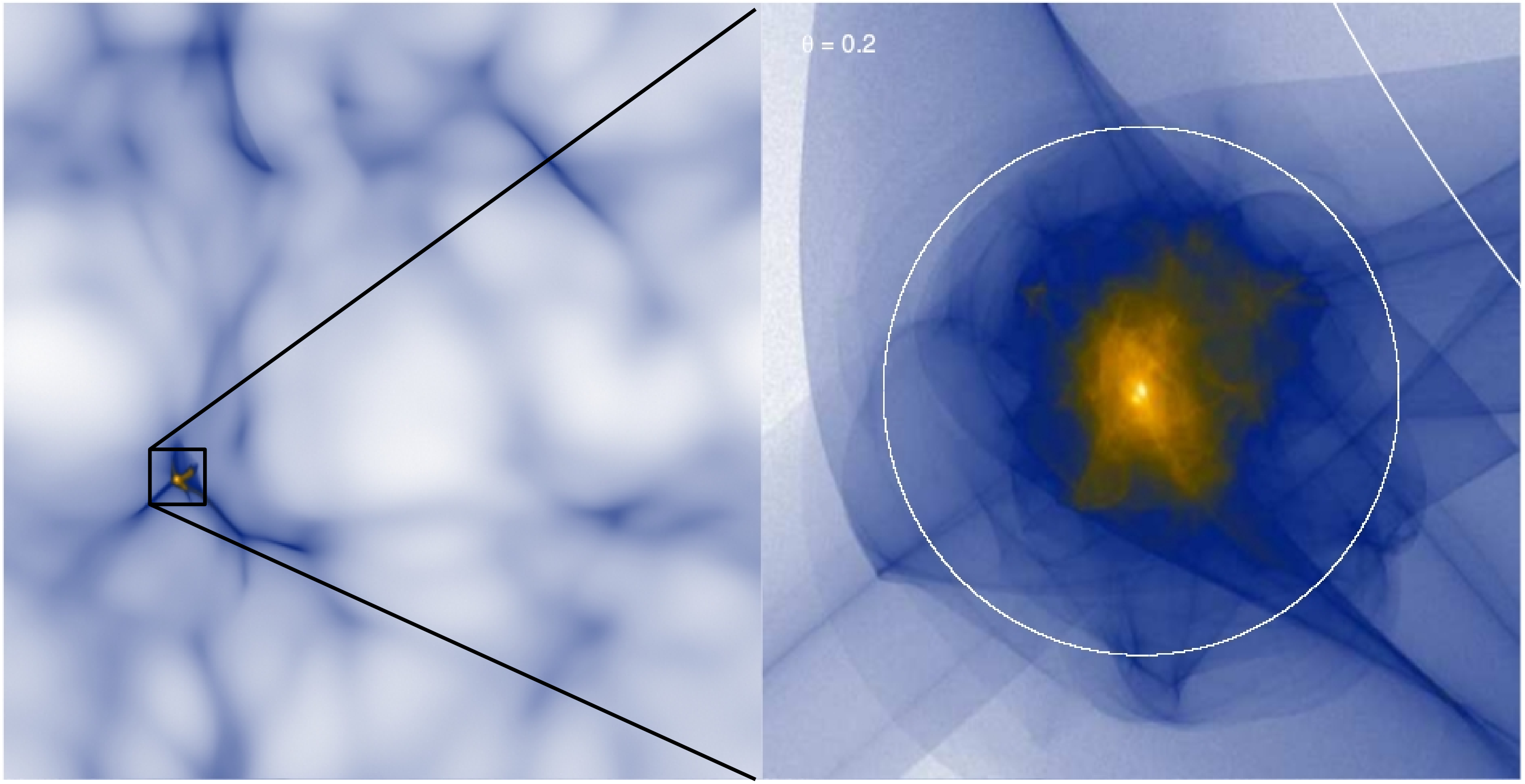

z = 1.0
z = 4.0
PM
Tetrahedra
Quadratic, 3 ref. levels
z = 1
P(k) [(h/Mpc)^3]
P(k)/P$_{pm}$(k)
k [h/Mpc]
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
8.00
6.00
4.00
2.00
0.00
1
10



The hexahalo Project

Angulo et al (in preparation)

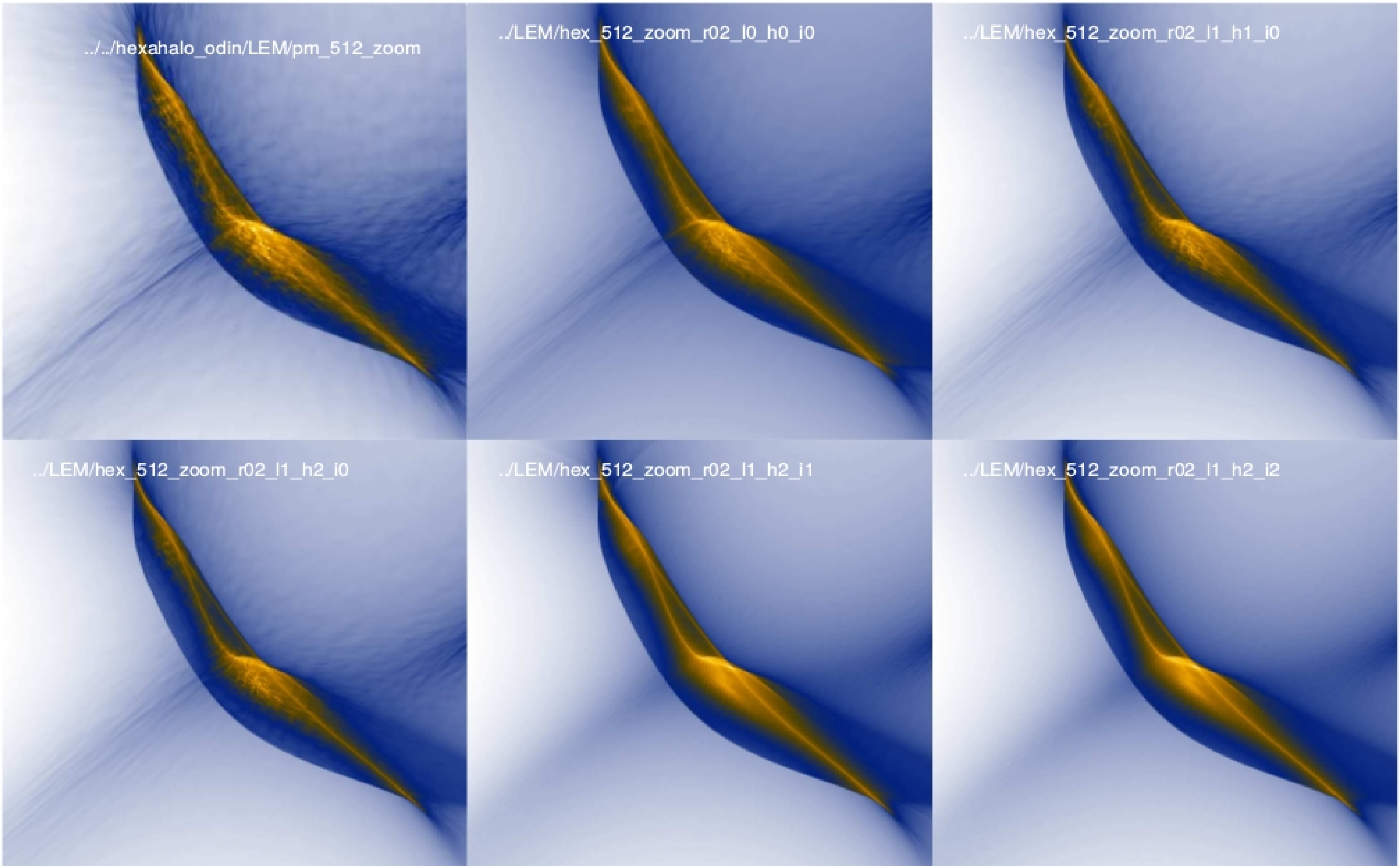
A zoom in simulation of the formation of a MW size halo in WDM With adaptively refined hexahedra

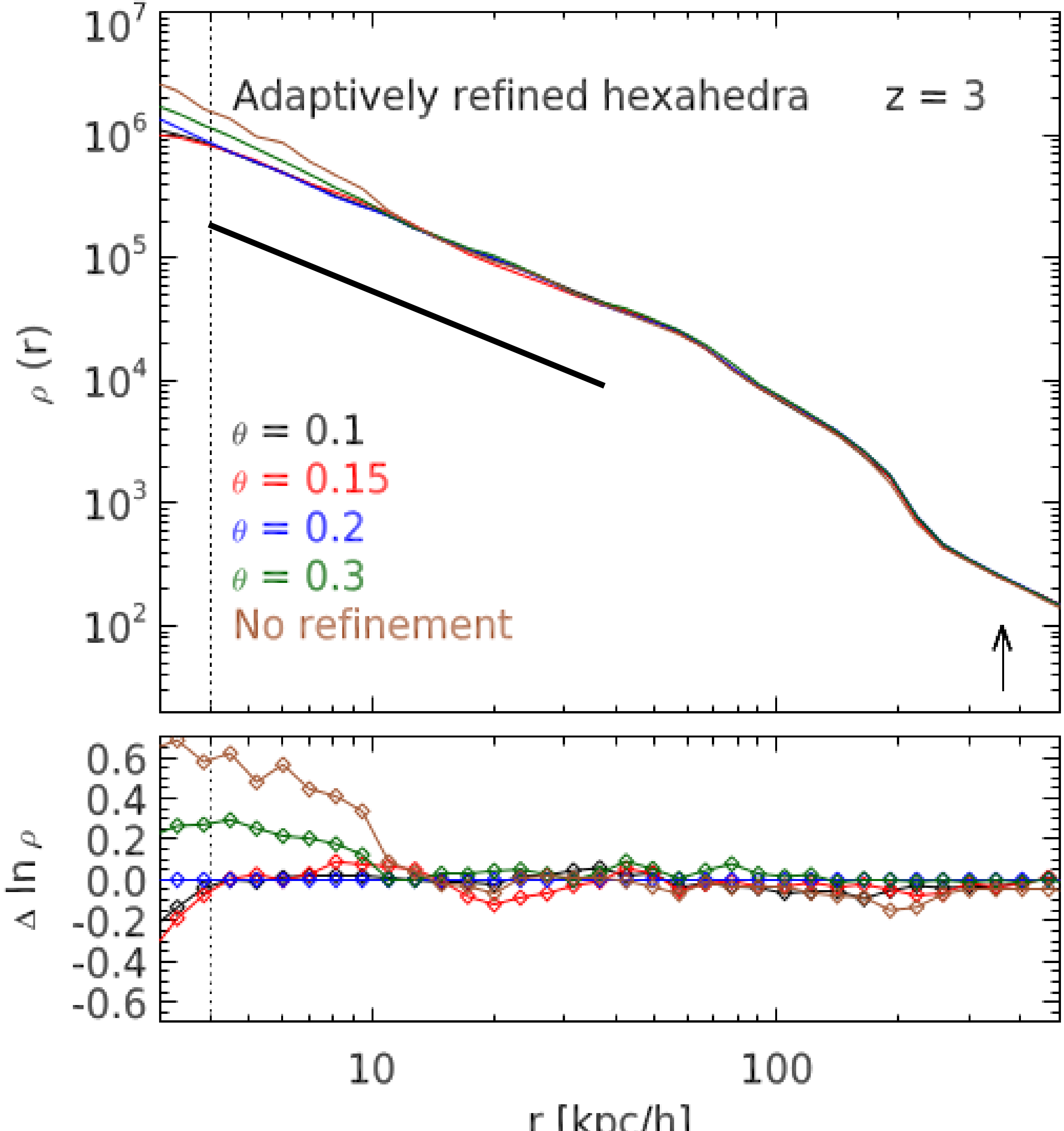


200,000 1.5e7 Msun flow tracers
700,000,000 final (600 Msun) flow tracers
100,000,000,000 (2 Msun) mass carriers
1024^3 + 4093^3 PM force mesh

$a = 0.13631347$

Final results are converged with respect to refinement threshold, Mass deposit level and initial resolution





Adaptively refined hexahedra
z = 3
θ = 0.1
θ = 0.15
θ = 0.2
θ = 0.3
No refinement
ρ (r)
$\Delta \ln \rho$
r [kpc/h]

Summary

Cosmological simulations in the highly nonlinear regime with Lagrangian Phase-space elements are possible.

They promise to increase the reliability and scope of computational cosmology.

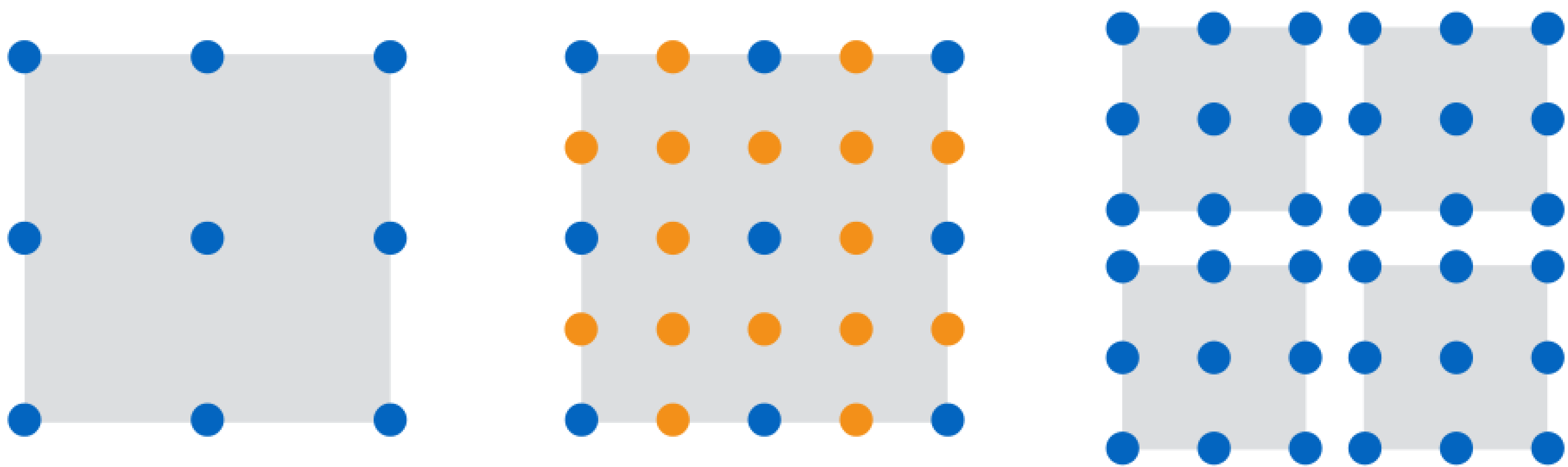
First application to the collapse of the First WDM halo shows a density profile steeper than a NFW.

Adaptive refinement of Lagrangian Elements

$$\frac{\eta}{\delta} = \frac{1}{2^{\ell}} \frac{\left|-\frac{1}{2}f_{i-2} + f_{i-1} - f_{i+1} + \frac{1}{2}f_{i+2}\right|}{\left|-\frac{1}{2}f_{i-2} + f_{i-1}\right| + \left|-f_{i+1} + \frac{1}{2}f_{i+2}\right| + \epsilon F}$$

$$\Theta = \left(\frac{\eta_0^2 + \eta_1^2 + \eta_2^2}{\delta_0^2 + \delta_1^2 + \delta_2^2}\right)^{1/2}$$

$$F = \left|\frac{1}{2}f_{i-2}\right| + |f_{i-1}| + |f_{i+1}| + \left|\frac{1}{2}f_{i+2}\right|,$$

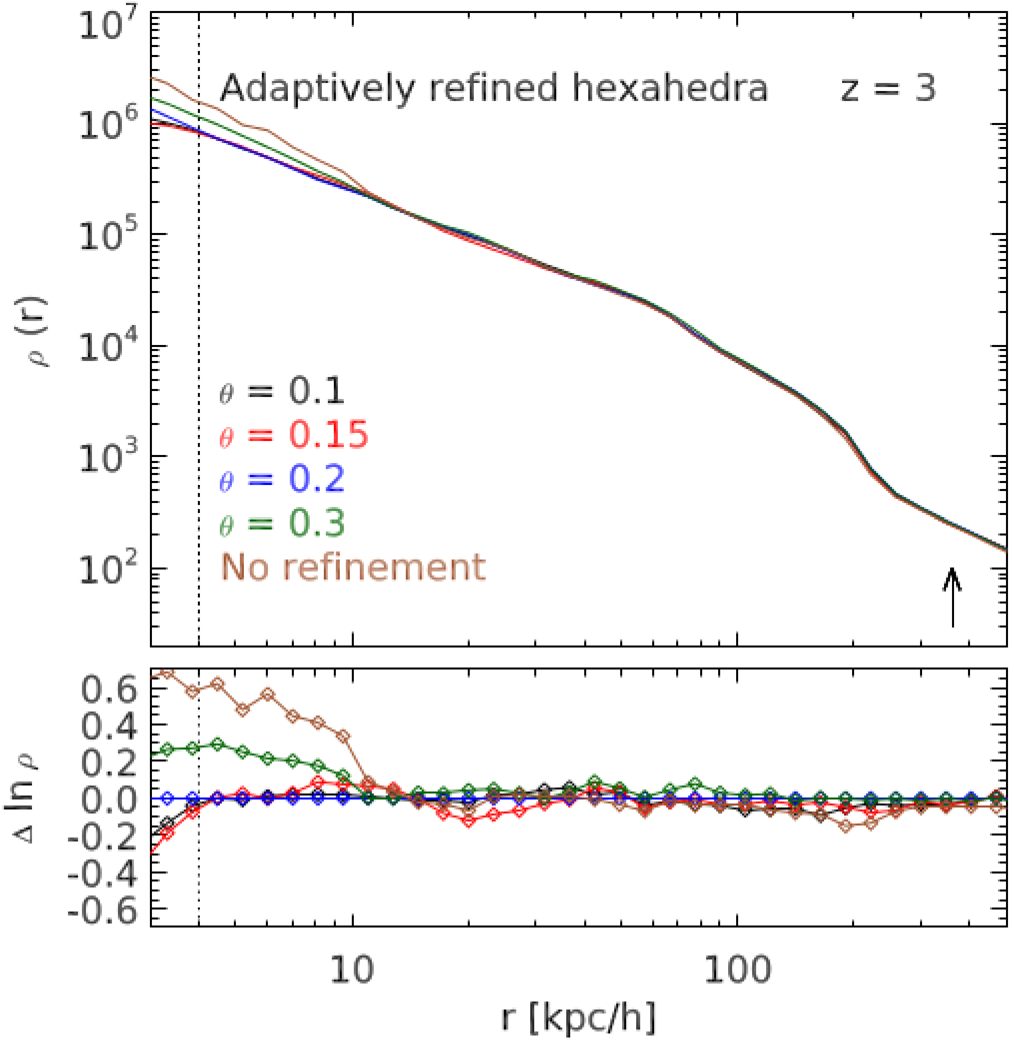


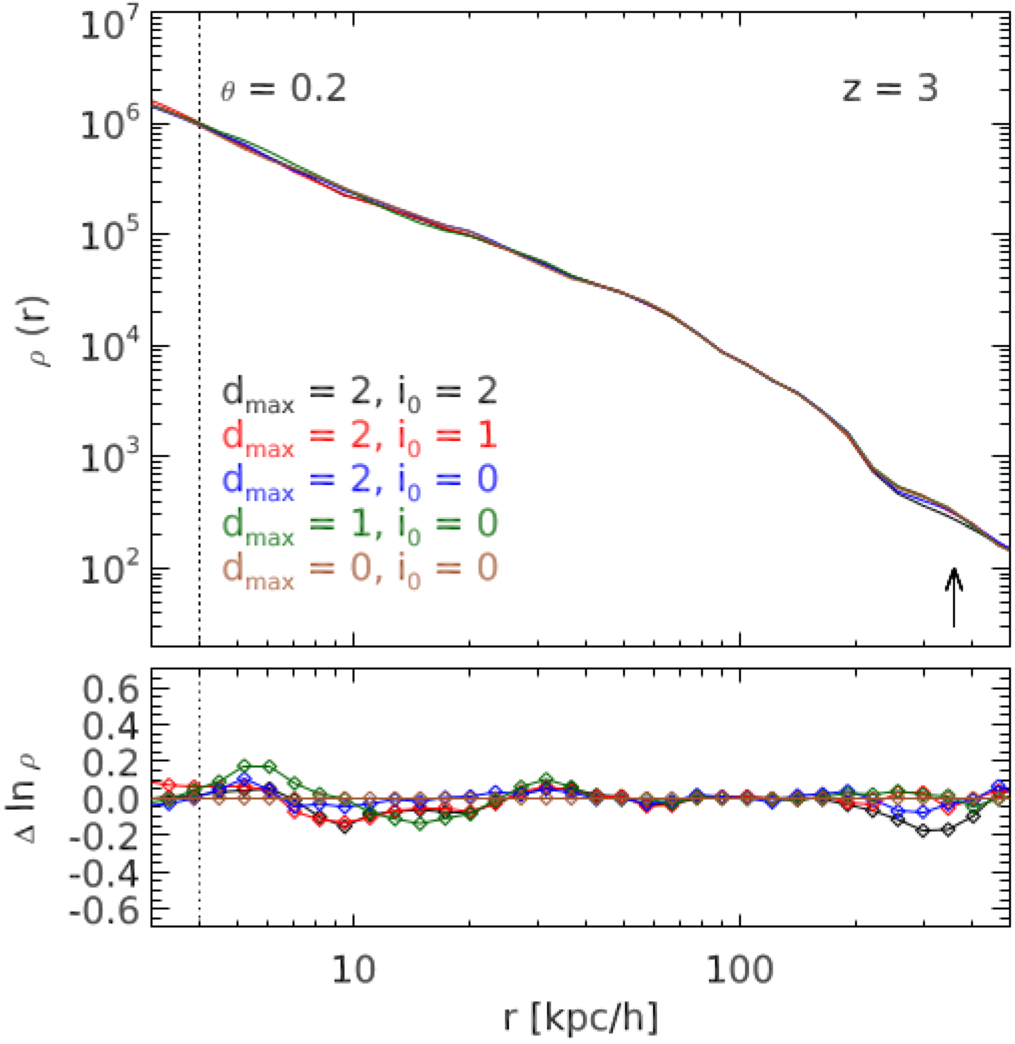
a. element is flagged for refinement

b. positions and velocities are determined at mid-points

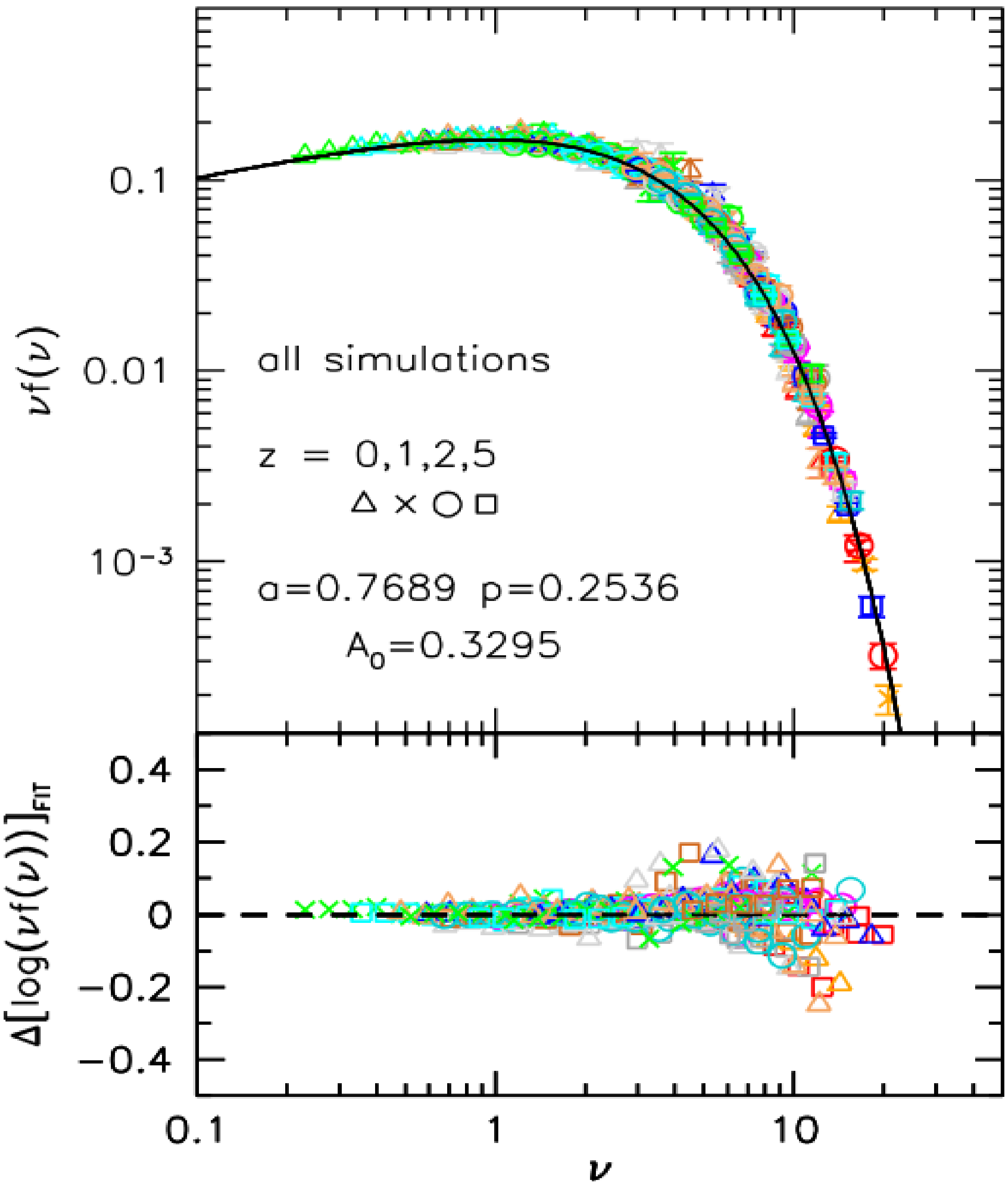
c. new elements are created using the mid-point values

Density profiles of a hexahalo



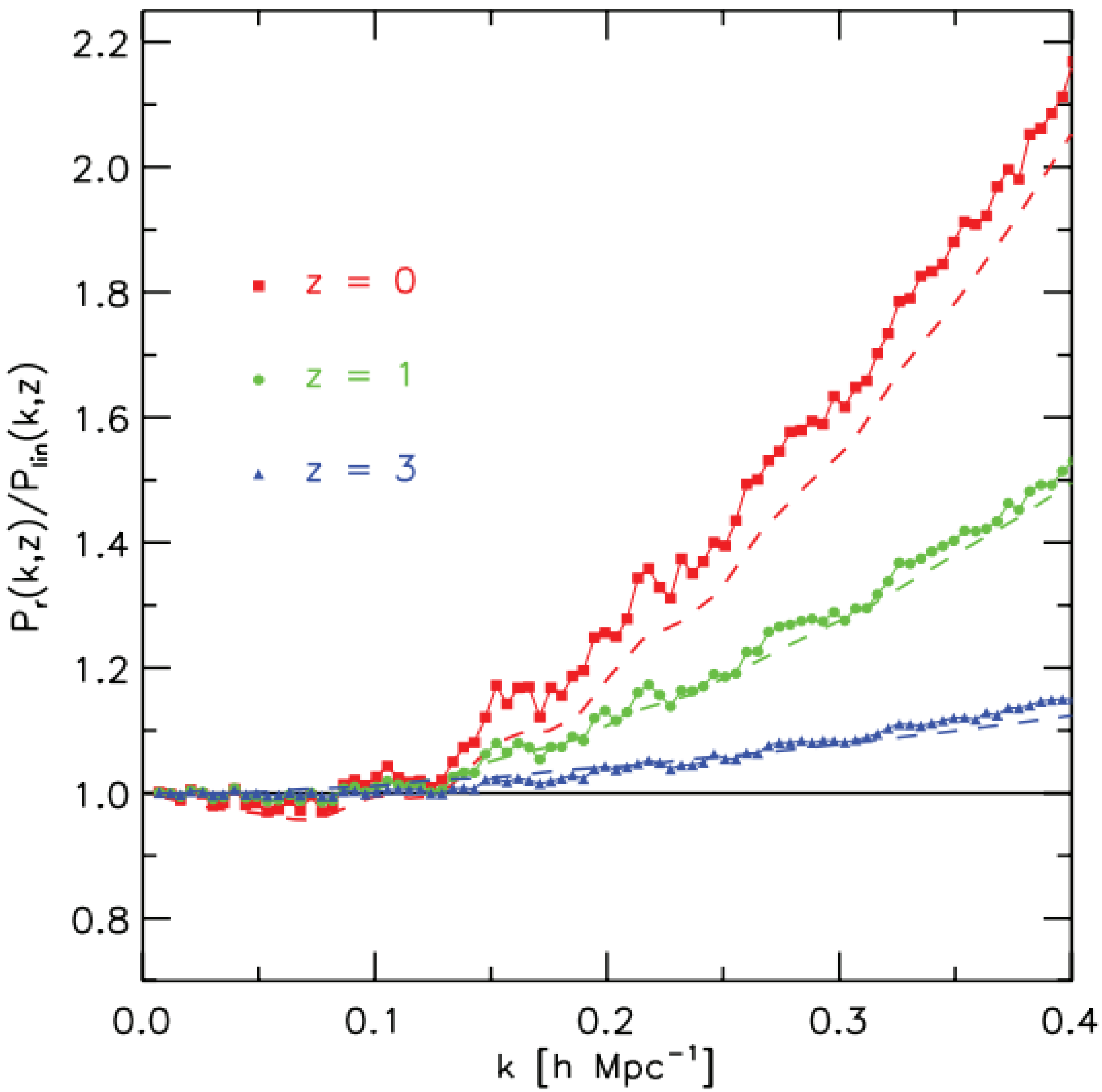


The abundance of collapsed structures



Despali et al 2015

The nonlinear evolution of the mass clustering



Angulo et al 2008