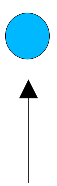
1. Accretion process – key parameters and their values

Imagine a following problem. We observed a globular cluster in X-ray band. We have optical observations of the cluster, so we can determine the distance to the cluster. We measure the X-ray flux, so knowing the distance we are able to determine the X-ray source absolute luminosity. Let us say that it is equal to 5×10^{37} erg/s. What is it? This is a difficult question. But let us say that we can also measure the X-ray spectrum, and we learn that the X-ray source is radiating as a black body of the temperature of 1.5 keV. Does that help? So what it is – a white dwarf? A neutron star? A binary system with a stellar-mass black hole? An intermediate mass black hole?

In order to answer more easily such questions it is important to know some typical values which are characteristic for an accretion-powered source.

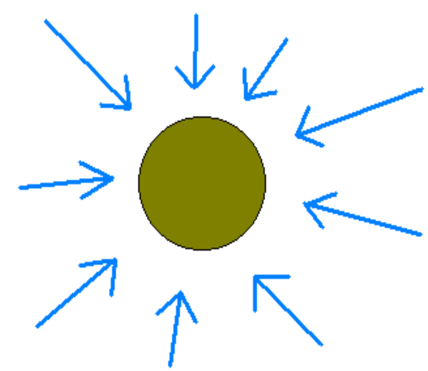
Eddington luminosity



electron

$$F = \frac{L}{4\pi r^2}$$

F radiation flux



Can this radiation flux be arbitrarily large? Radiation pressure may prevent further inflow and subsequent dissipation of the energy.

Digression: photon

Moving particle can be characterized by the mass m, energy E, and momentum p.

In special relativity:

$E^2 = p^2c^2 + m^2c^4$

For photon m = 0 so E = pc

Now we need a support of a quantum theory. Planck constant has been introduced by Planck in 1901 to solve the problem of a spectral shape of the black body radiation. The energy of the emitted radiation cannot be arbitrary but it is set by the frequency (wavelength) of the radiation. In 1905 it was confirmed by Einstein in his explanation of photoelectric effect.

A single photon has the energy: $E = h\nu$

where ν is the frequency of the photon wave, and the wavelength is given by $\lambda = c/\nu$.

Radiation flux F of a stream of photons is measured as energy/second/cm^2. In this simple approach we do not care about the wavelength of radiation. The force of radiation action on an electron is given by the gain of the momentum. If electron scatters or absorbs photons, it gains their momenta.

So the available momentum flux density is F/c. Now we need to know how large is an electron, or more precisely what is it range of photon interception. This is given by the Thomson cross-section, σ_T.

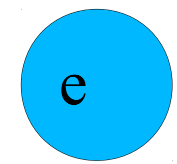
$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$

Note that there is no quantum trace in this value.

Digression: Thomson cross-section

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$ or stressing the radius involved as $$\sigma_T = \frac{8\pi}{3} r_e^2$$

Here the radius r_e is the classical radius of the electron. If we try to intuitively understand this quantity then two quantities come to our mind:
mc^2 – the rest energy of the electron
e^2/r_e – the potential energy of the interaction of an electron with itself.



The classical picture goes like this: we take an extended uniform charged medium of total charge e, compress it till the energy of the self-interaction becomes comparable to the rest mass energy of the electron, and this gets us the size. We can imagine also that virtual particles are also there, and form a wall, and this wall scatters photons independent from their energy.

In numbers: $r_e = 2.818 \times 10^{-15}$ m

Classical electron radius appear in Thomson (elastic !) scattering, in relativistic Klein–Nishina formula, it also marks the length scale where renormalization (gauge theory) becomes important in quantum electrodynamics.

Note 1: this does not mean that the electron is extended. It just appears like that to a photon.

Note 2: classical derivation is not exact (wrong by a factor 8/5, roughly 50 %). So shortcuts are good for intuition and rough estimates, if you need precision you should do things better…

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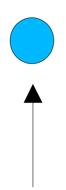
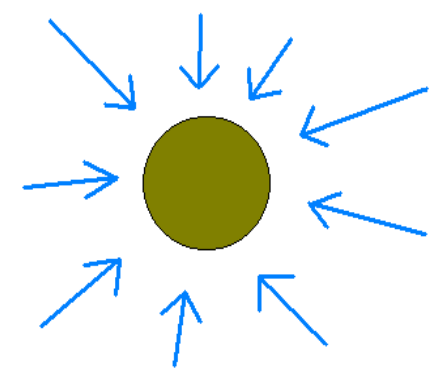


electron $\quad F = \frac{L}{4\pi r^2}$

F radiation flux



radiation force = absorbed momentum = $\frac{F}{c}\sigma_T$

$$\frac{F}{c}\sigma_T \leqslant \frac{GMm_p}{r^2}$$

Here we use proton mass since electron would drag proton (plasma is neutral)

From that we obtain a limiting value L

$$L_{Edd} = \frac{4\pi GM m_p c}{\sigma_T} = 1.38\times10^{38}\frac{M}{M_s} \quad [erg/s]$$

This is the limit of the luminosity for a stationary spherical gasous cloud, fully ionized (electron scattering !). Eddington luminosity depends on the mass of the body, but not on its radius.

Note: a person shines with the Eddington luminosity!

1. Accretion process – key parameters and their values

Radius

$$R = \frac{R}{R_{Schw}} R_{Schw} \qquad R_{Schw} = 2.95 \times 10^3 \frac{M}{M_S} [m]$$

does not depend on radius

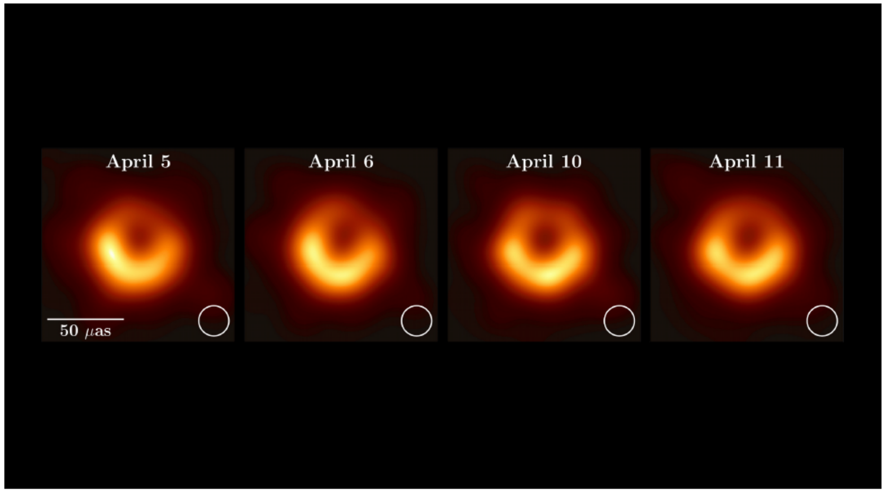
Minimum timescale

Coherent changes in the object can happen if the region is in a causal connection, and assuming the fastest signal propagation (speed of light) we get

$$\tau_{min} = \frac{R}{c} = 10^{-5} \frac{R}{R_{Schw}} \frac{M}{M_s} \quad [s]$$

And it is of order of microseconds for a neutron star, and 1000 seconds for a supermassive black hole.

This is the likely explanation why Event Horizon Telescope succeeded to get an image of M87, but failed so far to get an image of Sgr A*.



M87: the first image of the black hole (Event Horizon Telescope Collaboration et al. 2019)

The angular size of a black hole in Sgr A* is somewhat larger than angular size of the M87. But the mass in M87 is much larger, 6.5e9 Ms, while it is only 4.0e6 Ms for Sgr A*. Exposure time has to be equally long to get the signal, and much faster variability of the material in Sgr A* lead to smeared image.

Galactic black holes cannot be imaged, the required exposure time would be too short, and of and of course angular size prevents any attempt.

1. Accretion process – key parameters and their values

Radius

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Accretion efficiency

$$\eta=\frac{1}{2}\frac{R_{Schw}}{R}$$ does not depend on mass

Eddington accretion rate

because $\eta\dot{M}c^2=L\leqslant L_{Edd}$ we introduce $\dot{M}_{Edd}=\frac{4\pi GMm_p}{c\sigma_T\eta} = \frac{1}{\eta}1.3\times10^{14}\frac{M}{M_s}\quad[kg/s]$

The Eddington accretion rate corresponding to the Eddington luminosity depends on the accretion efficiency, i.e. whether we discuss the accretion onto a white dwarf, or neutron star, and in the case of a black hole depends on spin. Thus some people drop this efficiency factor (i.e. assume efficiency that is equal 1) which in my opinion is misleading.

1. Accretion process – key parameters and their values

Maximum energy of the photons emitted by accretion gas

Assuming that the whole energy of a single particle falling radially onto the body is dissipated at the impact and changed into 1 photon we obtain:

$$E_{max} = \frac{GMm_p}{R} \quad = \quad 470\ MeV\ \frac{R_{Schw}}{R}$$

Or, if we use the relation E=kT,

$$T_{max} = 6 \times 10^{12} \frac{R_{Schw}}{R} \quad [K]$$

This mechanism assume that the accreting plasma is optically thin, escaping photons do not interact with the accreting matter, and then the shape of emission is certainly NOT that of a black body. This upper limit does not depend on the mass of the object, so it is the same for stellar mass systems and in active galaxies. This is the reason why hard X-ray emission is the same in galactic black holes and in AGN, although the observed temperature is never that high (we see emission from electrons).

Minimum temperature of the photons emitted by accreting gas

Black body radiation is the most efficient way to emit, and the emitting body, at a fixed radiation flux, has the lowest temperature when emitting as a black body. This is a good approximation for emission of a medium which is optically thick, and the radiation is in equilibrium with the matter

$$L = \sigma T_{bb}^4 \pi R^2$$

which we can convert to

$$T_{bb} = 4 \times 10^7 \left(\frac{L}{L_{Edd}}\right)^{1/4} \left(\frac{R_{Schw}}{R}\right)^{1/2} \left(\frac{M_s}{M}\right)^{1/4} \quad [K]$$

(or 4 keV)

Now we can answer the question about the X-ray source in the globular cluster which has been posed at the beginning of the lecture. This has to be a neutron star, or a black hole, since in those two cases the temperature and the luminosity agree with expectations. Further differentiation between the neutron star and the stellar mass black hole is difficult. We will talk about it later.

1. Accretion process – key parameters and their values

Magnetic field

We can estimate the characteristic value of the magnetic field assuming the energy equipartition with the radiation field

$$\frac{B^2}{8\pi} = \frac{L}{4\pi R^2 c}$$

which can be finally expressed as

$$B = 4\frac{R_{Schw}}{R}\left(\frac{L}{L_{Edd}}\right)^{1/2}\left(\frac{M_s}{M}\right)^{1/2} \quad [G]$$

Therefore the magnetic field scales not with the mass, but with the square root of the mass!

Evolutionary timescale

The rate at which the mass of the central body increases with time in a stationary process is

$$\tau_{ewol} = \frac{M}{\dot{M}}$$

which can be expresses as

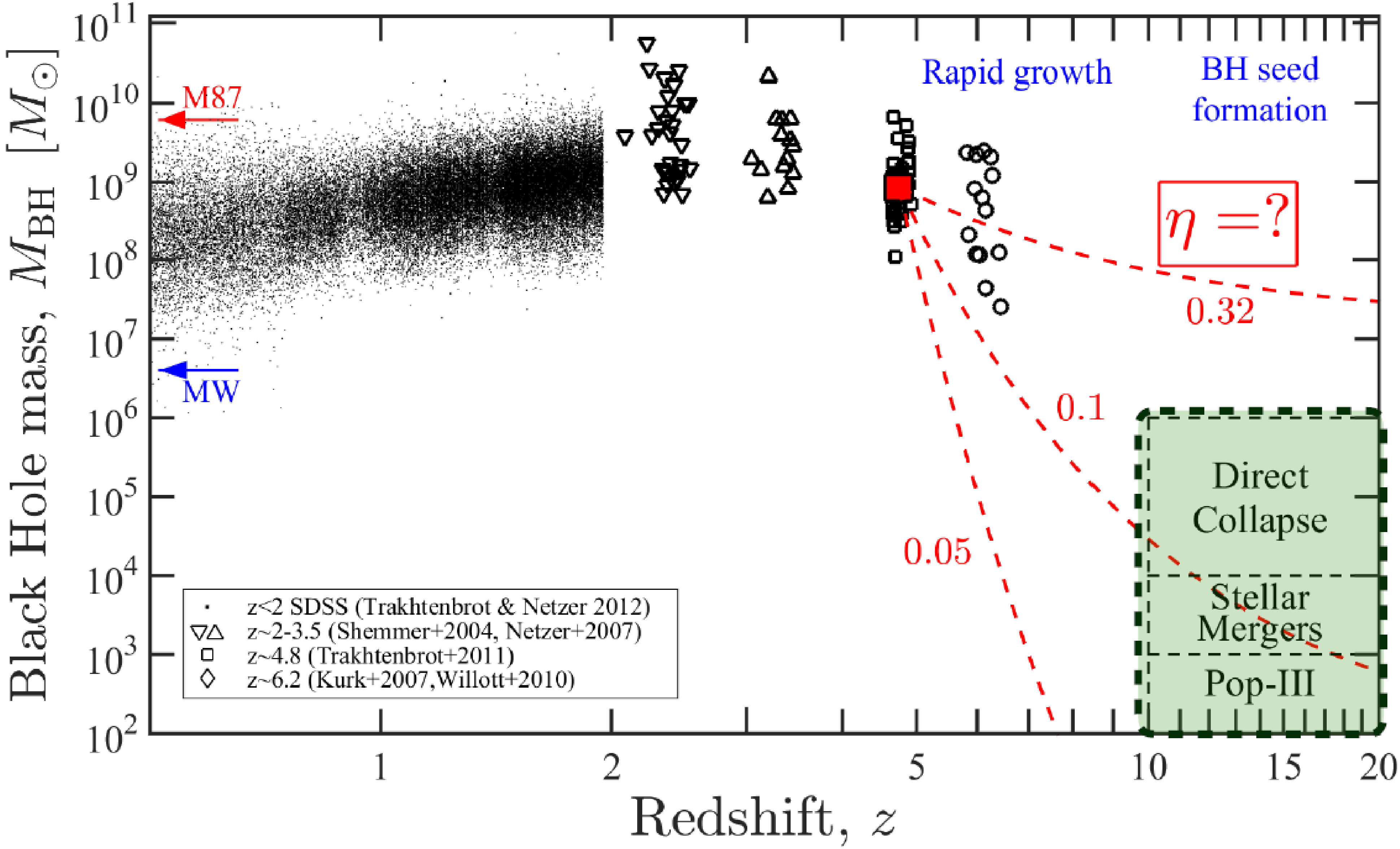
$$\tau_{ewol} = 3\times 10^8 \frac{1}{\eta}\frac{L_{Edd}}{L} \quad [lat]$$

Salpeter timescale

This rate does not depend on central mass, it is the same for Galactic black holes and for AGN. It is only somewhat shorter than the age of the Universe (13.7 billions of years) for a standard accretion efficiency of 10%.

This is a problem for very massive, high redshift quasars. The record holder is the quasar ULAS J1342+0928 at redshift z = 7.54; light observed from this quasar was emitted when the universe was only 690 million years old. Salpeter timescale for 10% efficiency is 3 Giga years, no time for a black hole mass to grow that big.

1. Accretion process – key parameters and their values



Schawinski 2016, presentation "The missing seed problem for massive black holes at high redshift

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DIGRESSION: In practice there may be complications. For example, accreting white dwarfs mostly loose mass in the accretion process because in the nova episodes they can eject more mass than they accreted between the outbursts.

Those relations allow to estimate source parameters if we can measure something from observational data.

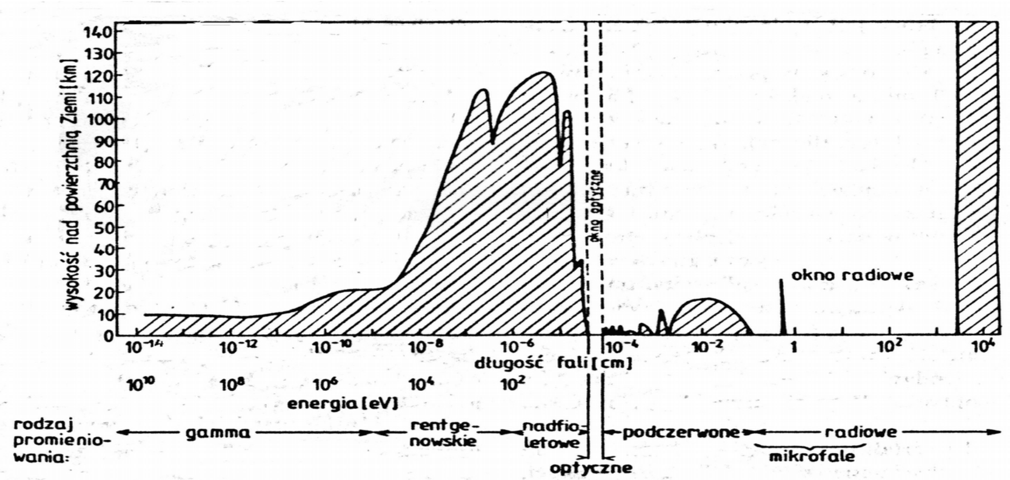
2. Determination of the basic accretion parameters from observations

Luminosity

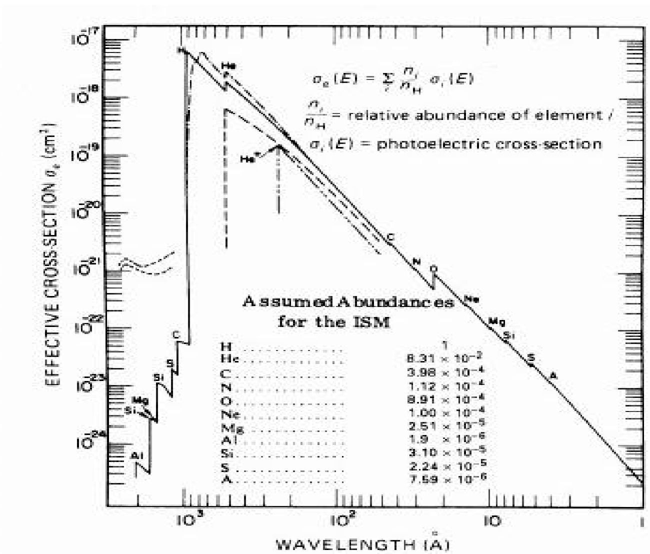
The largest problem is the measurement of the distance – this is in general the largest problem in astronomy.

Example 1rzykład. The discussion about the distances to the gamma-ray bursts (erratic bursts of gamma radiation, lasting from a fraction of a second to hundreds of seconds, with sources distributed randomly on the sky). A range of possibilities wer suggested, including the orin in the Solar System. Later the sudy of the source counts as a function of luminosity started to limit possibilities to a Galactic halo, i.e. 100 kpc (1pc = 3×10^{18} cm), or from cosmological distances, i.e. 10 Gpc. First option implied the typical luminosity 10^{41} erg/s, second option 10^{51} erg/s – a difference of 10 orders of magnitude. The solution came from telescope Beppo-SAX which lead to precise determination of the position, and identification with counterparts (e.g. for GRB970508), in other wavelengths, including optical band. This reveiled the so called afterglows. The identification of absorption lines was possible, and then distance determination. Cosmological explanation was the correct one.

Second problem is the incomplete spectra coverage which usually does not allow the determine precisely the ***bolometric luminosity.*** The problem lies in extinction. The issue of extinction in the Earth atmosphere can be solved by the use of satellites. But we cannot do much about the interstellar extinction!



Absorpcja promieniowania elektromagnetycznego w atmosferze ziemskiej. Krzywa odzwierciedla wysokość, do jakiej dociera 1% promieniowania o danej długości fali



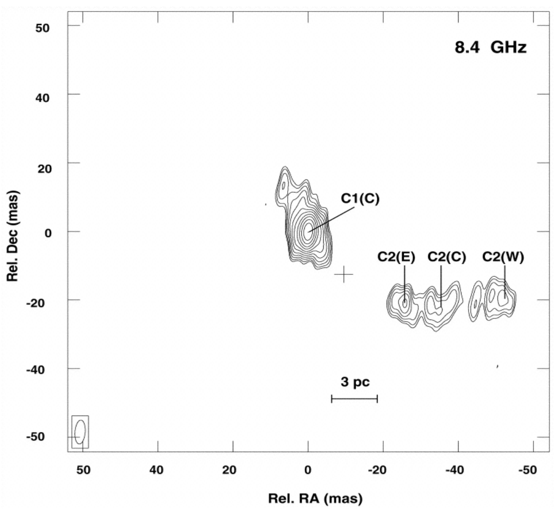
Zombeck Handbook

2. Determination of the basic accretion parameters from observations

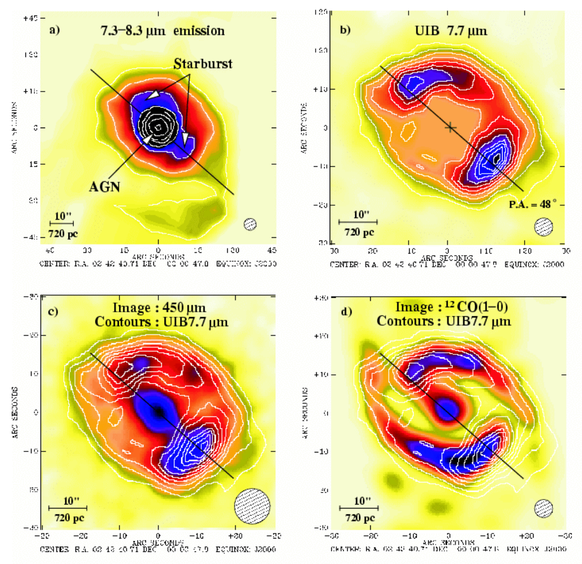
Radius

Radius (or more generally, size) is generally difficult to measure directly for accreting sources since we are limited by the spatial resolution.

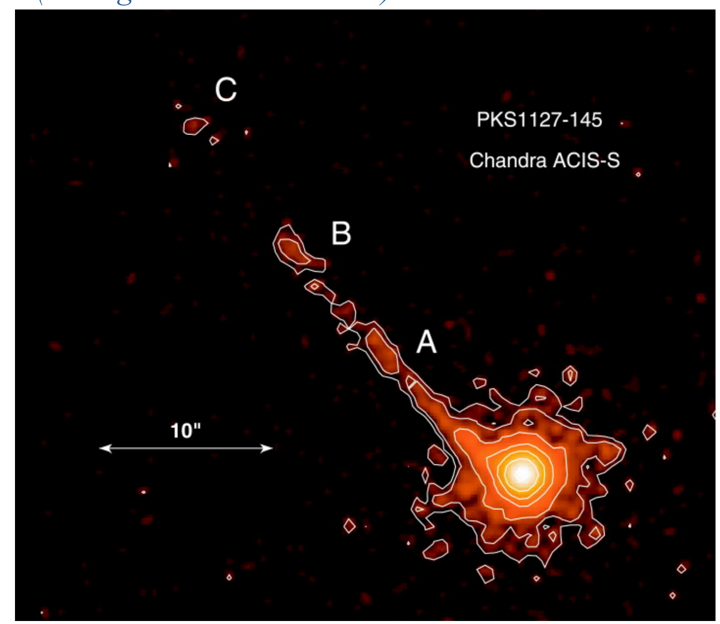
VLBI image in water maser line of a galaxy NGC 5793 (nucleus plus a jet in parsec scale (Hagiwara et al. 2001)



Mid-Infrared, ISOCAM, SCUBA i CO2 ISOCAM of a Seyfert galaxy NGC1068 (from Le Floch et al. 2001)



PKS 1127-145 ACIS-S Chandra – a rare case of the X-ray image of a jet extending up to 300 kpc (Siemiginowska et al. 2002)



Typical spatial resolution in the object corresponding to 1" angular resolution:

Object	Mass/Ms	Distance	1"[m]	1"[R_{Schw}]	
GBH	10	10 kpc	10^{15}	$3x10^{11}$	*(Galactic Black Hole)*
Milky Way/Sgr A*	$4x10^{6}$	10 kpc	10^{15}	10^{6}	
MBH	10^{7}	50 Mpc	$5x10^{18}$	10^{9}	*(Massive Black Hole)*
MBH	10^{9}	1 Gpc	10^{20}	$2x10^{9}$	
M87	$5.6x10^{9}$	16 Mpc	10^{15}	10^{6}	practically the same as for Sgr A*

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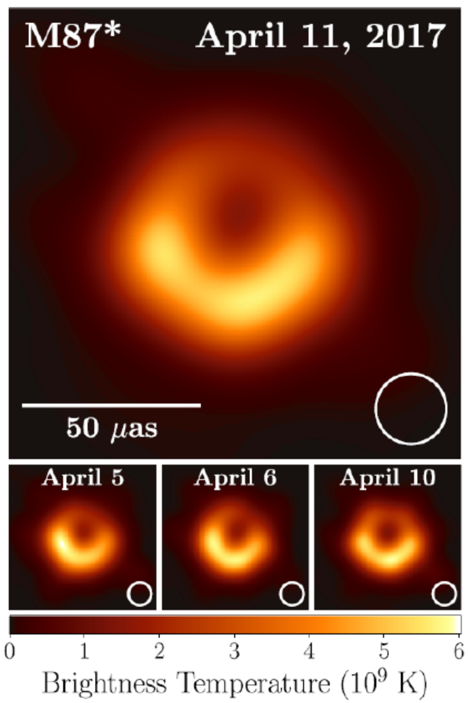
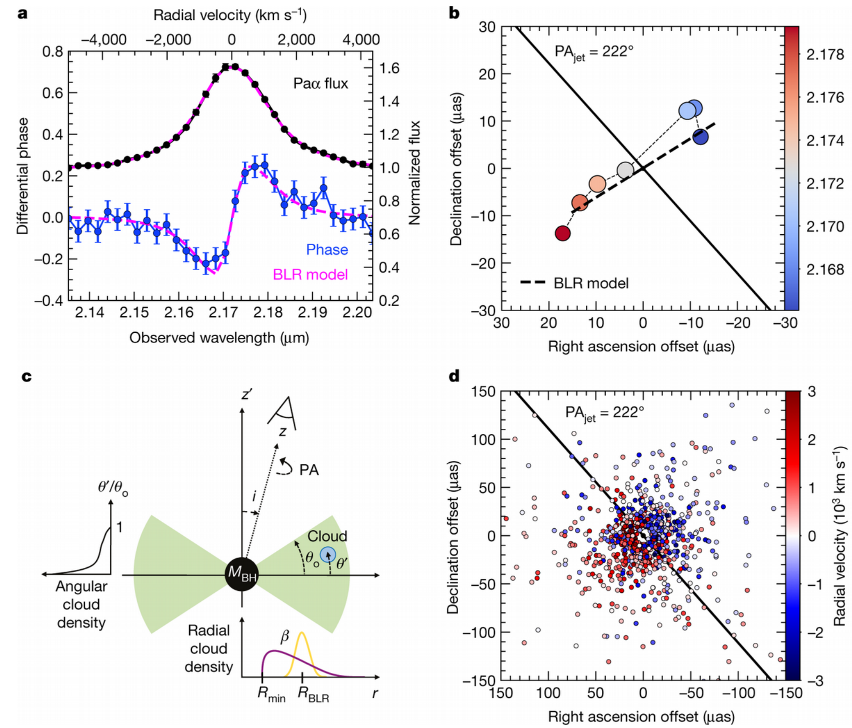


Most spectacular recent images of active galaxies possible to achieve with micro arc seconds spatial resolution.
Left: M87 (from Event Horizon Telescope) in mm band
Right: resolving Broad Line Region with GRAVITY on VLT in near-IR



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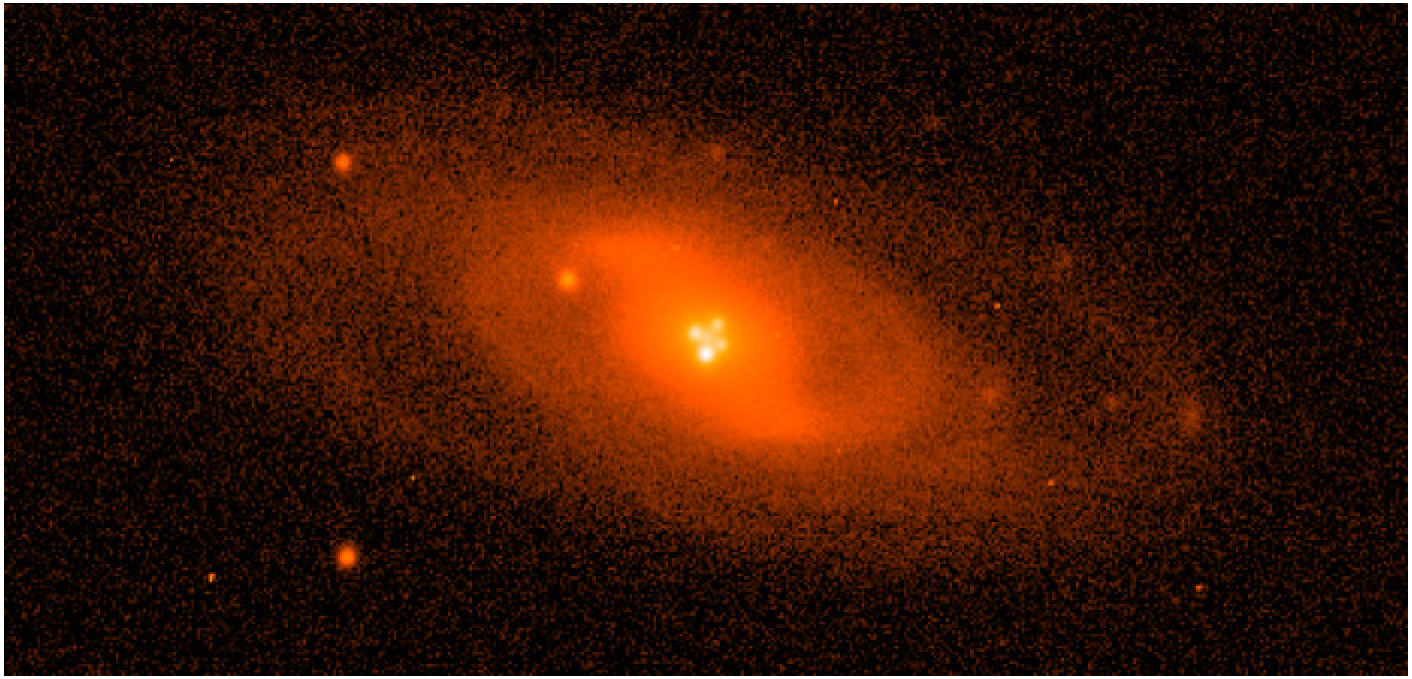
M87	5.6×10^{9}	16 Mpc	10^{15}	10^{6}	practically the same as for Sgr A*

2. Determination of the basic accretion parameters from observations

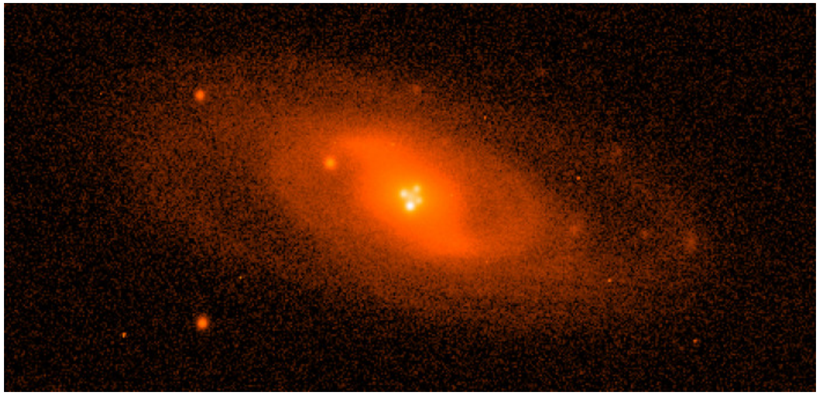
Thus in most cases we try to determine the size indirectly, for example (i) from the eclipse geometry in binary stars (ii) from the temperature and luminosity, if the emmiting radiation is close to a black body (iii) from the minimum variability timescale.

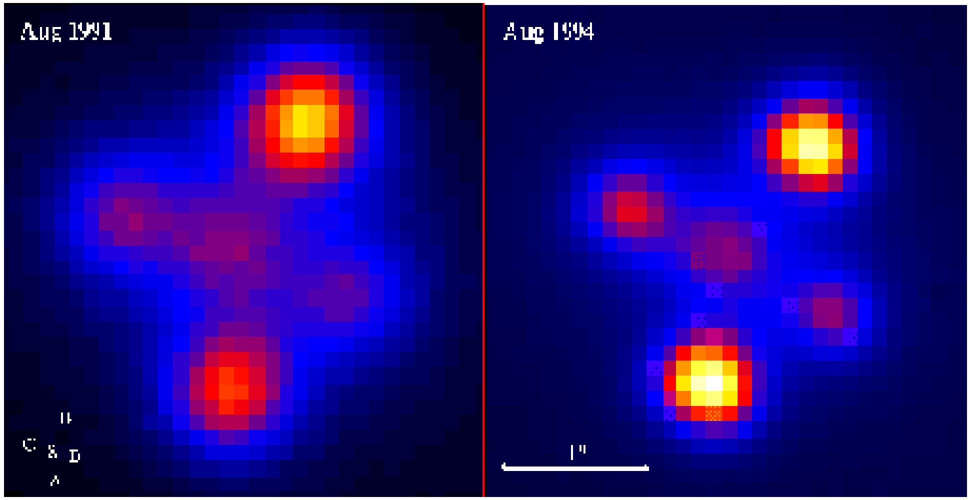


Einstein cross with a lensing galaxy (AAS Nova, E. Mediavilla et al 2015 ApJ 814 L26

But there are also other methods. One such example gaining importance now in the context of accretion disk size measurements and in cosmology is the use of the microlensing phenomenon. The most spectacular example of the lensing is the quasar Einstein cross Q2237+0305 (z=1.659). It forms four basic images due to the normal lensing by the intevening galaxy, but apart from that all images twinkle because of the microlensing by individual stars in the lensing galaxy.

2. Determination of the basic accretion parameters from observations

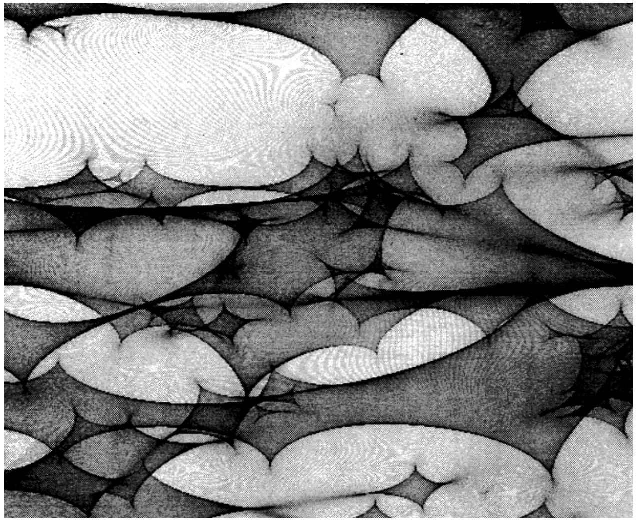




The quasar also vary, so the whole image luminosity vary, and on the top we have the flickering, and the character of this flickering depends on the size of the emitting region – for larger emitting regions the flickering washes out. In the old paper by Jaroszyński only the limit was reported: R < a few times 10^{15} cm.

More recent result for this source: (2.1 ± 1.0) 10^{15} cm (Mediavilla et al. 2015). For a black hole mass of 1.2 10^{9} Ms for this source this corresponds to a size of 5.3 ± 2.7 R_{Schw}, quite close to ISCO.

Caustic map for Q2237+0305 (Jaroszyński i in. 1994)



2. Determination of the basic accretion parameters from observations

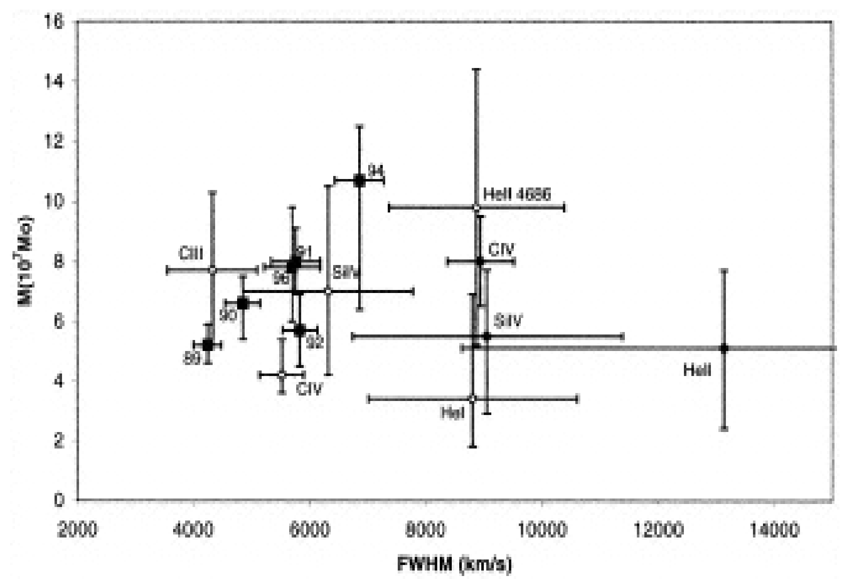
Mass

Mass determination in astronomy most frequently relies on the measurement of the motion of a satellite around the studied object. If we are able to measure the velocity of the satellite and the size of the orbit (or the period) we assume a Keplerian motion on a circular orbit:

$$\frac{GM}{R} = v^2$$

The problem reduces to the proper choice of the ‘satellite’. In the case of binary stars the companion seen in the optical band and providing the mass plays that role. We measure the velocity from the line shifts due to the Doppler effect, and we measure the period. In the case of AGN the role of the satellite is played by the stars or gasous clouds emitting the broad emission lines (BLR). In the first case we measure the velocity dispersion as a function of radius (for Sgr A* we measure individual stars). In AGN we measure the velocity from the line width, and the orbit size from the delay of the lines with respect to the continuum. In both cases we frequently assume the random distribution of the orbital planes.

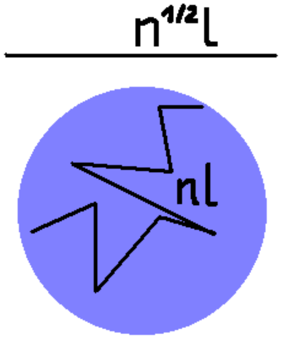
Determination of the central black hole mass $M=6\pm2\times10^7\,M_o$ (Wandel et al. 1999) for NGC 5548



2. Determination of the basic accretion parameters from observations

Accretion efficiency

This parameter can be also estimated directly from observations!



Example 1 (Fabian 1979)

Extended source with the luminosity L changed significantly its luminosity in
Let us know mark temporarily the unknown radius as R, and the optical depth
As tau.

Now we will estimate the source parameters.

Crossing time for the photon can be easily estimated in two extreme cases:

- tau << 1 then the photon passes through the medium without scattering T = R/c
- tau >> 1 then we can apply the diffusion approximation known from Brownian motion:

l - mean free path

after n scatterings the traveled path nl, but the systematic shift $n^{1/2}$ l

Thus the mean velocity drift is to $n^{1/2}l/(nl/c)=c/n^{1/2}$

Now we use the fact that the number of scattering n required to leave the cloud is $n^{1/2}l = R$, and from the fact that the optical depth tau is equal to n^2, we get the mean velocity c/tau, and the crossing time T=(R/c) tau.

Now we combine the two expressions into a single formula: $T = \frac{R}{c}(\tau + 1)$

Number density of the cloud can be calculated from expression

$$\tau = \sigma_T \rho R / m_p \qquad \rho = \frac{\tau m_p}{\sigma_T R}$$

Mass of the cloud is given as

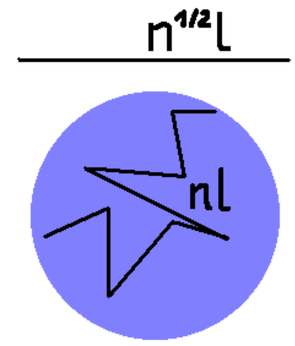
$$M = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi R^2 m_p \frac{\tau}{\sigma_T}$$

Radius can be expressed using the photon travel time above and now we are ready to calculate the ***efficiency of the process*** responsible for a change in the timescale T of the source with luminosity L:

2. Determination of the basic accretion parameters from observations

$$\eta = \frac{L\,T}{Mc^2} = \frac{3\,L\,\sigma_T}{4\pi\,c^4\,m_p\,T}\,\frac{(\tau+1)^2}{\tau}$$

function $\frac{(\tau+1)^2}{\tau}$ has the minimum at tau=1 (and the minimum value 4



Therefore in every case

$$\eta > \frac{L\,T}{Mc^2} = \frac{3\,L\,\sigma_T}{\pi\,c^4\,m_p\,T}$$

For example the core of the galaxy NGC 5548 in X-rays has the luminosity roughly 10^{44} erg/s, changes significantly in a timescale of 10^4 s, which gives the accretion efficiency 0.02. There are AGN with efficiency $\eta \gg 1$. This is possible if we deal with relativistic effects (jet emission).

Example 2 (Sołtan 1982)

Global way to determine the mean quasar efficiency was suggested by Paczyński and done in partise by Andrzej Sołtan (method is known as **Soltan argument**).

Mass, which accumulates at the center of a quasar of luminosity L (as a black hole) during its lifetime T

$$M = \frac{1}{\eta\,c^2}\int_0^T L(t)\,dt$$

We do not know T, but we can we can work around this problem. Instead of stydying a single quasar we calculate the total energy produced in 1 Gpc^3

$$E = \int\int \phi(L,t)\,L\,dL\,dt$$

where $\phi(L,t)$ is the number of quasars of luminosity L at any moment t, and we integrate over time from Bing Bang to now; integration over L: we use the observed luminosity range

2.

This still does not solve the isse since we do not know the time evolution of quasars. But assuming a cosmological principle, the number of quasars of luminosity L and age t is statistically the same as the number of quasars of luminosity L and of the redshift z corresponding to time t. This is also not known but we know the quasar luminosity functions for various redshifts, that is n(S,z), where S is the observed luminosity

$$L = 4\pi D^2 S$$

Thus we finally convert the expression for energy into integration over observed luminosity and redshift

$$E = 4\frac{\pi}{c}\int\int n(S,z)S(1+z)\,dz\,dS$$

In principle there are additional details about the counting quasars in a fixed wavelength band (color) instead of fixed bolometric luminosity. Andrzej Sołtan got

$$E = 8.5\times10^{66} \quad erg/Gps^3$$

$$M = \frac{1}{\eta}4.7\times10^{12} \quad M_s/Gps^3$$

Estimating the local black hole mass density, he got the result: $\eta \geqslant 0.1$

This method has been used later on, the values implied were in the range of 0.01 to 0.3. For example, Davis & Laor (2010) get strong dependence of η on black hole mass (rising from 0.03 for 10^7 Ms, high Eddington ratio sources to 0.4 for 10^9 Ms, lower Eddington sources, implying spin evolution).

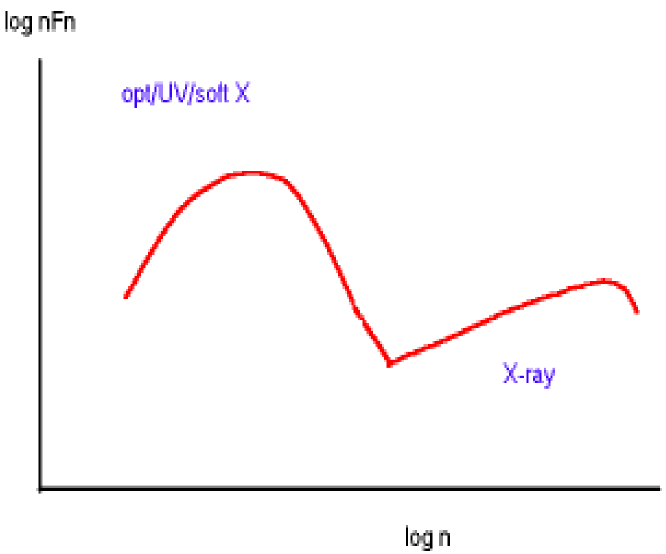
Accretion rate

Direct determination of the accretion rate is rather difficult. Sometimes we have good estimated of the accretion rate in the binary systems, from the stellar evolution. In the case of elliptical galaxies we usually estimate the accretion rate from the density of the interstellar medium, which is known from the X-ray emission profiles. But we cannot be sure that the process is stationary.

2. Determination of the basic accretion parameters from observations

Temperature and the maximum energy of photons

This parameters are determined directly from the data if we have a good spectra coverage. However, most of the accreting objects have complex spectra, not just a black body or (for optically thin plasma) bremsstrahlung (or free-free emission, if you prefer). For example, spectra of (bright) radio quiet AGN consist of two basic components. The two peaks imply two emission mechanisms. The problem is that the peak of the optical/UV part is usually placed in the unobserved part of the spectrum because of the interstellar extinction. On the other hand, high energy part extend to about 100 keV, not all instruments measure such high energies, and then also the number of photons detected is low.



Global evolution

Because the Salpeter timescale is large, we can study the global evolution only statistically, noting the number of objects at a given evolutionary stage.

Short term variability

All accreting objects vary in a broad range of timescales, and this variability is an increasing source of the information about the accretion process (and outflows). We will discuss that in more detail later on.

Homework:

1. Calculate your personal Eddington ratio
2. Wyprowadzić wzór na jasność Eddingtona dla atmosfery zdominowanej przez pary elektronowo-pozytronowe
3. Pokazać, jak przechodzimy od całkowania po czasie do całkowania po redshiftach we wzorach Sołtana opisujących wydajność akrecji w kwazarach.

Translation:2. Derive the formula for Eddington luminosit