Ankan Sur PhD report

Ankan Sur should be commended and congratulated on an excellent body of work, and a beautifully complete PhD thesis. The thesis covers a range of important sub-topics, with the overall goal of understanding the complicated equilibrium and evolution of magnetic fields in neutron stars, and their effect on a variety of observable properties of these enigmatic objects. The thesis is well written, and contains many published papers that Sur has led, all of which have made important contributions to this field. For this reason, I have no hesitation in recommending this dissertation be admitted for a public defense.

Chapter 1 - Neutron stars: An introduction:

Chapter 1 Provides a thorough introduction from the beginnings of neutron star astronomy and astrophysics, through to GRMHD equations and equilibria and gravitational-wave emission. While the bulk of the introduction is heavy on theory, it is nice to see a section on connecting that theory to observations. With that said, this section concentrates only on gravitational-wave emission, whereas magnetic field structure can, in principle, have significant effects on electromagnetic observations as well. It would have been nice to see a more detailed discussion of this in the introduction to provide context around the importance of the results obtained in the thesis.

I have a couple of minor queries and comments about this Chapter (including some typos which are included for completeness).

- Equation 1.1, missing a negative sign in front of the dt component.
- P4. "Similarly, the observation of gravitational waves from the binary neutron star merger GW170817 have put the maximum mass of NSs in the range 2.01 M ≤ M ≤ 2.17 M [MM17; RMW18]." – This is cherry-picking the literature: the constraint on the maximum mass deduced from GW170817 depends on the unknown outcome of the merger (hypermassive vs. supramassive neutron star), which is heavily debated in the literature.
- P4. "Furthermore, and most recently, millisecond pulsars have been studied using Shapiro delay to set the TOV limit to approximately 2.14M [Cro+20]." The one-sigma uncertainty on this measurement is approximately 0.1 Msun, so it is a little misleading the way this is written, especially given the context of the surrounding sentences.
- P4: "Atmosphere: This thin layer (few millimeters thick) of low density plasma is mostly responsible for the electromagnetic spectrum we observe, …" This is a little unclear, and potentially misleading; much of the electromagnetic emission comes from higher in the magnetosphere, e.g., radio.
- P5: typo "start tto drip"
- Equation 1.12 is not dimensionally correct. The terms in the square root have not been properly normalised.
- P15: typo "wehere"
- Eqn 1.52. The \Lambda term is included in this equation, which is presumably the ratio of toroidal to poloidal magnetic fields. This quantity is introduced in a paragraph on p.21 that seems to have been misplaced!
- Sec. 1.6 says the discussion connecting to observations will focus "on two main characteristics of magnetic field effects in NSs", however below this statement there is only a section on gravitational waves. What is the other?

Chapter 2 - Magnetic field configurations of neutron stars from MHD simulations
This Chapter, published in MNRAS, studies the important yet challenging problem of determining the equilibrium configuration of magnetic fields in neutron stars. Performing resistive and ideal MHD simulations of neutron stars, they start with simple initial configurations, whereby the field is entirely poloidal or entirely toroidal. Such a state is known to be dynamically unstable, which is quickly seen in the simulations as energy is transferred between these various components. The final "end state" is a non-stationary system that includes a turbulent cascade, albeit with relatively constant average energies in the toroidal and poloidal components of the field.

The simulations in this paper are performed without superfluidity, superconductivity, or a crust. There are many good reasons for doing this given the difficulty associated with the simulations, and these are articulated well throughout the Chapter. One of the primary motivations discussed in the introduction and conclusion of this Chapter is that the simulations are "applicable to the first few hours of life of the star, after differential rotation is dissipated." What are the effects of rigid rotation on the evolution of field instabilities and the steady-state solution? Such stars are likely still rapidly rotating, which presumably has a non-negligible effect on the magnetic-field evolution.

Chapter 3 - Long-term general relativistic magnetohydrodynamics simulations of magnetic field in isolated neutron stars
This Chapter advances on the previous one by presenting work in full general relativity, rather than Newtonian gravity. The simulations also last significantly longer than those presented in the previous Chapter, allowing for a more detailed exposition into the nature of the non-stationarity present at the end of the simulations. The conclusion here seems to be that these non-stationarities are not, in fact, turbulence, but are instead numerical artefacts. This is an incredibly challenging problem to solve given both the resolution and length of simulations required. Sur et al should be commended on such impressive results, despite the inconclusiveness of this one aspect.

Again, Sur et al found that the purely poloidal initial conditions quickly become unstable, converting magnetic energy into a toroidal component with comparable energy. The subsequent evolution here showed significant dissipation of magnetic energy, though. Much of the magnetic energy is either converted to an increase in internal energy, or "radiated to infinity in the form of electromagnetic radiation." I am cautious of this last statement for two reasons: first, the effect seems to be resolution dependent, and second, there is no reason I can see to believe this is not simply a boundary effect either at the surface of the star or the edge of the computational domain. It is noted in the work that this conclusion is, nonetheless, not realistic given the lack of a solid crust or resistivity. However, I would still be cautious of this conclusion in light of the previous Chapter that argued such simulations without crust are pertinent for newly-born neutron stars.

Chapter 4 - The impact of superconductivity and Hall effect in models of magnetised neutron stars
Chapter 4 takes a different approach to the problem of understanding magnetic fields in neutron star cores from the previous two chapters by calculating equilibrium field configurations with superconductivity in the neutron star core, and the Hall effect in the crust. This is excellent and important work, most notably because Sur & Haskell show that, if the core of the star is a type II superconductor, the toroidal field is entirely constrained to the stellar crust. This has potentially important consequences for understanding how glitches occur in neutron stars. In particular, many models that explain glitches through the pinning of superfluid vortices to superconducting flux tubes require a strong toroidal field in the stellar core. Whilte Sur & Haskell discuss in detail that their

equilibrium configurations can be used to study dynamics of glitches *without* strong toroidal fields, the paper also leaves me wondering whether there are modifications to their calculations that could still allow for toroidal fields in the core?

Chapter 5 - Gravitational waves from mountains in newly born millisecond magnetars
The final Chapter of the thesis presents evolutions of the gravitational-wave emission from newly-born neutron stars, particularly focussing on the effect of fallback accretion. The work calculates the spin evolution of these nascent stars by looking at the torques on the star from this accretion, and also magnetic-field, neutrino, and gravitational-wave radiation. The gravitational-wave emission is generated from magnetic mountains formed through the matter accreting onto the stellar poles. The work is very thorough considering the physical effects that are taken into account, and is well adapted to the two situations of a core-collapse supernova, where there is significant matter accretion, and binary neutron star mergers, where the amount of accretion is far-less substantial.

It is worth saying that there are many uncertainties in these calculations not taken into account. I do not mean this as a criticism; there are an incredible number of physical effects that can control the dynamics of the star, and many of them are ill-understood. In that respect, this work provides valuable calculations for some of those effects, which then need to be considered in the context of other calculations (e.g., r-mode and bar-mode evolutions, inclination-angle evolution, …).

Chapter 6 - Conclusions and future directions
This Chapter provides a nice summary of the various works accomplished and presented in the dissertation. It ends by mentioning some upcoming work that constrains the height above the surface of a particular pulsar at which the radio emission is produced – I look forward to seeing this work in the near future.

Summing up, I consider the doctoral thesis of Ankan Sur to be a valuable contribution and to meet the criteria prescribed by the law for a doctoral dissertation. Therefore, I request that this dissertation be admitted to a public defense.

Paul Lasky