“HIGH ENERGY ASTROPHYSICS” (A subjective introduction)

VII. Compact binary systems

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Story so far ...

- supernovae, supernova remnants, pulsars and pulsar wind nebulae

Introduction

- more than half of all stars form binary or even multiple systems (1/3 of observed “stars” are binary systems)

- binary systems that contain compact objects – white dwarf, neutron star, or black hole – are called compact binary systems

Dynamics

- two-body problem

$$m_1 \ddot{\vec{r}}_1 = -G \frac{m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{e}_r$$

$$m_2 \ddot{\vec{r}}_2 = G \frac{m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{e}_r$$

- center of mass and reduced mass

$$\vec{R}_{cm} \stackrel{\text{def}}{=} \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M} \quad , \quad \mu \stackrel{\text{def}}{=} \frac{m_1 m_2}{M} \quad , \quad M = m_1 + m_2$$

$$\vec{r}_1 = \vec{R}_{cm} + \frac{m_2}{M} \vec{r} \quad , \quad \vec{r}_2 = \vec{R}_{cm} + \frac{m_1}{M} \vec{r}$$

- reduces to one-body problem

$$\mu \ddot{\vec{r}} = \frac{G \mu M}{r^2} \hat{e}_r$$

- solution

$$r = \frac{p}{1 + e \cos \varphi} \quad , \quad p = \frac{L^2}{G \mu M} \quad , \quad e = \sqrt{1 + \frac{2 E L^2}{G^2 \mu^3 M^2}}$$

Dynamics

- solution

$$r = \frac{p}{1 + e\cos\varphi} \quad , \quad p = \frac{L^2}{G\mu M} \quad , \quad e = \sqrt{1 + \frac{2EL^2}{G^2\mu^3 M^2}}$$

- orbits:
  - *circular* for e=0
  - *elliptic* for 0<e<1
  - *parabolic* for e=1
  - *hyperbolic* for e>0

Dynamics

- Kepler's first law

  the orbit of each planet is an ellipse with the Sun in one focus

$$a=\frac{1}{2}(r_{\max}+r_{\min})=\frac{p}{1-e^2}=-\frac{G\mu M}{2E} \quad , \quad b=a\sqrt{1-e^2}=\frac{p}{\sqrt{1-e^2}}$$

- Kepler's second law

  line joining the planet and the Sun sweeps out equal areas during equal intervals of time

$$\dot{\varphi}=\frac{L}{\mu r^2} \quad \Rightarrow \quad \frac{dS}{dt}=\frac{L}{2\mu}=\text{const}$$

- Kepler's third law

  squares of the orbital periods are proportional to the cubes of semi-major axis

$$S=\int_0^T \frac{dS}{dt}\,dt=\frac{L}{2\mu}T=\pi a b$$

$$\frac{T^2}{a^3}=\frac{4\pi^2}{GM}=\text{const}$$

Mass function

- consider a binary system of masses m_1 and m_2 observed at angle i
  observed velocity (*single-line spectroscopic binary*)

$$v_1 = v_{1,\mathrm{true}} \sin i = \frac{2\pi}{T} a_1 \sin i$$

- *mass function*

$$f_1 \stackrel{\mathrm{def}}{=} \frac{T v_1^3}{2\pi G}$$

$$f_1 = \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{m_2 \sin^3 i}{(1+q)^2} \quad , \quad q \stackrel{\mathrm{def}}{=} \frac{m_1}{m_2}$$

  mass of the unseen companion star $m_2 > f_1$

- *double-line spectroscopic binary*

$$f_1 \text{ and } f_2 = \frac{m_1^3 \sin^3 i}{(m_1 + m_2)^2} \quad \Rightarrow \quad q = \left(\frac{f_2}{f_1} \right)^{1/3}$$

Gravitational wave emission

- energy of the binary system

$$E=\frac{\mu^3 G^2 M^2}{2 L^2}\left(e^2-1\right)$$

e=0 is the minimal energy – binary system that losses orbital energy tends to circularize

most old binary systems are close to circular

- the energy loss mechanism may be the emission of gravitational waves

Gravitational wave emission

Burke (1971), Journ. of Math. Phys., 12, 402
Lightman et al. (1975)

- *radiation reaction* Φ_{GW}

$$\Phi = \Phi_N + \Phi_{GW}$$

- orbit change

$$d\vec{F}_{GW} = -\vec{\nabla}\Phi_{GW}\,\rho\,d^3x$$

$$\frac{dE}{dt} = \int \vec{v}\circ d\vec{F}_{GW} = -\int \vec{v}(\vec{r})\circ\vec{\nabla}\Phi_{GW}\,\rho\,d^3x \quad , \quad \frac{d\vec{L}}{dt} = \int \vec{r}\times d\vec{F}_{GW}$$

$$e = \sqrt{1 + \frac{2EL^2M}{G^2 m_1^3 m_2^3}} \qquad a = -\frac{G m_1 m_2}{2E}$$

$$\frac{da}{dt} = -\frac{64\,G^3 m_1 m_2 M}{5\,a^3 c^5}\,\frac{1 + 73/24\,e^2 + 37/96\,e^4}{(1-e^2)^{7/2}}$$

$$\frac{de}{dt} = -\frac{304\,G^3 m_1 m_2 M}{15\,a^4 c^5}\,e\,\frac{1 + 121/304\,e^2}{(1-e^2)^{5/2}}$$

- *inspiral phase*

$$\tau_{insp} = 10^7\,\mathrm{yr}\left(\frac{P}{1\,\mathrm{h}}\right)^{8/3}\left(\frac{M}{M_{SUN}}\right)^{-2/3}\left(\frac{\mu}{M_{SUN}}\right)^{-1}(1-e^2)^{7/2}$$

Roche lobe

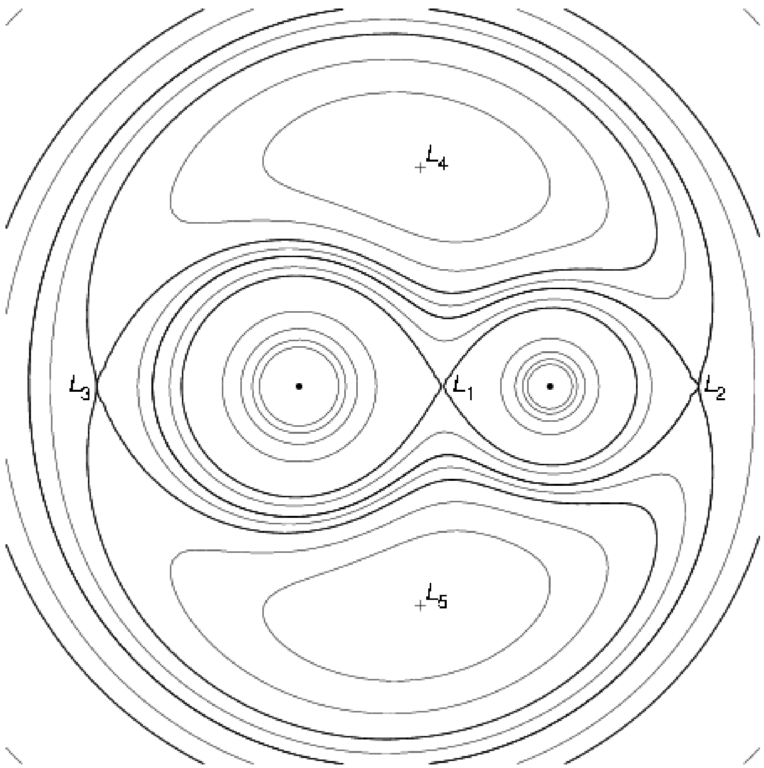
- center of mass reference frame

$$\vec{F}' = \vec{F} + \vec{F}_{\text{cen}} + \vec{F}_{\text{Cor}} = \vec{F} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}') - 2\text{m}\,\vec{\omega}\times\vec{v}'$$

- *Roche potential*

$$\Phi_{\text{Roche}}(\vec{r}) = -\frac{G m_1}{|\vec{r}-\vec{r}_1|} - \frac{G m_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}(\vec{\omega}\times\vec{r})^2$$

- *Roche lobes, Lagrange points*



Binary evolution

- *mass transfer* via strong winds or Roche lobe overflow
    1. for constant separation star evolves to fill its Roche lobe
    2. separation shrinks and Roche lobe of a star becomes smaller than the star size
    - mass transfer may reduce or enlarge the orbit

- supernova explosion in a binary

$$E_{\mathrm{tot}} = \frac{1}{2}\mu\left(\omega_{\mathrm{K}}\, a\right)^2 - \frac{G\, m_1 m_2}{a} = -\frac{G\, m_1 m_2}{2\,a} \quad , \quad \omega_{\mathrm{K}} = \sqrt{\frac{GM}{a^3}}$$

$$E'_{\mathrm{tot}} = \frac{1}{2}\frac{m_{\mathrm{r}} m_2}{m_{\mathrm{r}} + m_2} v^2 - \frac{G\, m_{\mathrm{r}} m_2}{a} = \frac{G\, m_{\mathrm{r}} m_2}{2\,a}\left(\frac{m_1 + m_2}{m_{\mathrm{r}} + m_2} - 2\right)$$

$$m_{\mathrm{r}} > \frac{m_1 - m_2}{2} \quad , \quad \Delta m < \frac{m_1 + m_2}{2}$$

- *common envelope evolution*

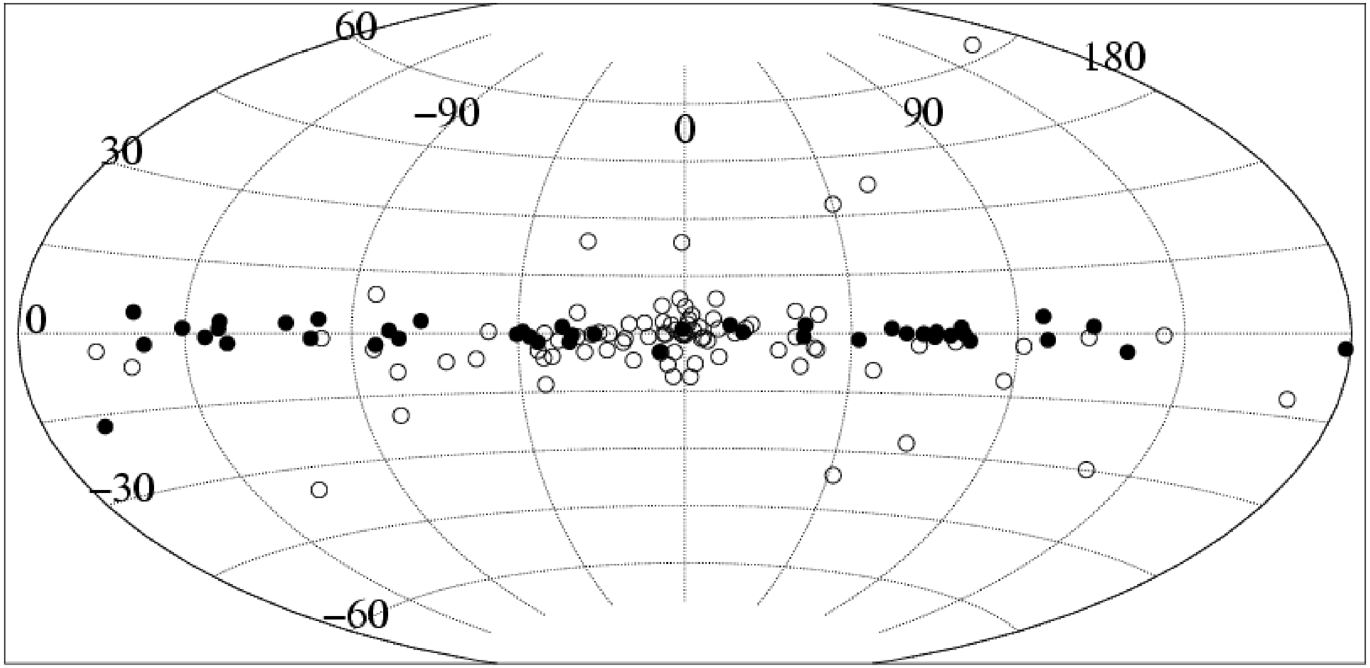
X-ray binary systems

- binary systems with X-ray emission
- neutron star or black hole receiving mass from non-compact companion star
- few hundred in our Galaxy (very rare, 10^8 stellar mass black holes)
- 90% belongs to either

high-mass X-ray binaries – HMXBs

or

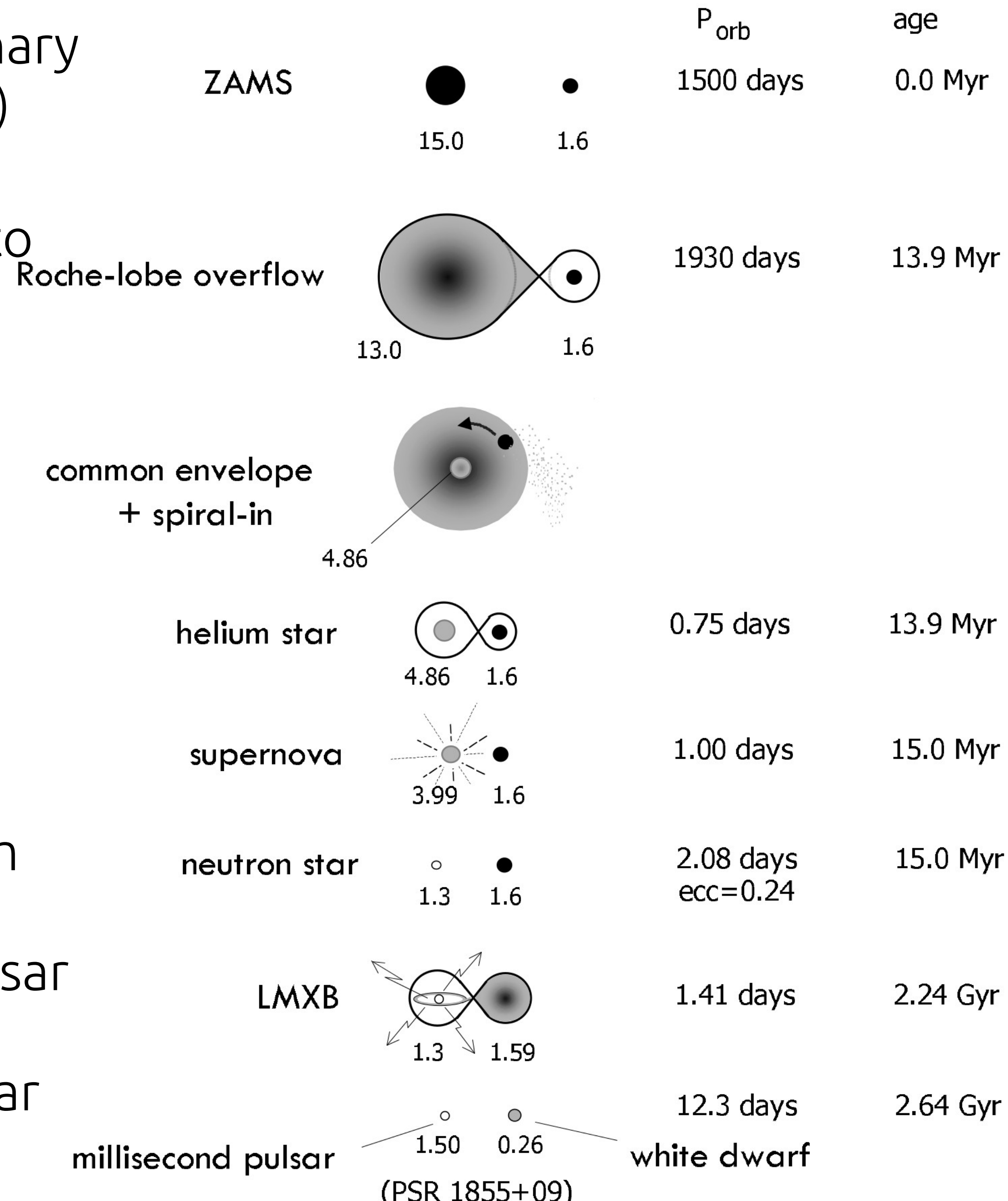
low-mass X-ray binaries – LMXBs



Low-mass X-ray binary systems

- neutron star or black hole in a binary system with a low-mass (<1.4 M_{SUN}) main sequence star
- orbital periods from minutes up to several days
- spectrum dominated by the emission from accretion disc
- L_{opt}/L_X<<0.1, $k_B T$~10keV
- accretion through Roche lobe overflow

- evolution
    - long LMXB phase
    - magnetic field decreases from 10^{12}G to 10^9G
    - accretion can spun-up the pulsar (recycled pulsars)
    - X-ray bursts (1s; thermonuclear explosions on the surface of NS)
    - transients (change of accretion rate)

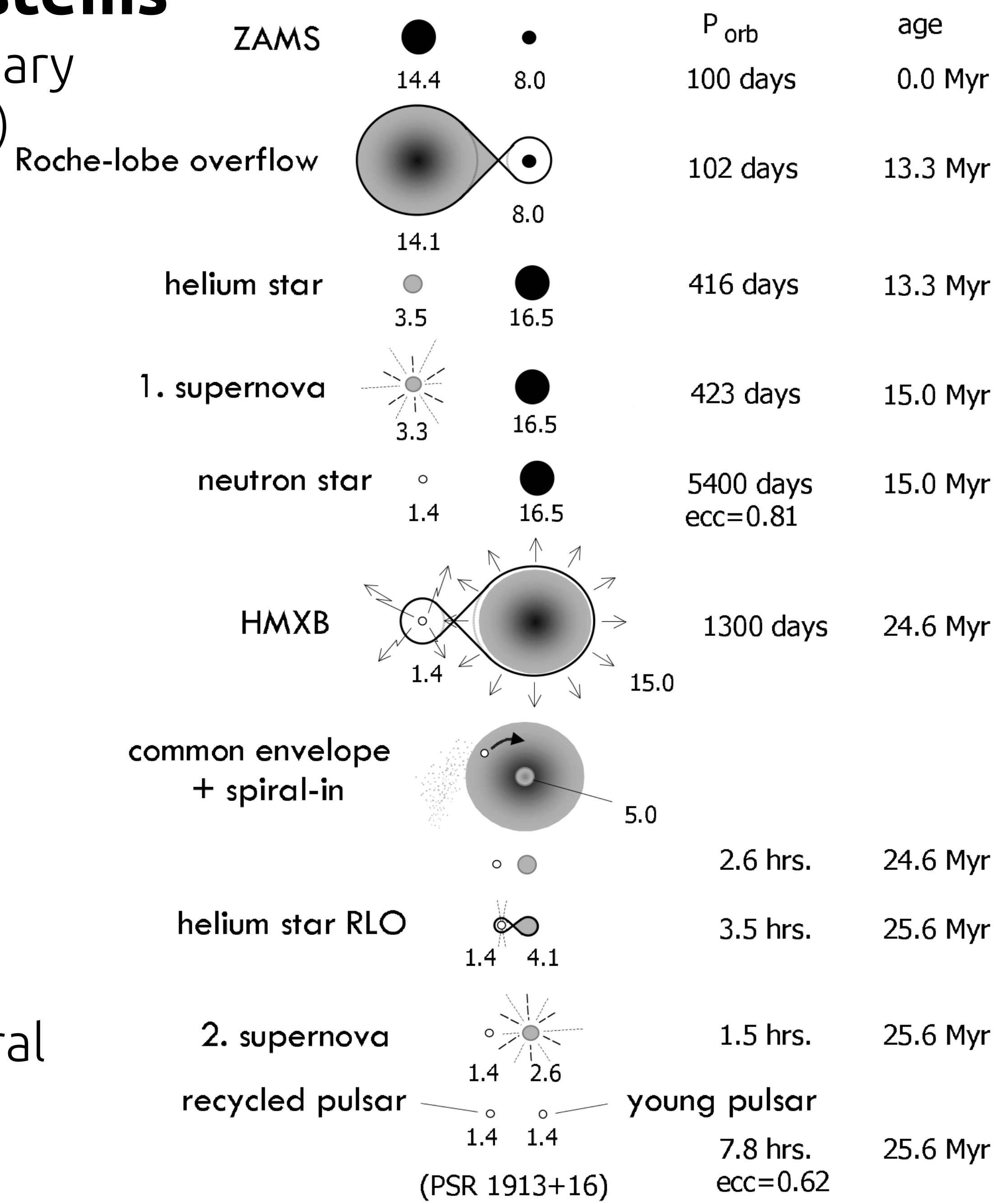


High-mass X-ray binary systems

- neutron star or black hole in a binary system with a high-mass (>10 M_{SUN}) O/B star
- orbital periods from hours up to several hundred days
- spectrum dominated by the emission from accretion
- $L>10^5 L_{SUN}$, $k_B T\sim 15$keV
- accretion from wind of a massive star

$$\eta=\frac{1}{4}\left(\frac{M_C}{M_*}\right)^2\left(\frac{R_*}{a}\right)^2\approx 10^{-4}\div 10^{-3}$$

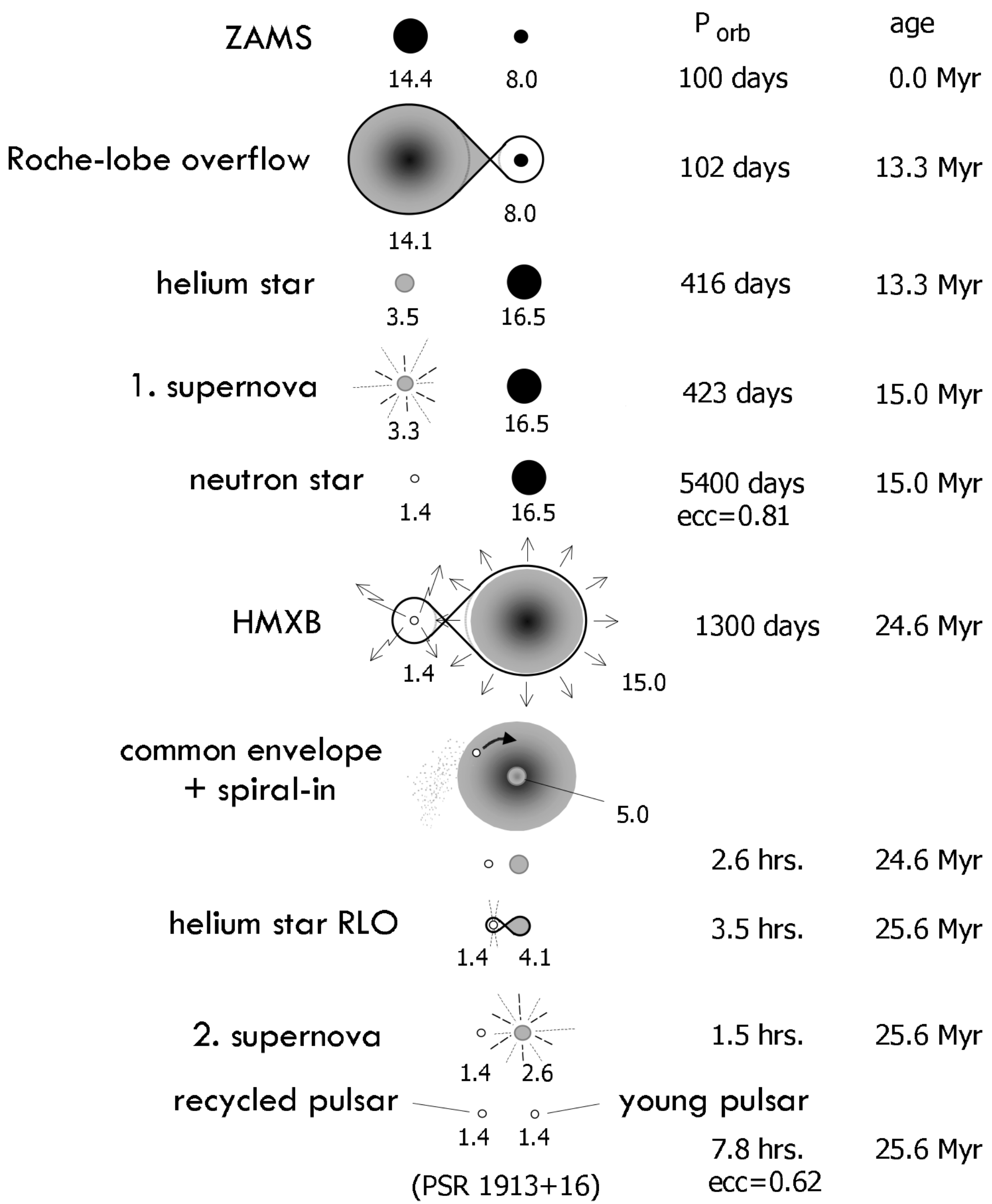
- evolution
  - requires both massive stars
  - magnetic field 10^{12}G may influence the accretion up to several hundred R_{NS} (*X-ray pulsars* – X-rays modulated by orbital period of neutron star)



Double neutron star systems

- PSR 1913+16 – Hulse-Taylor binary system consisting of a pulsar and neutron star discovered in 1974
- J0737-3039 A+B – double pulsar system discovered in 2003 with a period of only 2.45h

- formation
    - $M<8M_{SUN}$ – no SN, white dwarf
    - $M<25M_{SUN}$ – SN, neutron star
    - $M<40M_{SUN}$ – SN, black hole
    - $M>40M_{SUN}$ – no SN, black hole



Relativistic effects

- Newtonian description of the binary through five Keplerian parameters: (i) orbital period P, (ii) projected semi major axis a sin i, (iii) eccentricity e, (iv) longitude of periastron ω, (v) epoch of periastron passage T

- post-Keplerian parameters
  - (i) periastron advance

$$\dot{\omega} = 3 \left(\frac{P}{2\pi} \right)^{-5/3} \frac{(T_0 M)^{2/3}}{1 - e^2}$$

  - (ii) gravitational redshift and time dilation

$$\gamma = e \left(\frac{P}{2\pi} \right)^{1/3} T_0^{2/3} M^{-4/3} M_{\mathrm{c}} \left(M_{\mathrm{p}} + 2\mathrm{M}_{\mathrm{c}} \right)$$

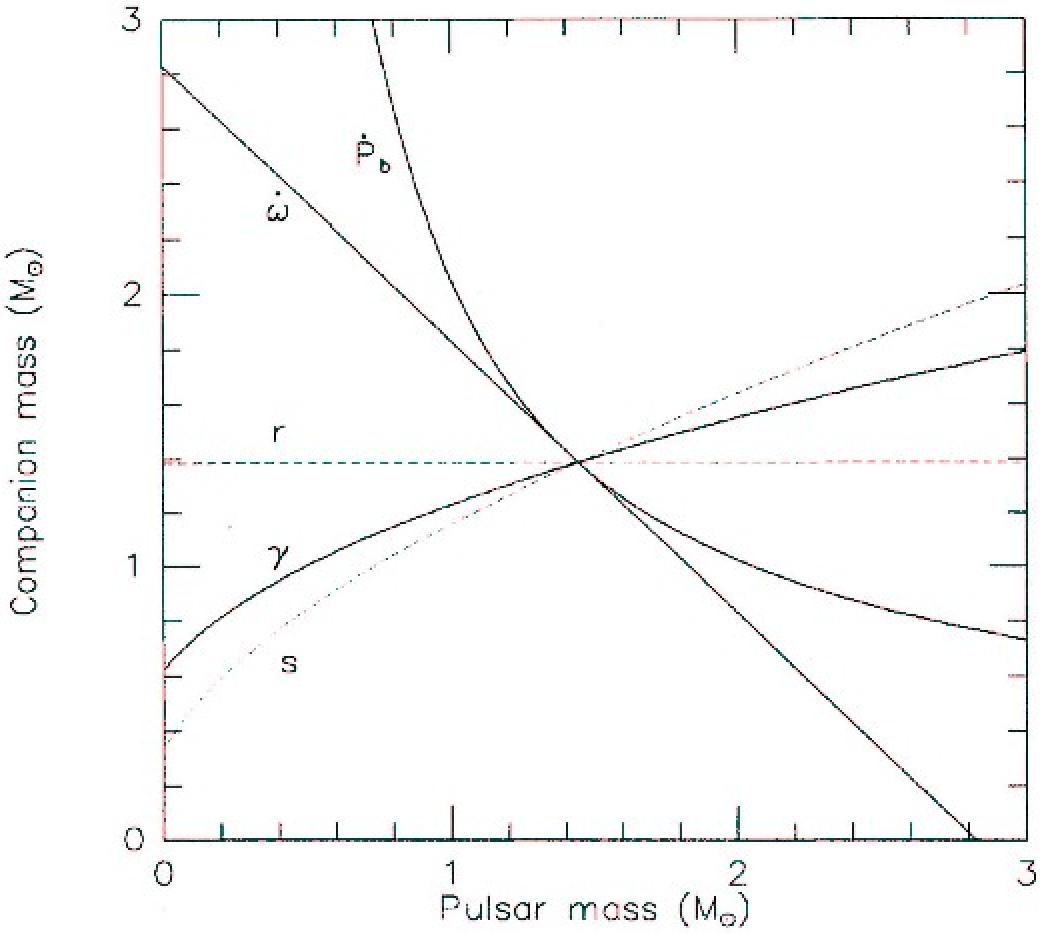
  - (iii) change in the orbital period

$$\dot{P} = -\frac{192\pi}{5} \left(\frac{P}{2\pi} \right)^{-5/3} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \left(1 - e^2 \right)^{-7/2} T_0^{5/3} M_{\mathrm{p}} M_{\mathrm{c}} M^{-1/3}$$

  - (iv) i (v) Shapiro-delay effect

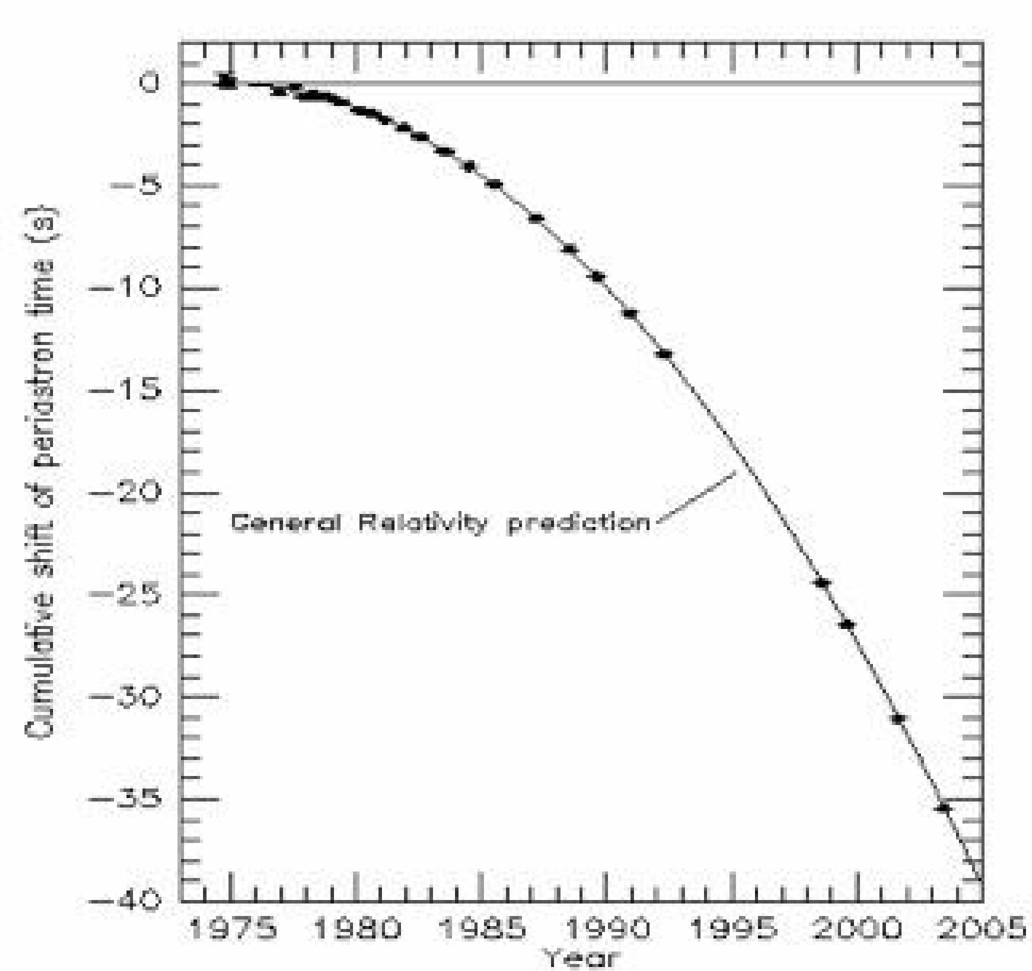
Relativistic effects

- mass determination in PSR 1913+16



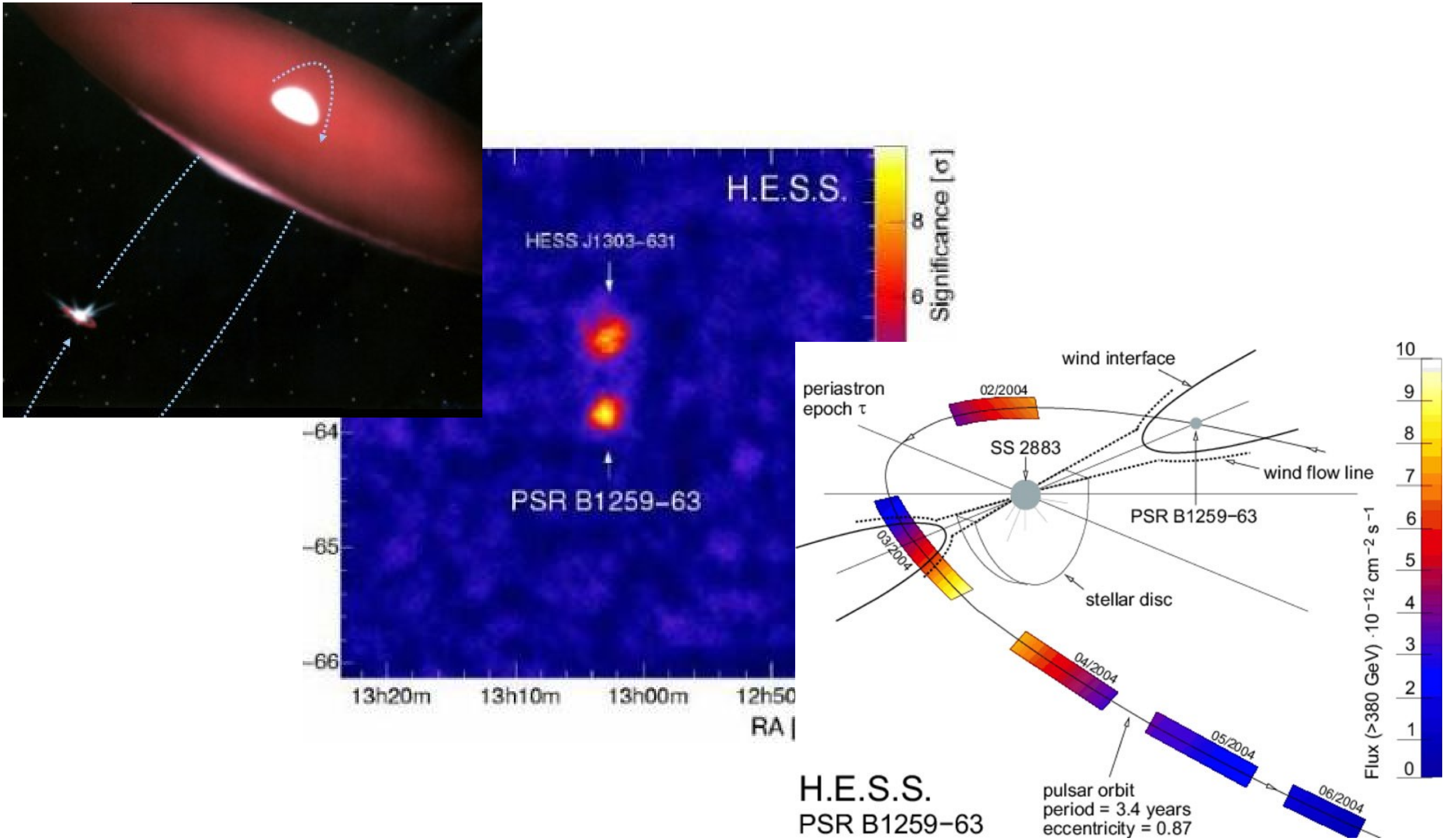
$$M_p = 1.4414 \pm 0.0002\, M_{SUN}$$
$$M_c = 1.3867 \pm 0.0002\, M_{SUN}$$

Relativistic effects – gravitational wave emission



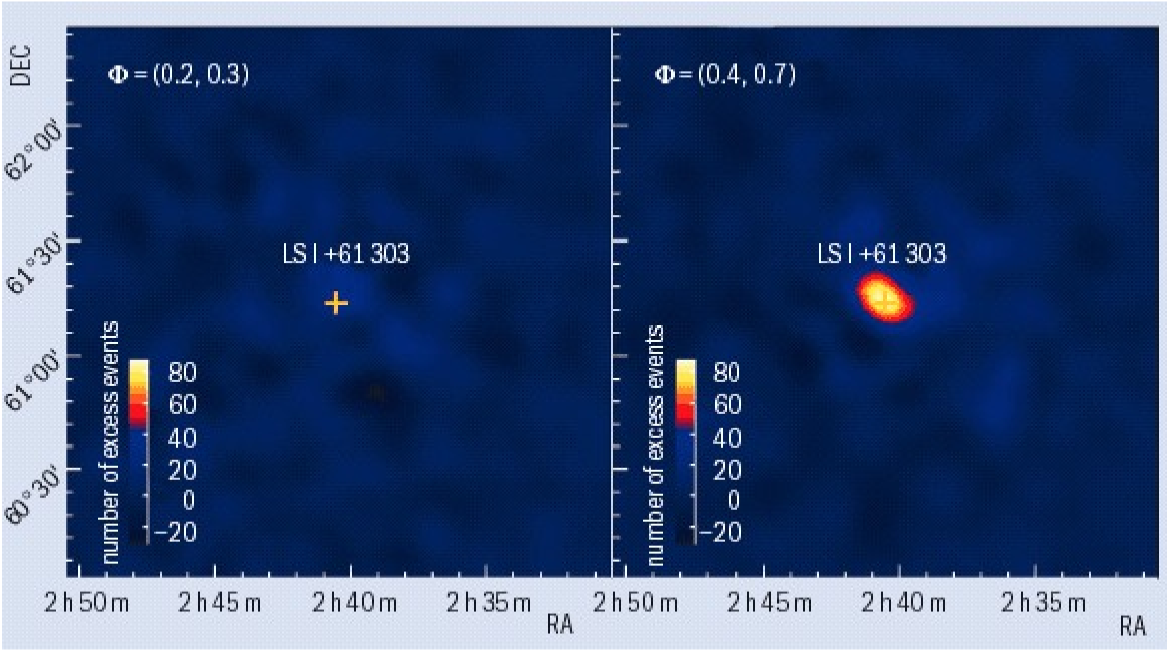
VHE emission from binary systems

- PSR B1259-63

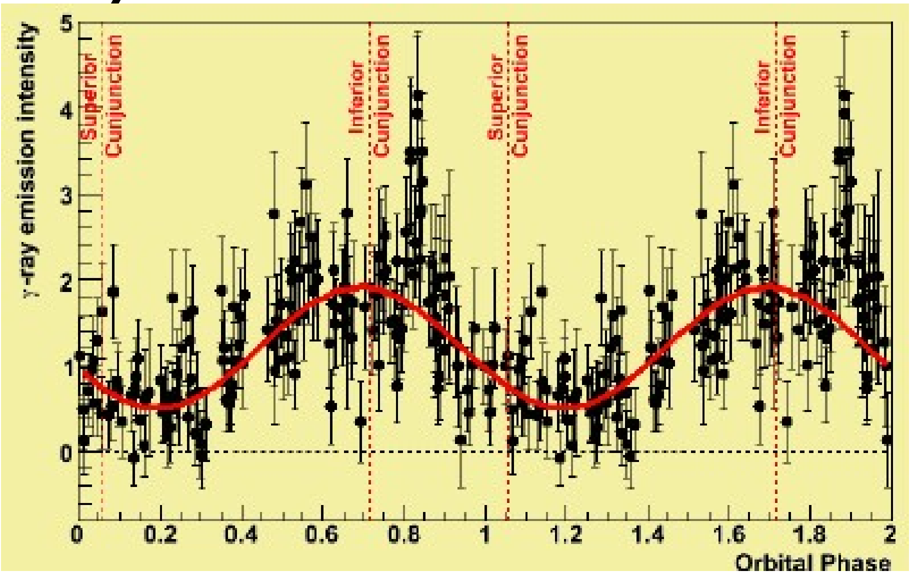


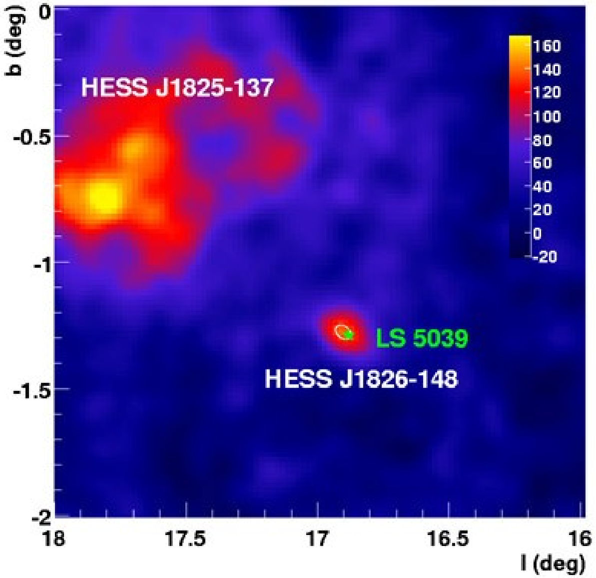
VHE emission from binary systems

LS I +61 303 (HMXB)



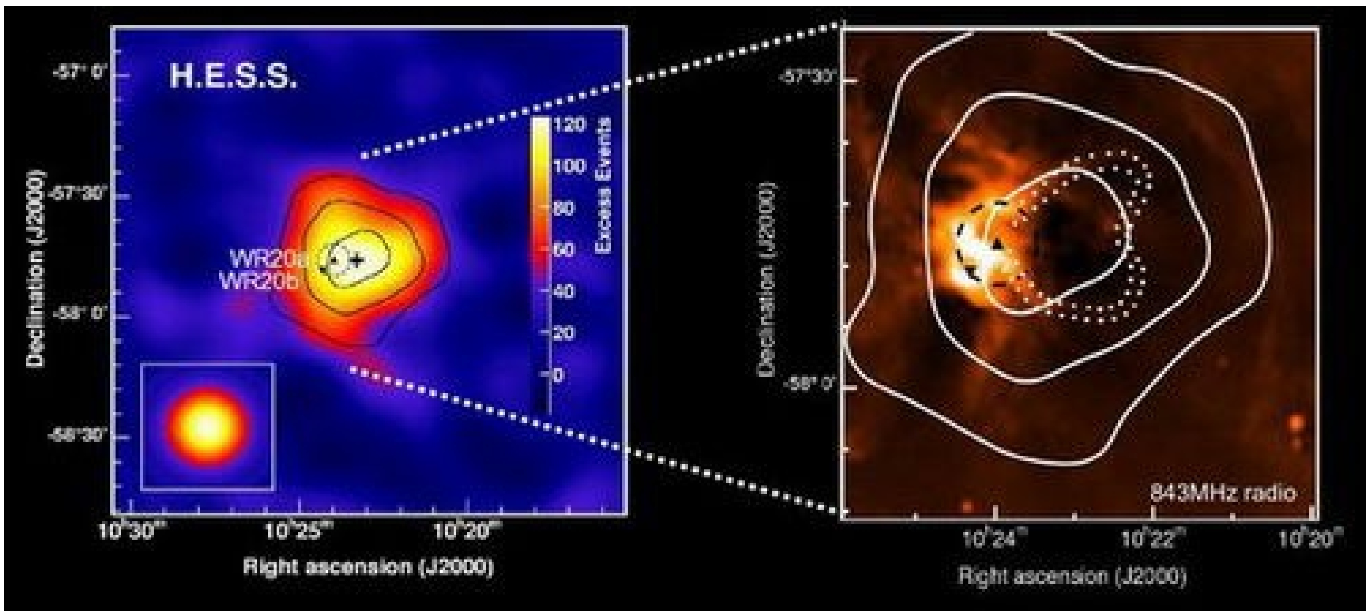
LS 5039 (HMXB)





VHE emission from star clusters

- Cygnus OB2, Westerlund 1 &2, W43, Pismis 22 and W49A



Exercises

1. a) calculate the separation of a circular neutron star binary with two neutron stars $1.4M_{SUN}$ each that has an orbital period of P=1.5ms

b) show that the inspiral time of such a binary is

$$\tau_{\mathrm{insp}} = \frac{5}{256} \frac{c^5}{G^3} \frac{a_0^4}{M^2 \mu}$$

c) calculate how long the system will be visible in frequency window from 10 to 1000Hz